target absorption, is measured by comparing pion yields from 10-, 20-, and 40-cm Be targets. In the same manner, multiple scattering in the spectrometer was measured by introducing various amounts of material between the counters.

The apparatus was quite close to the target, which made both lifetime and muon-contamination corrections very small. At low momenta, there was an appreciable yield of electrons; these were subtracted on the basis of Cherenkov curves.

We estimate the total uncertainty due to systematic errors to be about ±10%. Statistical errors are indicated in Table I for the kaon data but are negligible for the pion and proton data.

Various theoretical models (statistical, fireball, one-pion-exchange) have been proposed to interpret particle production and have been partially successful in indicating the over-all aspects, but detailed agreement is not good. A useful empirical expression proposed by G. von Dardel,[5]

$$\frac{d^2N}{dp d\Omega}=A^{\pm}p^2(P_0-p)\exp\left[-p\left(\frac{1}{T}+\frac{\theta^2}{p_0}\right)\right],$$

where

$d^2N/dpd\Omega$ = differential cross section [mb/(sr BeV/c)],

p = pion momentum [BeV/c],

θ = production angle [radian],

$P_0 = 13.4$ BeV/c,

$A^{\pm} = 16$ for π^+, 8 for π^-,

$T = 1.35$ BeV/c,

$p_0 = 0.057$ BeV/c,

is capable of representing our pion data within a factor of two over most of its range.

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[1]S. van der Meer and K. M. Vahlbruch, CERN Report No. 63-37 (unpublished), p. 97.

[2]W. F. Baker, R. L. Cool, E. W. Jenkins, T. F. Kycia, S. J. Lindenbaum, W. A. Love, D. Luers, J. A. Niederer, S. Ozaki, A. L. Read, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 101 (1961).

[3]A. C. Melissinos, G. G. Fazio, T. Yamanouchi, S. J. Lindenbaum, and L. C. L. Yuan, Phys. Rev. Letters 7, 454 (1961).

[4]D. Dekkers, J. A. Geibel, R. Mermod, G. Weber, T. R. Willitts, K. Winter, B. Jordan, M. Vivargent, N. M. King, and E. J. N. Wilson, Phys. Rev. 137, B962 (1965).

[5]G. von Dardel, CERN 1963 NPA Seminars 63-37 (unpublished), p. 98.

DYNAMICAL THEORY OF NONLEPTONIC HYPERON P WAVES IN SU(6) SYMMETRY

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On the basis of CP invariance and of transformation of the nonleptonic interaction as the $\underline{35}$ of SU(6), a definite understanding of nonleptonic hyperon S waves has recently been reached.[1] The presence of orbital angular momentum has not allowed so far for a simple understanding of nonleptonic hyperon P waves. Orbital angular momentum can be introduced following the general scheme proposed by Gell-Mann.[2] However, the large number of parameters in such a general treatment does not allow for useful predictions on the P waves. In this note we propose a dynamical theory of nonleptonic hyperon P waves. We supplement the assumptions of CP invariance and of behavior as $\underline{35}$ of the interaction with the assumption of dominance of the baryon intermediate states of $\underline{56}$ in a one-variable dispersion relation. We find the following relations for the P amplitudes (in addition to those following from $\Delta T = \frac{1}{2}$):

$$P(\Sigma_-^-) = 0, \tag{1}$$

$$P(\Lambda_-^0) = \sqrt{3}\, P(\Sigma_0^+), \tag{2}$$

$$\sqrt{2}\, P(\Xi_-^-) = P(\Omega^-_{\pi^-}). \tag{3}$$

$$P(\Omega^-_{\pi^-}) = (\tfrac{2}{3})^{1/2} P(\Omega^-_{K^-}). \tag{4}$$

Equations (1) and (2) are obtained also with the lower symmetry $SU(4)(T)\otimes SU(2)(X)\otimes W(Y)$,[3] instead of the full SU(6) symmetry—actually Eq. (1) is strictly inherent to the dynamical model.

Comparison with the data must be carried out for the same choice of amplitudes that was found to agree with the SU(6) predictions for S waves.[4] Equation (1) reads $-0.39 \pm 0.60 = 0$; Eq. (2) reads $2.0 \pm 0.25 = \sqrt{3}(3.6 \pm 0.35)$ for Solution (i), or $2.0 \pm 0.25 = \sqrt{3}(1.7 \pm 0.2)$ if one chooses Solution (ii). The latter solution is then preferred. It is known, however—risus dolore miscebitur—that neither Solution (i) nor (ii) agrees well with the $\Delta T = \frac{1}{2}$ relation $\sqrt{2}\,\Sigma_0^+ = -\Sigma_-^- + \Sigma_+^+$. Equations (3) and (4) can be used to derive lower limits to the rates of $\Omega^- \to \Xi^0 + \pi^-$ (briefly $\Omega^-_{\pi^-}$) and of $\Omega^- \to \Lambda^0 + K^-$ (briefly $\Omega^-_{K^-}$). We obtain[4]

$$\Gamma(\Omega^- \to \Xi^0 + \pi^-) \geq \Gamma_P(\Omega^- \to \Xi^0 + \pi^-) = \frac{1}{2}\frac{m_\Xi^{\,2}}{m_\Lambda m_\Omega}\left(\frac{K_\Xi}{K_\Lambda}\right)^3 2\Gamma_P(\Xi_-^-) \simeq 118 \times 10^7 \text{ sec}^{-1}, \tag{5}$$

$$\Gamma(\Omega^- \to \Lambda^0 + K^-) \geq \Gamma_P(\Omega^- \to \Lambda^0 + K^-) = \frac{1}{2}\frac{m_\Xi}{m_\Omega}\left(\frac{K_\Lambda'}{K_\Lambda}\right)^3 3\Gamma_P(\Xi_-^-) \simeq 64.5 \times 10^7 \text{ sec}^{-1}, \tag{6}$$

where Γ_P is the rate for decay into P wave; K_Ξ and K_Λ' are the momenta of the final Ξ and Λ in Ω^- decay, respectively; K_Λ is the momentum of the Λ emitted in Ξ decay. The predicted P-wave rates, Eqs. (5) and (6), are smaller than the estimates by Glashow and Socolow, who give $\Gamma_P(\Omega^- \to \Xi^0 + \pi^-) \simeq 178 \times 10^7 \text{ sec}^{-1}$ and $\Gamma_P(\Omega^- \to \Lambda^0 + K^-) \simeq 94 \times 10^7 \text{ sec}^{-1}$.[5]

To derive the above results we assume a one-variable dispersion relation in the squared initial four momentum and the dominance of the intermediate baryon states of $\underline{56}$. The general amplitude is a linear combination of the following four terms (corresponding to the four possible invariants that can be formed out of the baryon tensor $B^{\alpha\beta\gamma}$, the mesons $M_\alpha{}^\beta$, the intermediate baryons $\bar{B}^{\alpha\beta\gamma}$, and the orbital angular-momentum spurion $t_\alpha{}^\beta$):

$$\sum_P B_{\lambda\mu\nu}{}^\dagger M_\rho{}^\sigma t_\sigma{}^\rho t \bar{B}^{\lambda\mu\nu}\eta(\alpha\beta\gamma) \times \bar{B}_{\alpha\beta\gamma}{}^\dagger B^{\alpha\beta\delta} x_\delta{}^\gamma \delta_\lambda{}^\alpha \delta_\mu{}^\beta \delta_\nu{}^\gamma, \tag{7}$$

$$\sum_P B_{\lambda\mu\sigma}{}^\dagger M_\rho{}^\sigma t_\nu{}^\rho \bar{B}^{\lambda\mu\nu}\eta(\alpha\beta\gamma) \times \bar{B}_{\alpha\beta\gamma}{}^\dagger B^{\alpha\beta\delta} x_\delta{}^\gamma \delta_\lambda{}^\alpha \delta_\mu{}^\beta \delta_\nu{}^\gamma, \tag{7'}$$

$$\sum_P B_{\lambda\mu\rho}{}^\dagger M_\nu{}^\sigma t_\sigma{}^\rho \bar{B}^{\lambda\mu\nu}\eta(\alpha\beta\gamma) \times \bar{B}_{\alpha\beta\gamma}{}^\dagger B^{\alpha\beta\delta} x_\delta{}^\gamma \delta_\lambda{}^\alpha \delta_\mu{}^\beta \delta_\nu{}^\gamma, \tag{7''}$$

$$\sum_P B_{\lambda\sigma\rho}{}^\dagger M_\mu{}^\rho t_\nu{}^\sigma \bar{B}^{\lambda\mu\nu}\eta(\alpha\beta\gamma) \times \bar{B}_{\alpha\beta\gamma}{}^\dagger B^{\alpha\beta\delta} x_\delta{}^\gamma \delta_\lambda{}^\alpha \delta_\mu{}^\beta \delta_\nu{}^\gamma. \tag{7'''}$$

In Eqs. (7)-(7'''), $x_\alpha{}^\beta$ is the weak spurion, $\sum_P$ denotes summation on all permutations of the indices $\lambda\mu\nu$ into $\alpha\beta\gamma$, and the weight factor $\eta(\alpha\beta\gamma)$ accounts for the normalizations of the components of $B^{\alpha\beta\gamma}$; $\eta(\alpha\beta\gamma) = 1, 3, 6$ for $\alpha = \beta = \gamma$, or $\alpha = \beta \neq \gamma$, or $\alpha \neq \beta \neq \gamma$, respectively.[6] Carrying out the summation and noting that $\bar{B}^{\alpha\beta\gamma}\eta(\alpha\beta\gamma)\bar{B}_{\alpha\beta\gamma}{}^\dagger = 1$ (not summed on the indices), the above four amplitudes become

$$B_{\alpha\beta\gamma}{}^\dagger M_\rho{}^\sigma t_\sigma{}^\rho B^{\alpha\beta\delta} x_\delta{}^\gamma, \tag{8}$$

$$[B_{\alpha\beta\sigma}{}^\dagger t_\gamma{}^\rho + 2B_{\alpha\gamma\sigma}{}^\dagger t_\beta{}^\rho] M_\rho{}^\sigma B^{\alpha\beta\delta} x_\delta{}^\gamma, \tag{8'}$$

$$[B_{\alpha\beta\rho}{}^\dagger M_\gamma{}^\sigma + 2B_{\alpha\gamma\rho}{}^\dagger M_\beta{}^\sigma] t_\sigma{}^\rho B^{\alpha\beta\delta} x_\delta{}^\gamma, \tag{8''}$$

$$[B_{\alpha\rho\sigma}{}^{\dagger}M_{\gamma}{}^{\rho}t_{\beta}{}^{\sigma}+B_{\alpha\rho\sigma}{}^{\dagger}M_{\beta}{}^{\rho}t_{\gamma}{}^{\sigma}+B_{\gamma\rho\sigma}{}^{\dagger}M_{\beta}{}^{\rho}t_{\alpha}{}^{\sigma}]B^{\alpha\beta\delta}x_{\delta}{}^{\gamma}. \quad (8''')$$

The first term, Eq. (8), does not contribute; the second and the third, Eqs. (8′) and (8″), give equal contributions. Eliminating the two independent parameters one finds Eqs. (1), (2), (3), and (4).

Partial inclusion of symmetry breaking, by restricting the symmetry to the subgroup $SU(4)(T)\otimes SU(2)(X)\otimes W(Y)$, leaves Eqs. (1) and (2) unchanged. Under $SU(4)(T)\otimes SU(2)(X)$ the nucleons transform as $(\underline{20},\underline{1})$, Σ and Λ as $(\underline{10},\underline{2})$, π as $(\underline{15},\underline{1})$. The weak spurion x transforms as $(\underline{4},\underline{2})$, and the angular-momentum spurion t as $(\underline{15},\underline{1})\oplus(\underline{1},\underline{3})$. The main point is to observe that the $(\underline{1},\underline{3})$ component of t cannot contribute because of its $SU(2)(X)$ behavior—we call to mind the related selection rules following from conservation of G' parity.[3] The component $(\underline{15},\underline{1})$ of t introduces four invariants [corresponding to the fourfold appearance of $(\underline{1},\underline{1})$ in $(\underline{20},\underline{1})\otimes(\underline{15},\underline{1})\otimes(\underline{15},\underline{1})\otimes(\underline{20},\underline{1})$] in one-to-one correspondence with the analogous invariants in SU(6).

[1]G. Altarelli, F. Buccella, and R. Gatto, Phys. Letters 14, 70 (1965); K. Kawarabayashi, Phys. Rev. Letters 14, 86, 169 (1965); P. Babu, Phys. Rev. Letters 14, 166 (1965); R. Ferrari, M. Konuma, and E. Remiddi, to be published. Some conditons are also derived by S. P. Rosen and S. Pakvasa, Phys. Rev. Letters 13, 773 (1964); and by M. Suzuki, Phys. Letters 14, 64 (1965). Symmetry-breaking effects have been discussed by K. Sawarabayashi and R. White, to be published; and by W. Alles and G. Segré, to be published.

[2]M. Gell-Mann, Phys. Rev. Letters 14, 77 (1965).

[3]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

[4]See Table I in the first paper of reference 1; the data are taken from R. H. Dalitz, Proceedings of the International School of Physics "Enrico Fermi," Varenna Lectures, 1964 (to be published).

[5]S. Glashow and R. Socolow, Phys. Letters 10, 143 (1964).

[6]M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964).

COVARIANCE, SU(6), AND UNITARITY

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In a recent note[1] it has been shown how the method of relativistic completion of SU(6) leads to fully covariant and crossing-symmetric effective vertices and matrix elements. We continue to mean by SU(6) a group property of zero-three-momentum one-particle states. For brevity we denoted all these effective quantities as "S-matrix quantities," so that the term S matrix is used in a phenomenological sense. It was found that this completion procedure is in general not unique. The lack of uniqueness is due to the fact that there are (with the exception of the $\underline{6}$ representation) inequivalent ways in which an SU(6) representation can be boosted to momentum $\vec{p}$. The total set of ways in which this can be done is fully determined by the spin content of the SU(6) representation in hand. The inequivalent boosts are effectively indistinguishable when applied to bilinear forms (free particles), but they are effectively distinct when applied to n-point functions, $n>2$. For the meson($\underline{35}$)-baryon($\underline{56}$) three-point function, the set of covariant but inequivalent vertices was given. It was noted that the noncompact booster group $SU(12)_{\mathfrak{L}}$ provides a convenient way of keeping track of the inequivalent boosts of a given SU(6) representation. In particular it was found that a unique meson-baryon vertex emerges if one assigns the $SU(12)_{\mathfrak{L}}$ representations[1,2] $\underline{364}$ and $\underline{143}$ to the boosted $\underline{56}$ and $\underline{35}$, respectively. Similar results for this vertex have been obtained independently by several other authors.[2] It was further observed[1] that the same methods can be applied to any n-point function to yield covariant and crossing-symmetric answers.

Within the conventional framework of quantum mechanics and relativity theory, the description in terms of this covariant SU(6)-invariant effective S matrix is only approximate in a dynamical sense. It should indeed be recalled[1,3] that the completion procedures cannot be applied in general to a Lagrangian field theory with interaction, where the free kinet-