HARMONIC GENERATION AND FREQUENCY MIXING AT PLASMA RESONANCE

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We have observed a number of nonlinear phenomena associated with electron-plasma oscillations of a plasma column. They are the generation of the second harmonic of the excitation signal, and the generation of beat frequencies (linear sums) of the fundamental frequencies of two simultaneously excited oscillations. The oscillation modes used to elicit these effects are the Tonks-Dattner resonances,[1] which were recently shown[2] to be due to the excitation of radial plasma-electron waves. The strong coupling between electromagnetic and plasma waves which exists at resonance enables us to generate large-amplitude longitudinal (plasma) oscillations. These can exhibit strong nonlinear effects in the absence of a static magnetic field and at low excitation-power levels. The observations are found to be consistent with solutions of the nonlinear Vlasov equation, and to represent the inherent nonlinear properties of plasma-electron oscillations.

The oscillations were generated by microwaves beamed at the positive column of a low-pressure mercury-glow discharge, inserted across a waveguide with the discharge axis normal both to the electric field vector and to the direction of propagation of the microwaves in the TE_{01} mode. The microwave signals reflected, transmitted, and generated by the plasma were sampled by means of directional couplers, filtered to remove unwanted frequencies, and then measured by means of a narrow-band heterodyne detector system.

Measurements were performed at fixed values of the incident power levels and frequencies, while the plasma-electron density was continuously varied. Figure 1(a) shows a typical experimental record of the second-harmonic signal generated when a single microwave signal was incident on the plasma. The abcissa is proportional to the electron density, which was determined by means of a microwave cavity coaxial with the discharge, while the ordinate represents microwave power measured by the detector system. The solid line in this Figure shows an 8-Gc/sec signal generated when a 10-mW, 4-Gc/sec signal is incident on the plasma. The broken line shows the power reflected at the incident frequency. It is evident that the 8-Gc/sec signal is generated only in those ranges of electron density at which resonances in the reflected signal occur.

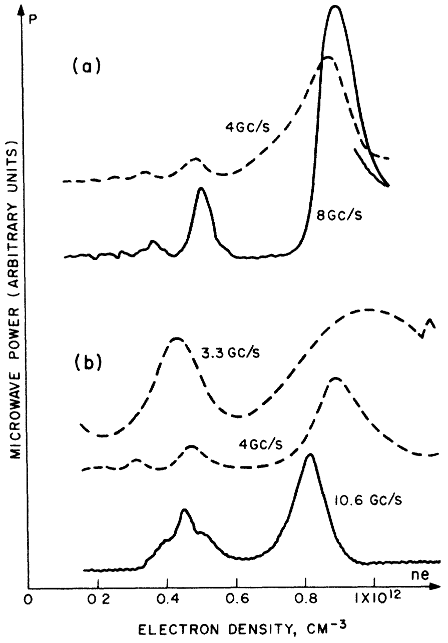

FIG. 1. Recorder traces of (a) second-harmonic and (b) side-frequency generation (solid lines) as function of electron density. Broken lines are simultaneous traces of reflected power at excitation frequencies. Vertical scale is not linear, and represents the microwave detector response.

As the incident power level was increased, the level of the 8-Gc/sec signal increased sharply within the resonant electron-density ranges, but no second-harmonic signal was detectable *outside the resonant ranges*. This indicates that phenomena other than the excitation of longitudinal plasma oscillation (such as the nonlinear coupling of the transverse incident electric field and the electron-density gradient in the plasma[3]) are ineffective in producing observable nonlinear effects in the present experiment. To determine the level of static electron-temperature changes due to the electric fields and the associated nonlinearities (Luxembourg effect[4]), the microwave emission of the plasma column as a function of electron density was measured, using a Dicke radiometer, while the plasma was being illuminated by 300-mW signals at the excitation frequencies 2-8 Gc/sec. Within the sensitivity of the radiometer (better than 1°K), no change in the static electron temperature was observed, showing that the Luxembourg effect was negligible in this case.

Figure 2(a) shows the dependence of the peak of the second-harmonic signal strength at 8 Gc/sec on the incident signal at 4 Gc/sec for an average plasma-electron density of 5.5×10^{11} cm^{-3}, corresponding to the peak of the first Tonks-Dattner resonance at 4 Gc/sec. Within the limits of experimental error the dependence is accurately quadratic, as expected.

A theoretical estimate[5] of the amount of harmonic generation due to the nonlinear properties of resonant plasma oscillations was obtained from an approximate solution of the Vlasov equation. The model considered the electron velocity distribution function in a one-dimensional homogenous plasma with an infinite ion mass, in the presence of a long-wavelength monochromatic applied electric field. The first nonlinear term was obtained by iterating the usual first-order linear solution for the distribution function in the linearized Vlasov equation. The second-harmonic field strength is, of course, proportional to the square of the applied field strength, in agreement with the results presented in Fig. 2(a). The ratio of the second harmonic to the first-order linear field depends on the applied field strength; if the latter is taken to be equal to the field of a 10-mW incident signal at 4 Gc/sec, the estimated ratio is 1×10^{-4}. The observed ratio of the field strength at 8 Gc/sec to the reflected field strength at 4 Gc/sec, for a 10-mW incident signal, is 3×10^{-5}, in reasonable agreement with the estimate.

When two microwave signals of frequencies f_1 and f_2 were incident on the plasma, signals at the sum (f_1+f_2) and side frequencies $(2f_1 \pm f_2)$ were observed. A typical side-frequency observation is given in Fig. 1(b). The solid line shows a 10.6-Gc/sec signal generated when two 10-mW signals at 3.3 and 4 Gc/sec are incident on the plasma. The broken lines show the power reflected at each incident frequency. In this case, the 10.6-Gc/sec signal is generated wherever resonances at the two incident frequencies nearly overlap. In addition to this signal, signals at the sum frequency 7.3 Gc/sec and at the other side frequencies 11.3, 4.7, and 2.6 Gc/sec were observed in the same resonant ranges. The difference frequency 0.7 Gc/sec, being lower than the waveguide cutoff frequency, was not observed.

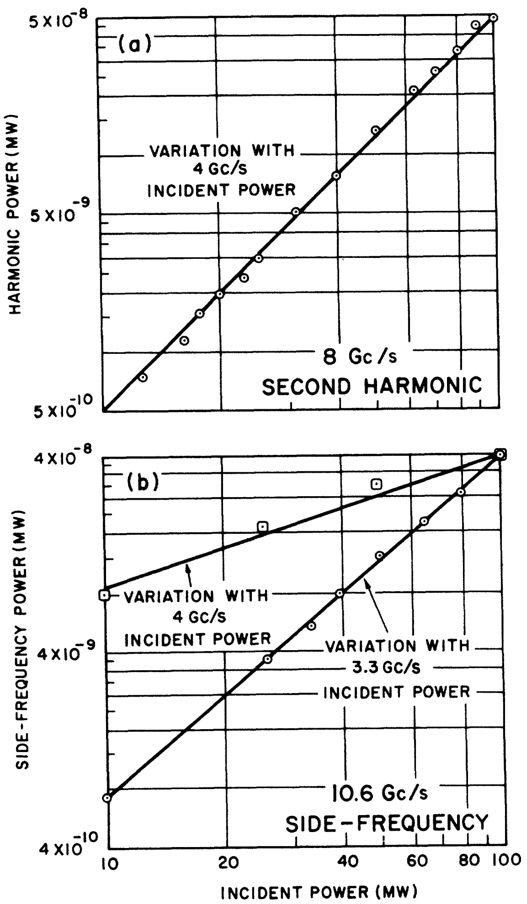

FIG. 2. Dependence of (a) second-harmonic and (b) side-frequency power on incident power. Calibration is obtained by repeating recorder traces (Fig. 1) with increased attenuation values in microwave detector input.

Figure 2(b) shows the dependence of the generated 10.6-Gc/sec side-frequency signal on the incident signals at 3.3 and 4 Gc/sec for an average electron density of 4.5×10^{11} cm^{-3}, corresponding to the simultaneous excitation of the first and second Tonks-Dattner resonances, respectively. In this three-photon process, one ideally expects the power at 10.6 Gc/sec to depend quadratically on the power at 3.3, and linearly on the power at 4 Gc/sec. Instead, the power at 10.6 Gc/sec appears to be best fitted by the form $P_{10.6} = P_{3.3}{}^{1.8} P_{4.0}{}^{0.9}$, where P is the signal strength, and the subscripts indicate the frequency. The slightly lower values of the observed exponents are caused by the dependence of the effective frequencies of the resonances on the excitation power levels. That is, the electron density at which the peak of one resonance occurs varies with the excitation power at the frequency of the other resonance, because of nonlinear effects.[6] In our experiment, the peaks of the resonances coincide (at the same electron density) at the lowest excitation power levels. This coincidence provides the most effective condition for side-frequency generation. With higher power levels at either frequency, the peaks of the resonances become displaced with respect to one another, so that the amount of side frequency generated is relatively less than at low power. As a result, the exponent of the power dependence will be less that that expected in the absence of the resonance displacement. When this effect is properly taken into account, the observed exponents are found to correspond closely to the ideally expected values.

The interaction does not affect the observed quadratic power dependence in harmonic generation, since the peaks of the resonances of the two photons involved are the same, and so will always coincide with each other and with the peak of the second-harmonic signal.

In conclusion, the exclusion of previously known nonlinear effects (Luxembourg, gradient coupling), and the agreement between theoretical estimates and measurement, indicate that the observed processes represent the inherent nonlinear properties of plasma-electron oscillations.

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OBSERVATION OF THE ANALOG OF THE ac JOSEPHSON EFFECT IN SUPERFLUID HELIUM

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We wish to report the first observation of the effect in superfluid liquid helium analogous to the alternating-current Josephson effect of superconductivity.[1-3] The possibility of the effect in He follows from the result of Beliaev[4] that the order parameter ψ in superfluid He includes the phase factor $e^{-i\mu t/\hbar}$, where μ is the chemical potential. This differs only by a factor of two from Gor'kov's[5] expression $e^{-2i\mu t/\hbar}$ for superconducting electron pairs.[6]

Our experiment is an exact analog of the Anderson-Dayem experiment for a superconductor.[3] Given two baths of superfluid at the same temperature weakly coupled through a small orifice, a difference, z, in the He head will cause a difference in chemical potential of mgz between the baths, just as a voltage V causes a chemical potential difference eV between two superconductors. Here m is the mass of a helium atom, g the gravitational acceleration. Thus the phase of the order parameter for one bath will slip at a mean rate $\omega = mgz/\hbar$ relative to the other. If the He in the orifice region is to remain superfluid, this phase slippage can best occur by means of the motion of vortices. For ψ to be single-valued, its phase must change by $2n\pi$ on going around a vortex, where n may be identified with the number of quanta of circulation enclosed. Thus the motion of vortices at the rate mgz/nh can account for the phase slippage. This motion can be in the form of vortex lines moving across the orifice, or "smoke rings" from the orifice moving into either bulk-He bath.