[12] B. Sakita, Phys. Rev. 136, B1756 (1964); L. Michel and B. Sakita, to be published; T. Fulton and J. Wess, Phys. Letters 14, 57 (1965); V. Kadyshevsky, R. Muradyan, A. Tavkhelidze, and I. Todorov, to be published; S. K. Bose and Yu. Shirokov, Phys. Rev. Letters 14, 398 (1965).

SELECTION RULES IN RELATIVISTIC SU(6) THEORIES

Harry J. Lipkin
Department of Physics, The Weizmann Institute of Science, Rehovoth, Israel
(Received 8 March 1965)

In a previous paper selection rules were obtained from SU(6) symmetry and found to give a consistent description of a number of known experimental facts.[1] Recently, some of these selection rules have been found to hold in particular relativistic SU(6) formulations.[2,3] We wish to show that the original derivation is valid relativistically under certain general assumptions shared by most of the relativistic formulations, and that all of the results obtained[1] are valid relativistically. In particular, the following decays and reactions are all forbidden:

$$\varphi \to \rho + \pi, \tag{1a}$$

$$\varphi \to 3\pi \text{ or } 5\pi, \tag{1b}$$

$$\pi + N \to \varphi + N + n\pi, \tag{1c}$$

$$N + N \to \varphi + N + N + n\pi, \tag{1d}$$

$$\bar{N} + N \to \varphi + n\pi, \tag{1e}$$

where n is any number.[4] Also the production of strange particles in πN, NN, and $\bar{N}N$ reactions is inhibited by a factor of at least four, on the average. The experimental validity of these selection rules can therefore be considered as evidence in favor of SU(6) symmetry, but cannot be used to distinguish between different relativistic formulations which should all give the same result. In particular, the results (1) should be obtained both from theories which embed SU(6) in a higher group including both SU(6) and the Lorentz group,[2] and theories which break SU(6) symmetry using kinetic-energy spurions.[3]

The selection rules were obtained using SU(2) subgroups of SU(6) called quark spins, $S_{p'}$, $S_{n'}$, and $S_{\lambda'}$, defined, respectively, as the total spin of the p', n', and λ' quarks in any multiquark state. These rules are still valid in any relativistic theory which satisfies the following condition:

The λ'-quark spin $S_{\lambda'}$ is conserved in all reactions involving only particles at rest and particles of finite momentum which contain no λ' quarks in a quark model. It is also conserved in reactions also involving particles of strangeness ±1 which contain one λ' quark or antiquark, where such particles have $S_{\lambda'} = \frac{1}{2}$, but the <u>orientation</u> of the quark spin may be different for a particle of finite momentum than for a nonrelativistic particle. Analogous conservation laws apply to $S_{p'}$ and $S_{n'}$.

It is easily seen that this condition is satisfied by theories of both types mentioned above.[2,3] The essential point is that the number of λ' quarks and antiquarks in a state cannot be changed by a Lorentz transformation nor by the application of the symmetry-breaking kinetic-energy operator. The only effect of such transformations on a multiquark state is to rotate and recouple the spins of the existing quarks. The transition matrix elements for all the reactions (1) vanish in nonrelativistic SU(6) because the φ has $S_{\lambda'} = 1$ and all the other particles contain no λ' quarks and have $S_{\lambda'} = 0$. This remains unchanged by Lorentz transformations of the individual particle states to finite momenta and by adding an arbitrary number of kinetic-energy "spurions" to the vertex function. These transformations do not affect the $S_{\lambda'} = 1$ of the φ, if the reactions are analyzed in a Lorentz frame in which the φ is at rest. They also do not affect the $S_{\lambda'} = 0$ of the other particles, since neither Lorentz transformations nor operation with kinetic-energy operators can produce states having $S_{\lambda'} \neq 0$, as this requires the creation of λ' quarks. Since the theory is Lorentz invariant, forbidding the reactions (1) in a particular Lorentz frame is sufficient to forbid it in all Lorentz frames.[5]

The selection rule against strange-particle production is based on similar considerations and also on the property that a state of two strange particles contains a λ' quark and antiquark, with uncorrelated spins of $\frac{1}{2}$, so that the probability of total $S_{\lambda'} = 0$ is only $\frac{1}{4}$. This is also unaffected by Lorentz transformations or kinetic-energy operators. They can change the relative orientation of the two λ'-quark spins by rotation, but cannot introduce correlations if the spins are initially randomly oriented.

Note that the undesirable selection rules of nonrelativistic SU(6) forbidding $\rho \to 2\pi$ and $N^* \to N+\pi$ are not carried over into the relativistic domain by these arguments. These selection rules depend upon the conservation of total quark spin, not $S_{\lambda'}$ alone. Lorentz transformations or kinetic-energy operators acting on a pion state introduce a component of total quark-spin one by recoupling the constituent quark spins.

Further conclusions regarding meson processes can be drawn from the observation that neither Lorentz transformations nor kinetic-spurion operations can change the number of quarks nor the SU(3) quantum numbers of a particular state. These operators can change a one-meson state in the SU(6) $\underline{35}$ only into another state with the same SU(3) quantum numbers which transforms under SU(6) like a linear combination of members of different $\underline{35}$ supermultiplets [plus a singlet for the SU(3) singlet vector meson], but cannot introduce higher SU(6) representations.[6] Thus predictions for processes involving only baryons at rest and mesons of finite momenta are apt to be insensitive to the details of the relativistic formulation. One example is the unpleasant value obtained by Harari and Lipkin[7] for the ratio of the cross sections $\sigma(\bar{p}+p\to K^+ + K^-)/\sigma(\bar{p}+p\to K^0+\bar{K}^0)=16$ for annihilation at rest. Another is the ratio of the $\rho\pi\pi/\omega\rho\pi$ couplings, where only a limited additional freedom is obtained in the spurion theories. It may be that a conclusive test between these theories is only obtainable in reactions containing relativistic baryons.

---

[1] H. J. Lipkin, Phys. Rev. Letters 13, 590 (1964).

[2] A. Salam, R. Delbourgo, and J. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); M. A. B. Bég and A. Pais, Phys. Rev. Letters 14, 269 (1965); B. Sakita and K. C. Wali, to be published. Earlier references are given in these papers. See Y. Ne'eman, Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January 1965 (to be published), for a general review of SU(6).

[3] K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 264 (1965).

[4] These reactions should be weak compared with similar allowed reactions; e.g. φ-decay and -production processes involving strange particles, ω-decay and -production processes analogous to the Reactions (1). This is verified by experiment.

[5] These results do not depend upon the existence of quarks, although they are conveniently expressed using a quark model. All that is required is that meson and baryon states should transform under SU(6) and Lorentz transformations in the same manner as states obtained from a quark model. Note that the requirement that the SU(6) classification is valid in the rest frame of a meson or baryon is nontrivial in a quark model although it is generally accepted. The rest frame of the particle is not the same as the rest frame of the constituent quarks.

[6] This is much more restrictive than the apparently reasonable assumption that the kinetic energy transforms under SU(6) like a member of a $\underline{35}$ and can produce all representations arising in the product $\underline{35}\times\underline{35}$ when operating on a $\underline{35}$ meson state, and that higher order spurion processes lead to even higher representations. Careful analysis is required in individual cases to verify that multispurion transitions really give higher representations. Similar restrictions without the quark picture are obtained by the observation that the kinetic-energy operator for fixed momentum is one of the generators of the U(12) group and therefore has vanishing matrix elements between states belonging to different U(12) representations. The author is indebted to Amnon Katz for elucidation of this point.

[7] H. Harari and H. J. Lipkin, to be published.