SPIN SUSCEPTIBILITY OF A SUPERFLUID FERMI LIQUID*

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The Bardeen-Cooper-Schrieffer (BCS) theory[1,2] and related theories[3,4] predict that the spin susceptibility of a Fermi system in the superfluid phase, $\chi(T)$, should be related to its value χ_n in the normal phase by

$$\chi(T)/\chi_n = f_{\text{eff}}(T), \tag{1}$$

where $f_{\text{eff}}(T)$ is the "effective fraction" of the excitations which can be spin-polarized by the external field, and is equal to $\rho_n(T)/\rho$ for singlet spin pairing and $\frac{2}{3}+\frac{1}{3}\rho_n(T)/\rho$ for triplet pairing.[4] These theories, however, all ignore the strong interparticle forces[5] which, for a normal system, are treated by Landau's theory of a Fermi liquid[6] and which among other things lead to the departure of the susceptibility from its "free-gas" value χ_0.[7]

The object of this Letter is to point out that the correct expression for the spin susceptibility of a superfluid Fermi liquid is

$$\chi(T) = \frac{\chi_0 f_{\text{eff}}(T)}{1 + f_{\text{eff}}(T) Z_0/4}, \tag{2}$$

where Z_0 is the constant which occurs in the usual Landau formula[8] for the normal-phase susceptibility [which in fact is a special case of (2) with $f_{\text{eff}}(T)=1$]. Thus, "Fermi-liquid" effects do not simply multiply the weak-coupling expression by a constant; indeed, they may change the behavior of $\chi(T)/\chi_n$ considerably (see Fig. 1). Some implications will be briefly considered at the end of this Letter.

To derive Eq. (2),[9] notice that for all real extended systems T_c, if it exists, is small compared to the temperature T_0 at which the Landau theory ceases to be useful for the normal phase. Therefore, superfluidity should be describable entirely in terms of (linear combinations of) Landau quasiparticle states. Accordingly, let us take as our Hamiltonian

$$H = H_w + \tfrac{1}{2}\sum_{\substack{pp'\\ \sigma\sigma'}} f(\vec{p}\vec{p}\,', \vec{\sigma}\vec{\sigma}\,')\delta n(p\sigma)\delta n(p'\sigma'), \tag{3}$$

$$H_w = \sum_{p\sigma} \epsilon(p\sigma)\alpha_{p\sigma}^{\dagger}\alpha_{p\sigma} + \sum_{\substack{pp'\\ \sigma\sigma'}} v(pp';\sigma\sigma')\alpha_{p\sigma}^{\dagger}\alpha_{-p\sigma'}^{\dagger}\alpha_{p'\sigma'}\alpha_{-p'\sigma}. \tag{4}$$

Here $\alpha_{p\sigma}^{\dagger}$ is the operator which creates a Landau quasiparticle, and $\delta n(p\sigma) = \alpha_{p\sigma}^{\dagger}\alpha_{p\sigma} - \theta(p_{\text{F}} - |p|)$, as in the usual Landau theory. Since $T_c \ll T_0$, we may take $f(pp')$ to be independent of $|p|$, $|p'|$; then the second ("Fermi-liquid") term in Eq. (3) depends only on the distortion of the "average" Fermi surfaces. Since formation of Cooper pairs does not affect these "average" surfaces, all one-particle properties, including $\rho_n(T)$, may correctly be calculated from H_w.[1-4]

Moreover, both in the normal and in the superfluid phase (however anisotropic) the only part of the "Fermi-liquid" terms relevant to the static susceptibility is that which depends on the total spin $\vec{S}$, producing thereby a molecular field. Using the standard definition[8] of Z_0 in terms of $f(pp', \sigma\sigma')$, we can therefore take as our model Hamiltonian in the external field $\mathcal{H}$ simply

$$H = H_w + \tfrac{1}{2}\rho_{\text{F}}^{-1} Z_0 S^2 - \beta\vec{S}\cdot\vec{\mathcal{H}}, \tag{5}$$

where ρ_{F} is the density of states of the normal liquid at the Fermi surface, and β is the appropriate gyromagnetic ratio. Let us call $\chi_w(T)$ the susceptibility obtained by keeping only H_w in (3); it is given by Eq. (1).[1-4] Then it follows immediately from (5) that

$$\chi(T) = \frac{\chi_w(T)}{1 + \beta^{-2}\rho_{\text{F}}^{-1} Z_0 \chi_w(T)}. \tag{6}$$

Using Eq. (1) and the fact that $\chi_w(T_c) \equiv \chi_0 = \frac{1}{4}\beta^2\rho_{\text{F}}$, we arrive at Eq. (2).

Notice that the derivation of Eq. (6) is entirely independent of the detailed "mechanism" of superfluidity, provided only that it conforms to the basic pattern of Cooper pairing. Thus, (6) is also valid in cases (such as superconductors with paramagnetic impurities) when $\chi_w(T)$ is not given by (1).

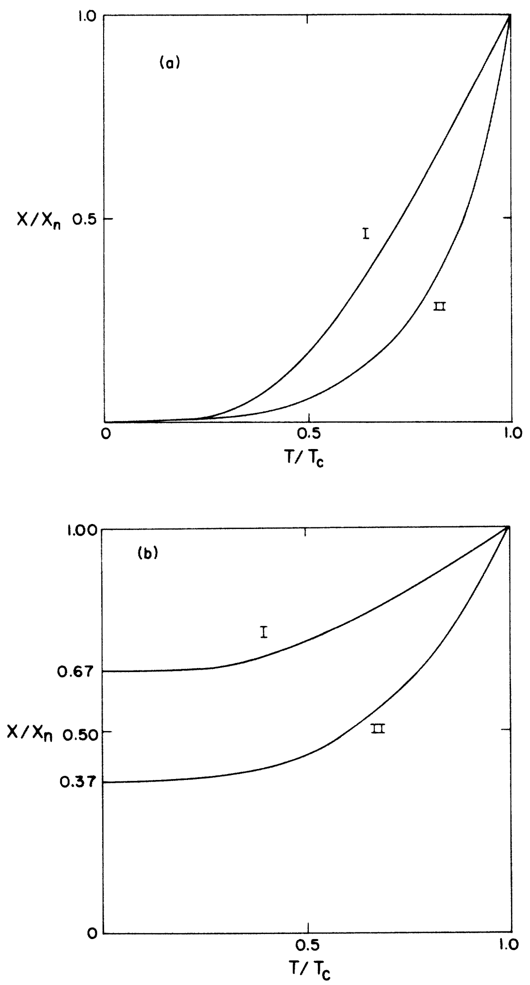

FIG. 1. $\chi(T)/\chi_n$ vs T/T_c for (a) S-state and (b) P-state condensation. (I): weak-coupling predictions (references 2 and 4, respectively); (II): present theory with $Z_0 = -2.8$.

In Fig. 1 is plotted $\chi(T)/\chi_n$, and for comparison also $\chi_w(T)/\chi_0$ [Eq. (1)] for S-state and P-state condensation; the value of Z_0 used is that appropriate to liquid He^3, namely -2.8.[10] Condensation into higher even (odd) orbital states will give graphs similar to that for the S (P) state, and identical values of $\chi(\theta)$, but both curves will be rather less sharply concave. In all cases the relative decrease of $\chi(T)$ just below T_c is enhanced, according to the present theory, by a factor $(1+Z_0/4)^{-1}$ relative to the weak-coupling predictions.

One immediate implication of this result is for the experimental detection of superfluidity in liquid He^3. Recently Wheatley and co-workers[11] have observed (among other things) no change in the susceptibility down to 3.5 mdeg. The condensation giving the weakest effect on $\chi(T)$ would be into the (spin-triplet) F state (assuming states with $l > 5$ to be unlikely). Assuming, by analogy with the P-state results,[4] that the gap in this case will have a D-wave dependence, and inserting into Eq. (2) typical D-state values[12] of $\rho_n(T)$, we find a decrease of 10% in $\chi(T)$ for $(T_c-T)/T_c \sim 7\%$. Thus it is extremely unlikely that a superfluid transition even very near the edge of the region investigated would have gone undetected, even in this "least favorable" case.

In principle it should also be possible to apply this result to the Knight shift in metallic superconductors; however, since the values of Z_0 encountered in metals are only ~ -0.1 or -0.2, it is unlikely that it would elucidate any of the major problems involved.

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