Press, Princeton, New Jersey, 1946), p. 265. The unitary trick is intimately connected with the Euclidean formulation of quantum field theories. See, e.g., J. Schwinger, Phys. Rev. 115, 721 (1959).

[4]We wish to thank Professor M. Gell-Mann for an interesting discussion on this point.

[5]It cannot be stressed too strongly that this subgroup is not identical with the physical Lorentz group. The generators of this subgroup are the "spin" part S_{ij} (as opposed to the "orbital" part L_{ij}) of the generators $M_{ij}=L_{ij}+S_{ij}$ of the Lorentz group. To the extent that S_{ij} is approximately conserved [which is implied by the approximate U(6) invariance], so is $L_{ij}=M_{ij}-S_{ij}$ (M. Gell-Mann, private communication). In a state in which $\langle L_{ij}\rangle=0$, we may utilize the transformation properties under U(6) to deduce the spin of the state.

[6]From here on, whenever we write a set of generators G_i we always refer to a group of transformations $\exp(ia_iG_i)$ with real parameters a_i.

[7]This is a consequence of the Baker-Hausdorff theorem; see, for example, D. Finkelstein, Commun. Pure Appl. Math. 8, 245 (1955). In a more pedestrian way the invariance of $\bar{\psi}\psi$ [$\bar{\psi}\gamma_5\psi$] under the transformations (4a) [(4b)] can be checked by direct calculation.

[8]We follow here the customary notations from the Fermi theory of beta decay.

[9]All W(6) and GL(6) groups considered in our work contain (as has been emphasized in I and in this paper) a U(6) subgroup with generators $\rho^0\otimes J_i$. This subgroup leaves the mass term invariant. The fact that the mass term does not violate U(6) invariance has been noted independently by M. A. B. Bég and A. Pais, to be published.

[10]Upon application of the unitary trick $M(12)$ leads to U(12). This symmetry could also play a role only in Euclidean field theory. Whether this has anything to do with physics in our Minkowskian world is an open question.

SU(6) AND SEMILEPTONIC INTERACTIONS

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In this note we study some consequences of the assumptions that the effective strongly interacting particle currents of both vector (V) and axial vector (A) kinds associated with the semileptonic processes each transform according to the adjoint representation of the group SU(6). We discuss furthermore the implications of more restrictive dynamical assumptions which relate to each other the adjoint representation for the V current and the one for the A current. In this latter context we also encounter the question of the SU(6) completion of the Goldberger-Treiman relations. In accordance with the general considerations[1] about the interpretation of SU(6), we restrict ourselves for the present to the low-frequency limit of these effective currents, taking into account only effects up to and including the first order in v/c.

(i) Vector current. Up to this order we must consider two kinds of terms: (a) the weak charge term proportional to the Fermi constant G_V which gives the allowed Fermi transitions, and (b) the weak magnetism term.[2] In the spirit of the proportionality assumptions between these terms and the corresponding electromagnetic ones, our assumptions will here be the straight transcriptions from those made earlier[3] for the electromagnetic case. Thus we postulate that the weak charge operator transforms like an $(\underline{8},\underline{1})$ member of a $\underline{35}$ and the weak magnetic moment operator like an $(\underline{8},\underline{3})$ member of a $\underline{35}$. Again, as for electromagnetism, we do not assume that the same $\underline{35}$ representation appears in both cases. The meaning of this last proviso will be discussed in more detail elsewhere.[4]

(ii) Axial vector current. At low frequencies we have here only the Gamow-Teller transition term proportional to G_A. This term is now assumed to transform like an $(\underline{8},\underline{3})$ member of still another $\underline{35}$.

With these specifications, we can now write down the effective low-frequency four-point vertex for the interaction between leptons and strongly interacting particles. We consider specifically the interaction with the baryons of the $\underline{56}$ representation[5] of SU(6), which may be written as[6]

$$3B_{\alpha\beta\gamma}{}^{\dagger}(p_2)B^{\alpha\beta\delta}(p_1)C_{\delta}{}^{\gamma}(q), \qquad (1)$$

$$C_{\delta}{}^{\gamma}(q)=\frac{G_V}{\sqrt{2}}[\delta_i{}^j(L_0)_A{}^B+i\mu_W(\vec{\sigma}\cdot\vec{q}\times\vec{L}_A{}^B)_i{}^j]$$
$$+\frac{3G_A}{5\sqrt{2}}(\vec{\sigma}\cdot\vec{L}_A{}^B)_i{}^j, \qquad (2)$$

$$\mu_W = \tfrac{3}{5}[\mu(p)-\mu(n)]/e. \tag{3}$$

The lepton variables are contained in the L symbols. We have $L_\mu = (\vec{L}, iL_0)$;

$$(L_\mu)_A{}^B = \sqrt{2}\begin{pmatrix} 0 & l_\mu \cos\theta & l_\mu \sin\theta \\ l_\mu{}^\dagger \cos\theta & 0 & 0 \\ l_\mu{}^\dagger \sin\theta & 0 & 0 \end{pmatrix}, \tag{4}$$

$$-il_\mu = \bar{\mu}(p_4)\gamma_\mu(1+\gamma_5)\nu_\mu(p_3) + \bar{e}(p_4)\gamma_\mu(1+\gamma_5)\nu_e(p_3),$$

with $q = p_2 - p_1 = p_3 - p_4$. The SU(3) tensor, Eq. (4), has been defined so as to have no neutral lepton currents. We have furthermore weighted the $\Delta S = 0$ relative to the $\Delta S = 1$ transitions by means of the Cabibbo angle[7] θ. Normalizations are such that (at low frequencies)

$$(n\,\tfrac{1}{2} \to p\,\tfrac{1}{2})_V = G_V/\sqrt{2}; \quad (n\,\tfrac{1}{2} \to p\,\tfrac{1}{2})_A = G_A/\sqrt{2}, \tag{5}$$

where we specify in terms of the neutron → proton transition amplitudes. The $\frac{1}{2}$'s denote the 3 component of spin; subscripts V and A refer to the vector and axial vector parts, respectively.

We proceed to state the consequences of our Ansatz. First we note relations due to SU(3) only:

$$(\Omega^- \to \Xi^0)\cot\theta = (N^{*-} \to n) = -(N^{*+} \to n)\sqrt{3}. \tag{6}$$

Next, the G_A term yields the ratio $(D/F) = \frac{3}{2}$, as stated earlier[8] and compared with experiment elsewhere.[1] It should be stressed that the value of this ratio is exclusively determined by the SU(6) structure of the A part of the interaction. It is in particular (a) independent of more specific connections between V and A currents, (b) independent of specific dynamical connections between the A current which appears in the semileptonic interactions and the corresponding current[8] in the strong interactions.

These same two independences also apply to the following two new and specific SU(6) relations:

$$(N^{*+}\,\tfrac{1}{2} \to n\,\tfrac{1}{2})_V = -\frac{2\sqrt{2}}{5}(n\,\tfrac{1}{2} \to p\,\tfrac{1}{2})_{V,\,\text{magn}}, \tag{7}$$

$$(N^{*+}\,\tfrac{1}{2} \to n\,\tfrac{1}{2})_A = -\frac{2\sqrt{2}}{5}(n\,\tfrac{1}{2} \to p\,\tfrac{1}{2})_A. \tag{8}$$

In Eq. (7) the subscript "V, magn" means that one should only take the contribution from the μ_W term in Eq. (2).

Equations (7) and (8) can be tested from the 33-resonance production in neutrino-nucleon collisions. For the vector part of $\nu + N \to N^* + l$, Eq. (7) is of course equivalent to the determination of the amplitude for $N^* \to N + \gamma$ as in reference 3 and subsequent use of conservation of vector current. A comparison with experiment along these lines has meanwhile been made by Albright and Liu[9] in the course of a detailed study of N^* production by neutrinos.

Equations (6)-(8) imply furthermore that to a good approximation the β and the μ decay of the Ω^- are strongly dominated by the axial-vector contribution.[10]

All the foregoing conclusions are independent of the G_A/G_V ratio for the nucleon, as it occurs in Eq. (5). However, it is a natural assumption that $C_\delta{}^\gamma(q)$ given by Eq. (2) is the universal lepton-current combination, that is to say that the semileptonic couplings of all SU(6) multiplets come about via $C_\delta{}^\gamma(q)$. Then the axial-vector to vector ratio for different SU(6) representations can all be expressed in terms of G_A/G_V. In particular it follows from the structure of Eq. (2) that

$$(G_A/G_V)_{\text{nucleon}} = (5/3)(G_A/G_V)_{\text{sextet}}, \tag{9}$$

where the ratio on the left is the ratio of the matrix elements written down in Eq. (5) and the ratio on the right is the corresponding quantity for $\Delta S = 0$ transition elements for the fundamental sextet[11] (spin-$\frac{1}{2}$ triplet).

Therefore, if the semileptonic interaction of the fundamental sextet is V–A [in the SU(6) limit] then the semileptonic interaction of the nucleon has $G_A/G_V = 5/3$ [in the SU(6) limit], corresponding to the following more specific form of $C_\delta{}^\gamma(q)$:

$$C_\delta{}^\gamma(q) = \frac{G_V}{\sqrt{2}}[\delta_i{}^j (L_0)_A{}^B + i\mu_W(\vec{\sigma}\cdot\vec{q}\times\vec{L}_A{}^B)_i{}^j + (\vec{\sigma}\cdot\vec{L}_A{}^B)_i{}^j]. \tag{10}$$

It is of course very appealing to reduce the number of independent 35's appearing in Eq. (2). Equation (10) is an example of such a reduction.

Generally, it will be necessary to supplement SU(6) with specific dynamical considerations extraneous to SU(6), if one wishes to establish a relation between G_A and G_V. In order to gain some further insight on how this may come about, we consider a rather simplified model in which the weak-interaction currents satisfy[12]

$$\frac{i}{\mu_{00}}[\partial^\lambda J_\lambda{}^A(x)]_A{}^B = c(A,B)[P(x)]_A{}^B, \qquad (11)$$

$$[J_\lambda{}^V(x)]_A{}^B = d(A,B)[V_\lambda(x)]_A{}^B, \qquad (12)$$

where P is the pseudoscalar octet, V_λ is the vector nonet, $c(A,B)$ and $d(A,B)$ are numerical constants depending on the SU(3) indices A and B, and all operators are in the Heisenberg representation.

Now according to SU(6), the one-particle Fourier projections of P and V_λ are united in the adjoint representation 35. Thus if we make the Ansatz that the divergence of the axial-vector current is similarly united with the vector current, we obtain the constraint

$$c(A,B) = d(A,B). \qquad (13)$$

The conserved-vector-current hypothesis gives[13]

$$d(2,1) = \mu_{00}{}^2 G_V/g\sqrt{2}, \qquad (14)$$

and the partially conserved axial-vector current hypothesis may be formulated[14] such that

$$c(2,1) = \mu_{00}{}^2 G_A/g_A\sqrt{2}. \qquad (15)$$

Equation (13) then implies

$$G_A/G_V = g_A/g, \qquad (16)$$

a relationship between the strong and weak couplings. Note that Eq. (16) is valid in all SU(6) representations.[13] Thus $G_A/G_V=1$ for sextets, $G_A/G_V=5/3$ for nucleons, etc. Therefore, the model defined by Eqs. (11)-(13) is in accordance with, but more restrictive than, Eq. (10).

In the above argument the most important dynamical input has been the introduction of assumptions sufficient to guarantee the validity of the Goldberger-Treiman formulas[15] in the symmetry limit. Since SU(6) only unites states of the same parity, it can only relate pseudoscalar (0^-) and vector (1^-) couplings. Any relationship between pseudoscalar (0^-) and axial-vector (1^+) couplings is extraneous to this group.

It has been suggested in the past that if $G_A/G_V=1$ for bare nucleons, it may be enhanced to ~1.2 due to renormalization of the axial-vector current. The present model[16] reverses the situation. We now seek a reduction of $G_A/G_V=5/3$ to ~1.2. Therefore one cannot be confident that this model is meaningful unless one can show how this reduction comes about by the breakdown of SU(6).

An alternative way to relate G_A to G_V would be to embed SU(6) in a larger group. Such alternatives have been encountered before in the case of the D/F ratio. Thus one can attempt to find D/F by reference to specific dynamical models,[17] or else, as done here and in foregoing papers, by embedding SU(3) into SU(6). A possible embedding of SU(6) is provided[18] by the group U(6)⊗U(6). Until these points are clarified further, it is an open question whether a "good" value for G_A/G_V in a symmetry limit has definitively been recognized as yet.

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[1]M. A. B. Bég and A. Pais, Phys. Rev. (to be published).

[2]M. Gell-Mann, Phys. Rev. 111, 362 (1958).

[3]M. A. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964).

[4]M. A. B. Bég and A. Pais, to be published.

[5]Strictly speaking, it is an additional assumption that in the weak interactions at low frequencies it is again a good approximation to unite the baryons in this representation.

[6]At zero three-momentum the symbol $B^{\alpha\beta\lambda}$ is explicitly given in Eq. (10) of reference 3. It is explained in reference 1 how to go to nonzero three-momentum. We use the notations $\gamma=(j,B)$, $\delta=(i,A)$; see footnote 9 of reference 3.

[7]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[8]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

[9]C. H. Albright and L. S. Liu, Phys. Rev. Letters 13, 673 (1964). See also reference 2 of this paper for detailed references to earlier work on this reaction.

[10]Cf. S. Glashow and R. Socolow, Phys. Letters 10, 143 (1964).

[11]The sextet is discussed in some detail in reference 1.

[12]μ_{00} is the central mass of the 35.

[13]g and g_A are, respectively, the strength of the strong s-wave ρ_0 coupling and the strong p-wave π_0 coupling to any SU(6) representation. For nucleons, the normalization of g and g_A is as in reference 8.

[14]See, e.g., S. Adler, Phys. Rev. (to be published).

[15]J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento 17, 757 (1960).

[16]The induced pseudoscalar coupling is, of course, also predicted in this model, namely $G_P \approx (2M_{00}/\mu_{00})G_A = 3.5G_A$. However, the transition from the symmetry limit to the actual situation can not be made without mass corrections which play an essential role for this coupling. We do not therefore take this value very seriously.

[17]R. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963); S. Glashow and L. Rosenfeld, Phys. Rev. Letters 10, 192 (1963); A. Martin and K. Wali, Nuovo Cimento 31, 1324 (1964).

[18]This group was studied some time ago by F. Gürsey, A. Pais, and L. A. Radicati (unpublished) as the group of V and A charges and currents. One aim of the study was to find a group which not only contains the SU(6) of the strong interactions, but also a new group SU(6), the fundamental representation of the latter to be associated with the sextet of leptons with common lepton number. This is possible. [It was further noted that the introduction of intermediate bosons carrying a group representation seems to single out SU(6) representations of dimensions 15 or 21 as simple possibilities.] In the symmetry limit of a common zero lepton mass this new group contracts to a structure contained in $SU(4) \otimes SU(2) \otimes U(1)$. With respect to this new group one has $G_A/G_V = 1$ in the symmetry limit. However, this limit can only be taken by also dropping the baryon masses. As emphasized by F. Gürsey, this zero-mass limit radically alters the group structure (suppression of polarization states for spin $>\frac{1}{2}$!). This circumstance obscures a classification of particle states in terms of $U(6) \times U(6)$. This group has also been discussed recently by R. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964); K. Bardakci, J. Cornwall, P. Freund, and B. Lee, to be published.

ERRATUM

SCATTERING OF RUBY-LASER BEAM BY GASES. T. V. George, L. Slama, M. Yokoyama, and L. Goldstein, Phys. Rev. Letters 11, 403 (1963).

The graphs of Figs. 3 and 4 of this paper failed to take correctly into account the angular variations of the effective scattering volume of the gas.

The experimental differential scattering cross sections at 60° scattering angle, of Table I, had to be reduced by a factor of 0.835 which arises with the various interface reflections occurring in the experimental arrangement.

These corrections have been included in the detailed paper scheduled for publication in the 18 January 1965 issue of the Physical Review [Phys. Rev. 137, A369 (1965)].