the power at 10.6 Gc/sec appears to be best fitted by the form $P_{10.6} = P_{3.3}^{1.8} P_{4.0}^{0.9}$, where P is the signal strength, and the subscripts indicate the frequency. The slightly lower values of the observed exponents are caused by the dependence of the effective frequencies of the resonances on the excitation power levels. That is, the electron density at which the peak of one resonance occurs varies with the excitation power at the frequency of the other resonance, because of nonlinear effects.[6] In our experiment, the peaks of the resonances coincide (at the same electron density) at the lowest excitation power levels. This coincidence provides the most effective condition for side-frequency generation. With higher power levels at either frequency, the peaks of the resonances become displaced with respect to one another, so that the amount of side frequency generated is relatively less than at low power. As a result, the exponent of the power dependence will be less that that expected in the absence of the resonance displacement. When this effect is properly taken into account, the observed exponents are found to correspond closely to the ideally expected values.

The interaction does not affect the observed quadratic power dependence in harmonic generation, since the peaks of the resonances of the two photons involved are the same, and so will always coincide with each other and with the peak of the second-harmonic signal. In conclusion, the exclusion of previously known nonlinear effects (Luxembourg, gradient coupling), and the agreement between theoretical estimates and measurement, indicate that the observed processes represent the inherent nonlinear properties of plasma-electron oscillations.

The active interest and useful suggestions of S. J. Buchsbaum and especially the theoretical assistance of N. Tzoar are gratefully acknowledged.

[1]L. Tonks, Phys. Rev. 37, 1458 (1931); A. Dattner, Phys. Rev. Letters 10, 205 (1963).

[2]P. Weissglass, Phys. Rev. Letters 10, 206 (1963). J. C. Nickel, J. V. Parker, and R. W. Gould, Phys. Rev. Letters 11, 183 (1963); Phys. Fluids 7, 1489 (1964). F. C. Hoh, Phys. Rev. 133, A1016 (1964).

[3]V. L. Ginzburg and A. V. Gurevich, Usp. Fiz. Nauk 70, 201, 393 (1960) [translation: Soviet Phys.—Usp. 3, 115, 175 (1960)].

[4]J. M. Anderson and L. Goldstein, Phys. Rev. 100, 1037 (1955).

[5]To be published.

[6]R. A. Stern, Appl. Phys. Letters 4, 80 (1964).

OBSERVATION OF THE ANALOG OF THE ac JOSEPHSON EFFECT IN SUPERFLUID HELIUM

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We wish to report the first observation of the effect in superfluid liquid helium analogous to the alternating-current Josephson effect of superconductivity.[1-3] The possibility of the effect in He follows from the result of Beliaev[4] that the order parameter ψ in superfluid He includes the phase factor $e^{-i\mu t/\hbar}$, where μ is the chemical potential. This differs only by a factor of two from Gor'kov's[5] expression $e^{-2i\mu t/\hbar}$ for superconducting electron pairs.[6]

Our experiment is an exact analog of the Anderson-Dayem experiment for a superconductor.[3] Given two baths of superfluid at the same temperature weakly coupled through a small orifice, a difference, z, in the He head will cause a difference in chemical potential of mgz between the baths, just as a voltage V causes a chemical potential difference eV between two superconductors. Here m is the mass of a helium atom, g the gravitational acceleration. Thus the phase of the order parameter for one bath will slip at a mean rate $\omega = mgz/\hbar$ relative to the other. If the He in the orifice region is to remain superfluid, this phase slippage can best occur by means of the motion of vortices. For ψ to be single-valued, its phase must change by $2n\pi$ on going around a vortex, where n may be identified with the number of quanta of circulation enclosed. Thus the motion of vortices at the rate mgz/nh can account for the phase slippage. This motion can be in the form of vortex lines moving across the orifice, or "smoke rings" from the orifice moving into either bulk-He bath.

A head difference imposed on the system relaxes by fluid flow through the orifice, which is frictionless in superfluid He, except for the creation of vortices. In the present experiment we synchronize the vortex motion to an ultrasonic frequency ω by modulating this flow. It may be that vortices are created in or disengage themselves from the orifice only every second or third cycle of the ultrasonic transducer; or, several vortices may move in each cycle. Thus, there is a mechanism for the production of submultiples as well as multiples of the fundamental head difference $\hbar\omega/mg$. We monitor the pressure head as a function of time to see whether the system prefers the head-difference intervals

$$z = n\hbar\omega/n'mg, \qquad (1)$$

where n and n' are integers.

A schematic diagram of the apparatus is shown in Fig. 1. Quartz crystals designed for frequency-control use at 69, 105, and 153 kc/sec were used as transducers. The coaxial capacitor A used to measure the liquid level in the Dewar is open at the bottom. The bottom of the otherwise identical capacitor B is closed except for a ~15-μ-diameter orifice punched in a 1-mil-thick Ni foil with a sharpened needle. The top of each capacitor is sealed except for a thin tube which reaches high enough in the Dewar that its open end is warmer than 2.2°K. Flow into the capacitors by means of superfluid creep is thereby prevented. The coaxial capacitors form two arms of a commerical 100-kc/sec capacitance bridge. The off-balance voltage from the bridge, which is proportional to the He head difference z, goes through a lock-in amplifier to a chart recorder and also to a digital voltmeter coupled to a card punch.

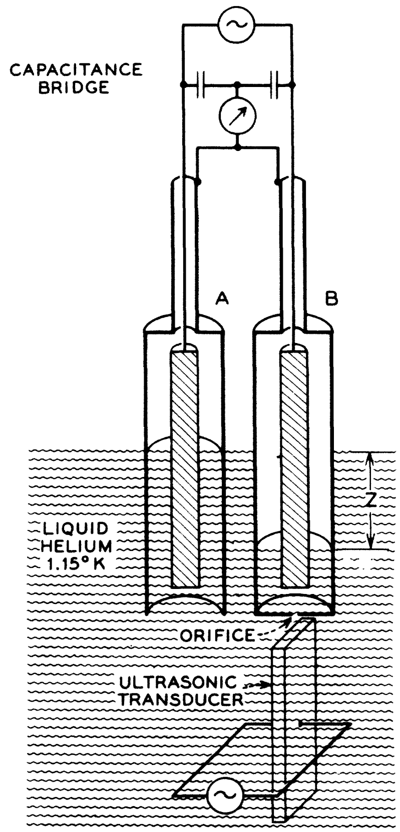

FIG. 1. Schematic diagram of apparatus. The two coaxial capacitors A and B form two arms of a capacitance bridge which measures the helium head difference z. Helium flow through the orifice is modulated by a quartz ultrasonic transducer.

When a head is produced by raising or lowering the apparatus in a quiet He bath, it decays nearly linearly, as is shown in Fig. 2(a). The flow corresponds to a slightly head-dependent critical velocity of ~27 cm/sec in the orifice when $z = 3$ mm and $T = 1.15$°K. This value is in good agreement with the results of Allen and Misener,[8] when the latter are extrapolated to zero capillary length. If the bath is filled with vortices by operating the ultrasonic transducer, the decay rate may be orders of magnitude slower than in a quiet bath, suggesting that vortex creation at the orifice is aided by the presence of vortices in the baths.

When the ultrasonic transducer is driven at its resonant frequency for longitudinal motion, a head develops, showing that He is being pumped out of capacitor B. The fountain effect can contribute to this pumping because of the heat dissipated by the transducer. When the same power is dissipated in the resistor, however, the pumping is reduced by ~10^2, indicating that this effect is not of major importance. Therefore the pumping action implies that vortices are being formed by the transducer. An estimate of the rms velocity of the end of the 105-kc/sec crystal (based on measurements of its Q in He and of the total power dissipated in the bath when it is driven at a typical level of 10 V) gives ~1 cm/sec, which is less than the critical velocity in the orifice. There is an amplification factor of $D^2/ld \sim 100$, however, due to the small distance l between the end of the transducer (of diameter D) and the ori-

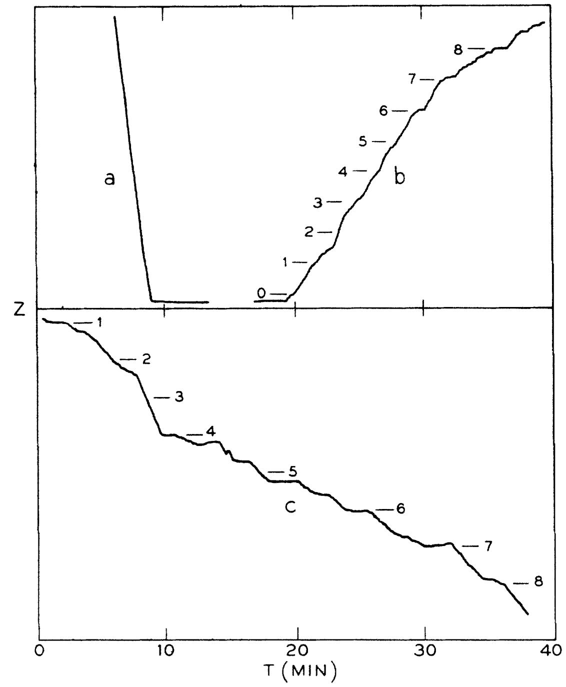

FIG. 2. Chart recordings of head difference versus time. (a) Decay of a head with no ultrasonic modulation; (b) increasing head produced by pumping action of a 69.3-kc/sec transducer; (c) decay of a head with ultrasonic modulation. Intervals of head $h = 0.705$ mm corresponding to $\omega = 69.3$ kc/sec are indicated where appropriate, and numbered in a time sequence. The zero head position recorded in (b) is not simply related to the steps because of a contribution to the head produced by the thermomechanical effect in the first few minutes after the transducer is turned on.

fice (of diameter d), so that the resulting velocity in the orifice is more than adequate to produce vortices.[8]

After the transducer is turned on, the head difference increases as is shown in Fig. 2(b). It rises smoothly at first and then slows and shows a series of steps. The initial rise is often too rapid to allow the steps to develop. The system does indeed seem to "stick" at certain values of head difference. The spacing of the steps usually fits the prediction of Eq. (1) quite well. Data can also be obtained by producing a head mechanically and setting the transducer level so that the head decays at a rate which allows the steps to develop. An example is shown in Fig. 2(c). A number of steps are seen at precisely the heads predicted by Eq. (1). Some subharmonics, especially half-steps, are also seen. The steep portion between steps 2 and 4 occurred when the transducer was turned off. Step 3 is missing as expected. Step 5 is also missing for no apparent reason.

A more revealing way to display these data is to plot the slope of the decay curve versus head, z. A step then shows up clearly as a head value at which $dz/dt \sim 0$. This has been done by digitizing the data at two-second intervals and using a computer to plot the slope. Data are shown in Fig. 3 for each of three different values of ω. The data at 69.3 kc/sec are the same as those shown in Fig. 2(c). Here the levels show much more clearly, and many more subharmonics are seen. The level separations predicted from Eq. (1) are indicated, and some of the sublevels are labeled. The data at 104.6 kc/sec are from a pumping curve similar to Fig. 2(b) but for which the thermo-

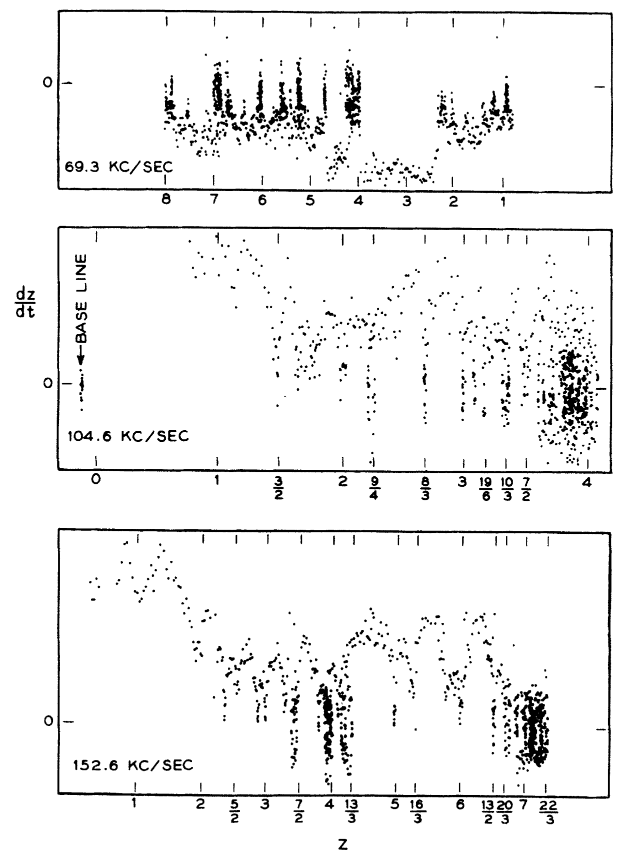

FIG. 3. The rate of change of head with time plotted versus head for three ultrasonic drive frequencies. The intervals of head difference expected by Eq. (1), as well as some subintervals, are indicated. The data for $\omega = 69.3$ kc/sec are the same as for Fig. 2(c). The data at 104.6 and 152.6 kc/sec are from pumping curves such as that shown in Fig. 2(b).

mechanical effect was small. In this case the abscissa corresponding to zero chemical potential difference is close to the base line established before the transducer was turned on. Step 1 hardly shows because the curve is rising rapidly. Above step 2, on the other hand, the rate of rise was slow, and many subharmonics are apparent. A plausible assignment for some of them is given. The data at 152.6 kc/sec are from another pumping curve. In this case, z rose smoothly for $\sim 5\hbar\omega/mg$ before steps began to develop. The data shown are from the top portion of this curve.

One possible source of the steps found here could be a series of ultrasonic standing wave resonances associated with the changing He level. This possibility can be eliminated since the head intervals between such resonances would depend on absolute head, not head difference, and would be inversely, not directly, proportional to frequency. All of the data shown in Figs. 2 and 3 were obtained at a temperature of 1.15°K. The steps have been observed up to 2.1°K, but disappear above the λ point. The pumping action, however, persists to higher temperatures as expected. We believe that the steps shown in Figs. 2 and 3 can only be explained by the existence of the ac Josephson effect in liquid helium.

As in the case of superconductivity, this effect in turn demonstrates the existence of coherent phase coupling of the particle fields in the two reservoirs. In general, the relative phase of ψ for two reservoirs at different μ is arbitrary, but in this type of experiment we synchronize the phase variation by means of an applied ac particle current which enforces phase coherence on the particle fields in the two baths. Once established, this coherence remains stable until disturbed by fluctuations. The synchronization by a strong external signal is the key to the observability of the effect.[9] It seems to us that the existence of discrete head levels satisfying the Josephson-like frequency condition (1) is difficult to interpret in any other way and therefore is very strong evidence for the essential correctness of our present model for superfluidity in liquid helium, where the evidence for quantization of circulation is less complete and direct than that for flux quantization in superconductivity.

[1]B. D. Josephson, Phys. Letters 1, 251 (1962).

[2]S. Shapiro, Phys. Rev. Letters 11, 80 (1963).

[3]P. W. Anderson and A. H. Dayem, Phys. Rev. Letters 13, 195 (1964).

[4]S. T. Beliaev, Zh. Eksperim. i Teor. Fiz. 34, 417 (1958) [translation: Soviet Phys.-JETP 34, 289 (1958)].

[5]L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 34, 735 (1958) [translation: Soviet Phys.-JETP 34, 505 (1958)].

[6]The significance of this expression may be clearer if we observe that the order parameter $\psi(r,t)$ is most simply interpreted as the mean value of the He-atom wave field. Naively, we might expect ψ to behave like a wave function whose phase factor comes from Schrödinger-like equation $i\hbar\dot{\psi} = (\frac{1}{2}mv^2 + V)\psi$, where V is the one-particle potential energy. Very general arguments[4] show, however, that it is the chemical potential $\mu = \partial E/\partial N$, where E and N are the system total energy and the number of particles, respectively, which appears in place of V in the complete phase factor $\exp[(i/\hbar)(\mu + \frac{1}{2}mv^2)t]$. [See also P. W. Anderson, N. R. Werthamer, and J. M. Luttinger, Phys. Rev. (to be published).] In stationary superfluid flow the supercurrent $\hbar\nabla p/m$ is time independent so $\mu + \frac{1}{2}mv^2$ must be independent of position.

[7]J. F. Allen and A. D. Misener, Proc. Roy. Soc. (London) A172, 467 (1939).

[8]It seems likely that the pumping is similar to the familiar Pitot tube effect of lowering of pressure in a side tube connected to a moving stream. There is no way to align the crystal to the orifice accurately enough to avoid fluid flow across the orifice; a reasonable estimate of the transverse velocity is ~10-50 cm/sec, corresponding to a head difference $\sim V^2/g$ of a few mm as observed. We assume that the ultrasonic flow, except near the orifice, is fairly well approximated by a stationary superfluid flow. No Bernoulli pressure head between two free surfaces where the fluid is not moving would appear in such a flow,[6] but we can expect the transverse flow to generate quantized vortices at one side of the orifice and carry them across it, allowing a Bernoulli pressure to appear. Because of this large transverse fluid velocity, the motion of the vortices may be nearly transverse to the axis of the orifice as in the superconducting experiment.[3] In general the motion of the vortices is probably quite complex, since several may be in the orifice region at one time. The distance of transverse fluid flow in one cycle is comparable to the orifice diameter so that vortices may or may not have time to completely cross the orifice, and may last for several cycles.

[9]B. D. Josephson, thesis, Trinity College, Cambridge University, 1962 (unpublished); P. W. Anderson, Lectures on the Many-Body Problem, edited by E. Caianello (Academic Press, Inc., New York, 1964), pp. 113-135.