and E supermultiplets might be almost equal. This is consistent with $K(725)$ being a member of the E supermultiplet, as the production cross section for this particle is low and its decay width is narrow. (However, the production of $\kappa\bar{\kappa}$ pairs would have to be important, which so far is not supported by the experimental evidence.)

One may also wonder if the mass formula might include terms proportional to the triality of SU(6), SU(3)(X). In such a case, if the coefficient of these terms is large, the value of the central masses of the B, C, F, and G supermultiplets might be very large.

SU(6) CLEBSCH-GORDAN COEFFICIENTS FOR THE PRODUCT $\underline{35}\times\underline{56}$†

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The unifying characteristics of the group SU(6)[1] have allowed the correlation of quantities such as masses[2] and electromagnetic interaction properties[3] for baryons and the $J^P=\frac{3}{2}^+$ baryon resonances, and for the pseudoscalar and vector mesons. Whereas the predictions given by SU(3) refer to only one of the above sets of particles at a time, SU(6) relates the properties of the baryons to those of the baryon resonances and those of the pseudoscalar mesons to the vector mesons. In computing scattering amplitudes, reaction amplitudes, amplitudes for photoproduction processes, etc., in SU(3),[4] relations are obtained only within one system, e.g., for

$$\text{meson}+\text{baryon}\rightarrow\text{meson}+\text{baryon}, \tag{1}$$

or for

$$\text{meson}+\text{baryon}\rightarrow\text{vector meson}+\text{baryon}. \tag{2}$$

Relations between these two systems are not obtainable from SU(3). The use of SU(6) offers the possibility of obtaining both relations within one system and also relations which connect this system with others. In order to carry out such studies, and also in the pursuit of the relativistic extensions[5] of SU(6), it is very useful to have available a set of Clebsch-Gordan coefficients for the SU(6) product of $\underline{35}\times\underline{56}$. Such a set has been obtained for the reduction of SU(6) according to SU(3)⊗SU(2).

The SU(3)⊗SU(2) constituents of the $\underline{35}$ are $\underline{8}^3$, $\underline{8}^1$, and $\underline{1}^3$ in a slightly different notation than usual. The SU(2) multiplicity has been written as a superscript in analogy with the notation of atomic and nuclear spectroscopy. Similarly, the constituents of the $\underline{56}$ are $\underline{10}^4$ and $\underline{8}^2$. As given previously,[6] the product

$$\underline{35}\times\underline{56}=\underline{56}\oplus\underline{70}\oplus\underline{700}\oplus\underline{1134}. \tag{3}$$

The contents of the 70-, 700-, and 1134-dimensional representations are

$$\underline{70}=\underline{8}^4,\ \underline{10}^2,\ \underline{8}^2,\ \text{and}\ \underline{1}^2; \tag{4}$$

$$\underline{700}=\underline{35}^6,\underline{10}^6,\underline{35}^4,\underline{27}^4,\underline{10}^4,\underline{8}^4,\underline{27}^2,\underline{10}^2,\underline{10}^{*2},\underline{8}^2; \tag{5}$$

$$\underline{1134}=\underline{27}^6,\underline{10}^6,\underline{8}^6,\underline{35}^4,\underline{27}_A{}^4,\underline{27}_B{}^4,\underline{10}_A{}^4,\underline{10}_B{}^4,$$
$$\underline{10}^{*4},\underline{8}_A{}^4,\underline{8}_B{}^4,\underline{8}_C{}^4,\underline{1}^4,\underline{35}^2,\underline{27}_A{}^2,\underline{27}_B{}^2,$$
$$\underline{10}_A{}^2,\underline{10}_B{}^2,\underline{10}^{*2},\underline{8}_A{}^2,\underline{8}_B{}^2,\underline{8}_C{}^2,\underline{1}^2. \tag{6}$$

Inasmuch as the product representations of (3) may be considered as the product of four quarks and one antiquark, the structure of the Young diagram and the use of SU(4) allows one to obtain the decomposition of the $\underline{700}$.[7] Since the decomposition of the $\underline{56}$ and $\underline{70}$ have been studied already,[1] the remaining states must lie in the $\underline{1134}$. Note that in the $\underline{1134}$, there are states which are repeated. The $\underline{27}^4$, $\underline{10}^4$, $\underline{27}^2$, and $\underline{10}^2$ occur twice; the $\underline{8}^4$ and $\underline{8}^2$ occur three times.

The product $\underline{35}\times\underline{56}$ may be rewritten as

$$(\underline{8}^3\oplus\underline{8}^1\oplus\underline{1}^3)\times(\underline{10}^4\oplus\underline{8}^2)$$
$$=(\underline{8}^3\times\underline{10}^4)\oplus(\underline{8}^1\times\underline{10}^4)\oplus(\underline{1}^3\times\underline{10}^4)\oplus(\underline{8}^3\times\underline{8}^2)_s$$
$$\oplus(\underline{8}^3\times\underline{8}^2)_a\oplus(\underline{8}^1\times\underline{8}^2)_s\oplus(\underline{8}^1\times\underline{8}^2)_a\oplus(\underline{1}^3\times\underline{8}^2). \tag{7}$$

The subscripts s and a refer to the symmetric and antisymmetric product representations of $(\underline{8}\times\underline{8})$. These are $\underline{27}$, $\underline{8}_s$, and $\underline{1}$ and $\underline{10}$, $\underline{10}^*$, and $\underline{8}_a$, respectively. The exact linear combinations of the products of factors in (7) which occur for any of the states in (4), (5), or (6) are listed in Table I.

For example, the $\underline{27}^2$ representation of the

Table I. SU(6)-multiplet coupling factors for the product $\underline{35}\times\underline{56}$. The rows are labeled by the SU(3)⊗SU(2) representations of each SU(6) representation. The columns are labeled by the source terms. Each entry in a row is to be divided by the normalization factor N listed in the last column. Each type of SU(3)⊗ SU(2) representation within each column is normalized separately. For example, the sum of the squares of the coefficients of the $\underline{8}^4$ terms in the first column is 1.

		$(\underline{8}^3\times\underline{10}^4)$	$(\underline{8}^1\times\underline{10}^4)$	$(\underline{1}^3\times\underline{10}^4)$	$(\underline{8}^3\times\underline{8}^2)_s$	$(\underline{8}^3\times\underline{8}^2)_a$	$(\underline{8}^1\times\underline{8}^2)_s$	$(\underline{8}^1\times\underline{8}^2)_a$	$(\underline{1}^3\times\underline{8}^2)$	N
$\underline{56}$	$\underline{10}^4$	$2\sqrt{5}$	$2\sqrt{3}$	$\sqrt{5}$		$2\sqrt{2}$				$3\sqrt{5}$
	$\underline{8}^2$	$-2\sqrt{5}$			$-\sqrt{10}$	$2\sqrt{2}$	0	$\sqrt{6}$	1	$3\sqrt{5}$
	$\underline{8}^4$	5	$\sqrt{15}$		$\sqrt{5}$	1			$\sqrt{2}$	$4\sqrt{3}$
	$\underline{10}^2$	$-4\sqrt{2}$		$-2\sqrt{2}$		$-\sqrt{2}$		$\sqrt{6}$		$4\sqrt{3}$
	$\underline{8}^2$	$2\sqrt{10}$			$-\sqrt{5}$	5	$-\sqrt{15}$	$\sqrt{3}$	$2\sqrt{2}$	$4\sqrt{6}$
	$\underline{1}^2$				$\sqrt{3}$		1			2
$\underline{700}$	$\underline{35}^6$	1								1
	$\underline{10}^6$	1		1						$\sqrt{2}$
	$\underline{35}^4$	$\sqrt{5}$	$\sqrt{3}$							$\sqrt{8}$
	$\underline{27}^4$	$\sqrt{5}$	$-\sqrt{3}$		-4					$\sqrt{24}$
	$\underline{10}^4$	1	$-\sqrt{15}$	-4		$2\sqrt{10}$				$6\sqrt{2}$
	$\underline{8}^4$	$\sqrt{30}$	$-3\sqrt{2}$		$\sqrt{6}$	$-\sqrt{30}$			$-2\sqrt{15}$	12
	$\underline{27}^2$	$\sqrt{2}$			-1		$\sqrt{3}$			$\sqrt{6}$
	$\underline{10}^2$	1		-1		1		$\sqrt{3}$		$\sqrt{6}$
	$\underline{10}^{*2}$					$\sqrt{3}$		-1		2
	$\underline{8}^2$	$-2\sqrt{2}$			7	$-\sqrt{5}$	$-3\sqrt{3}$	$\sqrt{15}$	$2\sqrt{10}$	12
$\underline{1134}$	$\underline{27}^6$	1								1
	$\underline{10}^6$	1		-1						$\sqrt{2}$
	$\underline{8}^6$	1								1
	$\underline{35}^4$	$\sqrt{3}$	$-\sqrt{5}$							$\sqrt{8}$
	$\underline{27}_A{}^4$	$\sqrt{3}$	$\sqrt{5}$		0					$\sqrt{8}$
	$\underline{27}_B{}^4$	$\sqrt{5}$	$-\sqrt{3}$		2					$\sqrt{12}$
	$\underline{10}_A{}^4$	1	$-\sqrt{15}$	4		0				$4\sqrt{2}$
	$\underline{10}_B{}^4$	$7\sqrt{5}$	$-3\sqrt{3}$	$-4\sqrt{5}$		$-8\sqrt{2}$				$4\sqrt{30}$
	$\underline{10}^{*4}$					1				1
	$\underline{8}_A{}^4$	0	0		0	$\sqrt{2}$			-1	$\sqrt{3}$
	$\underline{8}_B{}^4$	1	0		$-\sqrt{5}$	0			0	$\sqrt{6}$
	$\underline{8}_C{}^4$	$\sqrt{15}$	9		$\sqrt{3}$	$\sqrt{15}$			$\sqrt{30}$	12
	$\underline{1}^4$				1					1
	$\underline{35}^2$	1								1
	$\underline{27}_A{}^2$	0			$\sqrt{3}$		1			2
	$\underline{27}_B{}^2$	$2\sqrt{2}$			1		$-\sqrt{3}$			$\sqrt{1}$
	$\underline{10}_A{}^2$	0		2		-1		$\sqrt{3}$		$\sqrt{8}$
	$\underline{10}_B{}^2$	1		-1		-2		0		$\sqrt{6}$
	$\underline{10}^{*2}$					1		$\sqrt{3}$		2
	$\underline{8}_A{}^2$	0			0	1	$\sqrt{15}$	$-2\sqrt{3}$	$4\sqrt{2}$	$2\sqrt{15}$
	$\underline{8}_B{}^2$	$\sqrt{2}$			1	0	$2\sqrt{3}$	$\sqrt{15}$	0	$\sqrt{30}$
	$\underline{8}_C{}^2$	$2\sqrt{6}$			$-13\sqrt{3}$	$-7\sqrt{15}$	-3	$3\sqrt{5}$	$2\sqrt{30}$	$12\sqrt{10}$
	$\underline{1}^2$				1		$-\sqrt{3}$			2

$\underline{700}$ is given as the linear combination

$$\underline{27}^2(\underline{700}) = 6^{-1/2}[\sqrt{2}(\underline{8}^3\times\underline{10}^4) - (\underline{8}^3\times\underline{8}^2)_S + \sqrt{3}(\underline{8}^1\times\underline{8}^2)_S]. \quad (8)$$

The complete SU(6) Clebsch-Gordan coefficient for any state considered may be simply obtained by multiplying the appropriate SU(3) coefficients as taken from, say, the published tables of McNamee and Chilton[8] by the SU(6)-multiplet coupling factors listed in Table I.

To illustrate the use of Table I, we write the $S=\frac{1}{2}$, $K^+\Sigma^+$ wave function in terms of channel amplitudes $|\underline{70}\rangle$, $|\underline{700}\rangle$, $|\underline{1134}\rangle$:

$$|K^+\Sigma^+\rangle^{1/2} = (-1/\sqrt{2})(6^{1/2}/4\sqrt{3})|\underline{10}^2, \underline{70}\rangle$$
$$+ (1/\sqrt{2})(\sqrt{3}/6^{1/2})|\underline{27}^2, \underline{700}\rangle$$
$$+ (-1/\sqrt{2})(\sqrt{3}/6^{1/2})|\underline{10}^2, \underline{700}\rangle$$
$$+ (1/\sqrt{2})(\tfrac{1}{2})|\underline{27}_A{}^2, \underline{1134}\rangle$$
$$+ (1/\sqrt{2})(-\sqrt{3}/12^{1/2}|\underline{27}_B{}^2, \underline{1134}\rangle$$
$$+ (-1/\sqrt{2})(\sqrt{3}/8^{1/2}|\underline{10}_A{}^2, \underline{1134}\rangle. \tag{9}$$

The Y, I, I_Z quantum numbers, $1, \frac{3}{2}, \frac{3}{2}$, are the same for each channel amplitude. Equation (9) holds for both $S_Z = \frac{1}{2}$ and $S_Z = -\frac{1}{2}$.

The SU(6)-multiplet coupling factors have been obtained independently by two methods.[9] One method uses the explicit symmetries among the four quarks and one antiquark which are required by the Young diagram of the product representation. This is somewhat analogous to the method by which Sawada and Yonezawa[10] constructed their SU(3) states for the Sakata model. The second method is a more "brute force" one in which one operates on the state of the highest weight in each of the four product representations with various SU(6) generators so as to generate the entire product space.

An interesting problem occurs in the $\underline{1134}$ representation where there are repeated states. This repetition is not analogous to the occurrence of two eights in the SU(3) product of $\underline{8}\times\underline{8}$, but rather to the situation in atomic spectroscopy where the seniority[11] quantum number is required to separate the two ^2D states which occur when three d particles are coupled together. The analogy with seniority is rather marked in the case of the $\underline{1134}$ representation, because the number of times that a repeated state arises is exactly equal to the number of source terms of (7) in which an $\underline{8}^3$ state is present (as may be seen from Table I). Although we have not made explicit use of this concept of $\underline{8}^3$ seniority in the construction of the mutually orthogonal repeated states, we have allowed it to guide us, in the sense that each repeated state contains a different number of $\underline{8}^3$ terms.

Using the coefficients so obtained, we consider next the construction of scattering amplitudes for processes of the form

$$(\underline{35}\times\underline{56}|\underline{35}\times\underline{56}), \tag{10}$$

in which the incident channel consists of pseudoscalar mesons striking proton targets. For this channel, the spin $S = \frac{1}{2}$. If we neglect considerations of orbital angular momentum, then for pure SU(6), there are three classes of scattering amplitudes which we may consider, namely those with meson-baryon, vector-meson-baryon, and vector-meson–baryon-resonance final states. As in previous SU(3) work,[4] one introduces energy- and angle-dependent amplitudes in terms of which the scattering amplitudes (10) are expressed. Only four amplitudes, $A^{(56)}$, $A^{(70)}$, $A^{(700)}$, and $A^{(1134)}$ are needed. An important difference between the SU(3) calculations and those of SU(6) is that SU(6) requires far fewer amplitudes than does SU(3). This simplification leads to the following interesting new SU(6) relationships among scattering amplitudes, as may be seen also from Table II:

$$(K^+p|K^{*+}p) = -(\sqrt{2}/\sqrt{3})(K^+p|K^{*0}N^{*++})$$
$$= -(8\sqrt{3})(K^-p|\pi^0\Sigma^0)$$
$$= (8/3)(K^-p|\pi^0\Lambda), \tag{11}$$

$$(\pi^+p|\rho^+p) = -(1/2\sqrt{3})(\pi^+p|\rho^0N^{*++})$$
$$= -(1/\sqrt{3})(\pi^+p|K^+\Sigma^+), \tag{12}$$

$$(K^-p|\eta\Sigma^0) = 1/\sqrt{3})(K^-p|\eta\Lambda), \tag{13}$$

$$(\pi^-p|K^0\Lambda) = (6^{1/2}/2)(K^-p|\bar{K}^0n), \tag{14}$$

Table II. Scattering amplitudes for meson-baryon-initiated amplitudes according to SU(6).

Representation / Process	$\underline{56}$	$\underline{70}$	$\underline{700}$	$\underline{1134}$
$(K^+p\|K^+p)$			$\frac{1}{2}$	$\frac{1}{2}$
$(K^-p\|K^-p)$	2/45	$\frac{1}{12}$	2/9	13/20
$(\pi^+p\|\pi^+p)$	0	$\frac{1}{16}$	$\frac{1}{2}$	$\frac{7}{16}$
$(\pi^-p\|\pi^-p)$	1/45	5/48	13/36	41/80
$(K^0p\|K^0p)$			$\frac{3}{8}$	$\frac{5}{8}$
$(\bar{K}^0p\|\bar{K}^0p)$	1/45	1/24	17/72	7/10
$(K^+p\|K^{*+}p)$			$-1/2\sqrt{3}$	$1/2\sqrt{3}$
$(K^+p\|K^{*0}N^{*++})$			$1/2\sqrt{2}$	$-1/2\sqrt{2}$
$(K^-p\|\pi^0\Sigma^0)$			$\frac{1}{16}$	$-\frac{1}{16}$
$(K^-p\|\pi^0\Lambda)$			$\sqrt{3}/16$	$-\sqrt{3}/16$
$(K^-p\|\eta\Sigma^0)$		$-\sqrt{3}/48$	$\sqrt{3}/48$	
$(K^-p\|\eta\Lambda)$		$-\frac{1}{16}$	$\frac{1}{16}$	
$(K^-p\|\bar{K}^0n)$	−1/45	−1/24	1/72	1/20
$(K^-p\|\pi^+\Sigma^-)$	−1/45	1/48	1/72	−1/80
$(K^-p\|K^+\Xi^-)$	−2/45	1/24	1/36	−1/40
$(\pi^+p\|\rho^+p)$		$-1/16\sqrt{3}$		$1/16\sqrt{3}$
$(\pi^+p\|\rho^0N^{*++})$		$\frac{1}{8}$		$-\frac{1}{8}$
$(\pi^+p\|K^+\Sigma^+)$		$-\frac{1}{16}$		$\frac{1}{16}$
$(\pi^-p\|K^0\Lambda)$	$-6^{1/2}/90$	$-6^{1/2}/48$	$6^{1/2}/144$	$6^{1/2}/40$
$(\pi^-p\|K^+\Sigma^-)$	−1/45	1/48	1/72	−1/80

$$(\pi^- p \,|\, K^+\Sigma^-) = (K^- p \,|\, \pi^+\Sigma^-) = (\tfrac{1}{2})(K^- p \,|\, K^+\Xi^-). \quad (15)$$

Also listed in Table II are the elastic amplitudes considered by Johnson and Treiman[12] in their very successful analysis of total cross sections. These amplitudes clearly satisfy their condition that

$$\tfrac{1}{2}[(K^+p \,|\, K^+p) - (K^-p \,|\, K^-p)] = (\pi^+p \,|\, \pi^+p) - (\pi^-p \,|\, \pi^-p) = (K^0p \,|\, K^0p) - (\bar{K}^0p \,|\, \bar{K}^0p). \quad (16)$$

Because of the neglect of orbital angular momentum in our analysis, it is difficult to say exactly how a comparison of the new SU(6) relations with experiment should be carried out. It is hoped that the use of relativistic extensions[5] will allow the incorporation of the orbital angular momentum into these scattering amplitudes in a consistent manner and will allow an eventual correlation with processes which have pseudoscalar-meson-baryon-resonance final states. In any case, SU(6) offers the remarkable possibility of relating processes which involve widely differing systems of particles.

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