MESON-BARYON SCATTERING IN THE INTRINSICALLY BROKEN $M(12)$ SYMMETRY SCHEME

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(Received 8 March 1965)

The group $M(12)$[1,2] has been proposed to underlie the symmetry of strong interactions. $M(12)$ is a 144-parameter noncompact group that upon application of Weyl's unitary trick goes into the compact group U(12). $M(12)$ symmetry is "intrinsically broken" by the effects of the kinetic energy (i.e., Feynman propagators), and consequently "exact" $M(12)$ invariance is only meaningful in a highly idealized, explicitly dynamics-dependent strong-coupling limit. Furthermore, $M(12)$ being noncompact, its finite-dimensional representations are not unitary and it is by no means necessary that all components of an $M(12)$ representation correspond to particles encountered in nature. In fact the maximal compact subgroup of $M(12)$ is $W(6)=U(6)\otimes U(6)$,[1,3] and one might at best hope that W(6) multiplets will actually have particle content. However, the intrinsic symmetry-breaking mechanism will split these W(6) multiplets in U(6) and then into $SU(3)\otimes SU(2)$ multiplets, and some of these multiplets might be shifted to very high masses and consequently get "isolated" from their relatives in the low-mass region. One may consider therefore only the low-mass U(6) multiplet in an $M(12)$ representation and "project out" all other multiplets by symmetry-breaking subsidiary conditions. A convenient (though not necessarily unique) projection procedure[2] is to require that the $M(12)$-multiplet fields satisfy the Bargmann-Wigner–type[4] wave equation. In view of the encouraging results obtained with this procedure for baryon-meson couplings, it may be interesting to apply it to the problem of meson-baryon scattering. This we shall do in the present paper and, indeed, a very rich body of sum rules for scattering amplitudes appears to emerge. We shall interpret the significance of these sum rules as tests for the $M(12)$ symmetry scheme.

It is very important to point out that the simple requirement of unitarity for the S matrix conflicts with $M(12)$ symmetry. This type of symmetry breaking is one and the same as the kinetic-energy symmetry breaking mentioned earlier.[1] The sum over intermediate states which occurs in the equation for the absorptive part of a scattering amplitude includes a sum over spin states; this sum over spin states is just the absorptive part of the propagator, which interferes with exact $M(12)$ symmetry (for spin $\geq \frac{1}{2}$). It is unclear that the rules used to construct vertex graphs can be combined to give meaningful scattering graphs. Nevertheless, we shall assume that the entire S matrix has the structure of the Born approximation. There is, of course, no theoretical justification for this procedure; our objective is merely to test a well-defined set of rules against experiment, without prejudice as to the ultimate significance of these rules.

We describe the baryons by $M(12)$ spinors[5]

$$\psi_{\alpha\beta\gamma}=\psi_{abc;ABC}(p)$$
$$=[(1+\gamma p/M)\gamma_5 C]_{ab}u_c(p)\epsilon_{ABM}b_C^{\ M}$$
$$+\text{cyclic permutations}+\cdots, \quad (1)$$

where $\alpha,\beta,\gamma=1,\cdots,12$; $a,b,c=1,\cdots,4$; and $A,B,C=1,\cdots,3$; $b_C^{\ M}$ is the usual SU(3) baryon matrix; M is the central baryon mass, and the dots stand for spin-$\frac{3}{2}$ decuplet whose production we do not intend to study here. Mesons are then described by [1,2,6]

$$M_\alpha^{\ \beta}=M_{ab;A}^{\ \ B}(k)=[(1+\gamma k/m)\gamma_5]_{ab}P_A^{\ B}+\cdots, \quad (2)$$

where $P_A^{\ B}$ is the SU(3) meson matrix, m is the central meson mass, and the dots stand for omitted vector-meson terms. Intrinsically broken $M(12)$ then restricts the form of the matrix element for the reaction between an incoming meson of momentum k with a baryon of momentum p, resulting in an outgoing meson of momentum k' and an outgoing baryon of momentum p', to[7]

$$T_{\underline{1}}(s,t)\bar\psi^{\alpha\beta\gamma}(p')\psi_{\alpha\beta\gamma}(p)\bar M_\rho^{\ \sigma}(k')M_\sigma^{\ \rho}(k)$$
$$+T_{\underline{143}}(s,t)\bar\psi^{\alpha\beta\rho}(p')M_\rho^{\ \sigma}(k)\bar M_\sigma^{\ \lambda}(k')\psi_{\alpha\beta\lambda}(p)$$
$$+T_{\underline{143}}'(s,t)\bar\psi^{\alpha\beta\rho}(p')\bar M_\rho^{\ \sigma}(k')M_\sigma^{\ \lambda}(k)\psi_{\alpha\beta\lambda}(p)$$
$$+T_{\underline{5940}}(s,t)\bar\psi^{\alpha\mu\nu}(p')M_\mu^{\ \rho}(k)\bar M_\nu^{\ \sigma}(k')\psi_{\alpha\rho\sigma}(p), \quad (3)$$

Table I. Meson-baryon reaction amplitudes in terms of the four $M(12)$-invariant amplitudes.[a,b]

Process	$a(s,t)$				$b(s,t)/F$		
	$T_{\underline{1}}$	$T_{\underline{143}}$	$T_{\underline{143}}'$	$T_{\underline{5940}}$	$T_{\underline{143}}$	$T_{\underline{143}}'$	$T_{\underline{5940}}$
$\pi^- + p \to \pi^- + p$	1	A_u	$A_s + B_s$	$-C + 3D_s$	-8	-32	-12
$\pi^0 + n$		$B_u/\sqrt{2}$	$-B_s/\sqrt{2}$	$3(D_u - D_s)/\sqrt{2}$	$20\sqrt{2}$	$20\sqrt{2}$	$12\sqrt{2}$
$K^+ + \Sigma^-$				$(D_s - 2D_u)$			-12
$K^0 + \Sigma^0$			$-A_s/\sqrt{2}$	$(C - D_u - D_s)/\sqrt{2}$		$-4\sqrt{2}$	
$K^0 + \Lambda$			$-(A_s + 2B_s)/\sqrt{6}$	$(C + D_u - 5D_s)/\sqrt{6}$		$12\sqrt{6}$	$4\sqrt{6}$
$\eta + n$		$B_u/\sqrt{6}$	$B_s/\sqrt{6}$	$(D_u + D_s)/\sqrt{6}$	$40/\sqrt{6}$	$-40/\sqrt{6}$	
$\pi^+ + p \to \pi^+ + p$	1	$A_u + B_u$	A_s	$-C + 3D_u$	32	8	12
$K^+ + \Sigma^+$			$-A_s$	$C - 3D_u$		-8	-12
$K^- + p \to K^- + p$	1		$A_s + B_s$			-32	
$\bar{K}^0 + n$			B_s			-40	
$\Xi^- + K^+$				$-D_u - D_s$			
$\Xi^0 + K^0$				$D_u - 2D_s$			12
$\Sigma^+ + \pi^-$		$-A_u$		$C - 3D_s$	8		12
$\Sigma^- + \pi^+$				$D_u - 2D_s$			12
$\Sigma^0 + \pi^0$		$-A_u/2$		$(C + D_u - 5D_s)/2$	4		12
$\Lambda + \pi^0$		$-(A_u + 2B_u)/2\sqrt{3}$		$(C - 5D_u + D_s)/2\sqrt{3}$	$-12\sqrt{3}$		$-4\sqrt{3}$
$K^+ + p \to K^+ + p$	1	$A_u + B_u$			32		

[a]We only consider reactions with the proton as target.
[b]The kinematic functions A_s, A_u, B_s, B_u, C, D_s, D_u, and F are listed in Table II.

where $s = (p+k)^2$ and $t = (k-k')^2$ and the amplitudes have been labeled by the $M(12)$ representation of the t channel. The matrix element c for any given pseudoscalar meson-baryon reaction can be described in terms of two amplitudes

$$c = a(s,t) + b(s,t)\gamma Q/m, \tag{4}$$

with $Q = k + k'$. The power of $M(12)$ makes itself felt when we realize that the amplitudes a and b for any meson-baryon reaction (e.g. for all 16 reactions listed in Table I) can be expressed in terms of the four amplitudes $T_{\underline{1}}, \cdots, T_{\underline{5940}}$ introduced in (3), with the use of some "kinematical" functions of s, t which come from expanding the products in Eq. (3). In Table I, we list all the experimentally accessible scattering reactions in terms of these kinematical functions; Table II lists the various kinematical functions. It should be noted that the kinematical function F multiplying $\gamma Q/m$ is universal. Tables I and II are the main result of this paper. We give below a selection of sum rules that follow from them. First, in addition to the well-known SU(3) relation[8]

$$d\sigma(K^- + p \to K^0 + \Xi^0) = d\sigma(K^- + p \to \Sigma^- + \pi^+), \tag{5a}$$

we find

$$d\sigma(\pi^- + p \to K^+ + \Sigma^-) = d\sigma(K^- + p \to \Sigma^- + \pi^+). \tag{5b}$$

Furthermore, we find that there should be no polarization of the out-coming baryons for the following reactions:

$$\pi^- + p \to K^+ + \Sigma^-, \tag{6a}$$

$$K^- + p \to \bar{K}^0 + n, \tag{6b}$$

Table II. Kinematic functions.[a]

$$A_x = 56 + \frac{32(M^2 + m^2 - x)}{mM} - \frac{8(x - M^2)}{m^2} - 2t\left(\frac{3}{M^2} + \frac{5}{m^2} + \frac{8}{mM}\right) - \frac{6xt}{m^2M^2}$$

$$B_x = 8 - \frac{16(M^2 + m^2 - x)}{mM} + \frac{40(x - M^2)}{m^2} - 2t\left(\frac{3}{M^2} - \frac{7}{m^2} - \frac{4}{mM}\right) - \frac{6xt}{m^2M^2}$$

$$C = -8 + 2t\left(\frac{1}{m} + \frac{1}{M}\right)^2 + \frac{2[(s - m^2 - M^2)^2 + st]}{m^2M^2}$$

$$D_x = \frac{4(x - m^2 - M^2)}{mM} + \frac{3(x - M^2)}{m^2} - \frac{t}{M^2} - \frac{(x - m^2 - M^2)^2 + (x - M^2)(t - M^2)}{m^2M^2}$$

$$F = \left(1 - \frac{t}{4M^2}\right)\left(1 + \frac{M}{m}\right)$$

[a]To evaluate A_s and A_u of Table I, insert $x = s$ and $x = u$, respectively, in A_x; same for B_x and D_x; $u = 2m^2 + 2M^2 - s - t$.

$$K^- + p \to \Xi^- + K^+, \tag{6c}$$

$$K^- + p \to \Xi^0 + K^0, \tag{6d}$$

$$K^- + p \to \Sigma^- + \pi^+. \tag{6e}$$

We also find the relation

$$\sigma_{\rm tot}(K^-p) - \sigma_{\rm tot}(K^+p) = 2[\sigma_{\rm tot}(\pi^-p) - \sigma_{\rm tot}(\pi^+p)], \tag{7}$$

derived earlier from nonrelativistic SU(6) considerations.[9]

Finally, we wish to give some relations between amplitudes for elastic and charge-exchange meson-baryon scattering, that we think should be tested:

$$\frac{b(K^- + p \to K^- + p)}{b(K^- + p \to \bar{K}^0 + n)} = \frac{4}{5}, \tag{8a}$$

$$\begin{aligned} a(K^+ + p \to K^+ + p) - a(K^- + p \to K^- + p) &= [(A_u + B_u)/32F] b(K^+ + p \to K^+ + p) \\ &+ [(A_s + B_s)/32F] b(K^- + p \to K^- + p), \end{aligned} \tag{8b}$$

$$\begin{aligned} b(\pi^- + p \to \pi^- + p) + b(\pi^+ + p \to \pi^+ + p) \\ = \tfrac{3}{4}[b(K^- + p \to K^- + p) + b(K^+ + p \to K^+ + p)], \end{aligned} \tag{8c}$$

for a and b defined above [Eq. (4)] and with A_s, A_u, B_s, B_u defined in Table II.

When compared with existing experiments, (6c) does not fare well at $P_{\rm Lab} = 1.95$ BeV/c where the average Ξ^- polarization is 0.77 $\pm$ 0.24.[10] But then it should be kept in mind that even the SU(3) relation (5a) conflicts with experiment at low energies. The main success of SU(3), as clearly emphasized in reference 8, is in correctly connecting large elastic-scattering amplitudes

$$\frac{d\sigma_{K^-p}}{dt} \cong \frac{d\sigma_{K^+p}}{dt} \cong \frac{d\sigma_{\pi^-p}}{dt} \cong \frac{d\sigma_{\pi^+p}}{dt}$$

and

$$\sigma_{{\rm tot}K^-p} \sim \sigma_{{\rm tot}K^+p} \sim \sigma_{{\rm tot}\pi^-p} \sim \sigma_{{\rm tot}\pi^+p}.$$

It may therefore be worth while to concentrate on relations of the type (7) and (8) which contain elastic amplitudes and are consequently expected to hold better than the relations of type (5) and (6). The relations (5) and (6) connect small reaction amplitudes which are more susceptible to serious symmetry- [even SU(3) symmetry-] breaking effects.

We wish to thank Professor J. R. Oppenheimer for the hospitality of the Institute for Advanced Study.

Note added in proof.—After the completion of this work we received a preprint by R. Blankenbecler, M. L. Goldberger, K. Johnson, and S. B. Treiman in which some of the results of this paper are also obtained.

*The study was supported by the U. S. Air Force Office of Scientific Research, Grant No. AF-AFOSR-42-64.

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