REGGE POLES AND THE PHASE OF THE FORWARD-SCATTERING AMPLITUDE*

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There is currently great interest in the phase of forward elastic-scattering amplitudes at high energy. In a simple diffraction picture, one expects a purely imaginary amplitude. However, recent Coulomb interference measurements[1-6] in pp and πp scattering have shown that the real part is substantial, of order 20 to 30%, up to 26 GeV/c. Various dispersion-relation calculations[7-9] have confirmed that something of this order may be expected, but they do not explain what mechanism produces it.

The present note points out that simple Regge-pole models predict a substantial real part at the energies in question, in surprisingly good agreement with experiment. We employ the latest πN, NN, and $\bar{N}N$ total cross-section data[5,10] from 6 to 26 GeV/c to fix the parameters of the forward-scattering amplitudes, taking three Regge poles for πN and four for NN. The ratio of the real to the imaginary part of the forward-scattering amplitudes is illustrated and compared with experiment in Figs. 1 and 2. Now Mandelstam[11] has shown that there are probably also branch points in the complex angular-momentum plane, which cannot be ignored asymptotically. Nevertheless, there appears to be a good chance that over a wide range—perhaps up to 100 GeV—these branch points are not yet important and the Regge poles dominate.[12] Our models assume this is so.

Let us define and normalize the spin-averaged forward elastic amplitude $A(0)$ for each process, such that the optical theorem reads

$$\sigma_T = \mathrm{Im}\,A(0), \tag{1}$$

where σ_T is the corresponding total cross section. Then in a high-energy approximation each Regge pole contributes to $A(0)$ a term of the form[13]

$$A_i(0) = B_i\{[1 \pm \exp(-i\pi\alpha_i)]/\sin\pi\alpha_i\}(E/E_0)^{\alpha_i - 1}. \tag{2}$$

Here i labels the Regge pole, α_i is its trajectory at squared momentum transfer $t=0$, B_i is a real coefficient measured in millibarns, E is the total laboratory-system energy of the bombarding particle, and E_0 is an arbitrary scale parameter which we choose to be 1 GeV. Note that the phase of $A_i(0)$ is determined by α_i, through the "signature factor" (in braces),

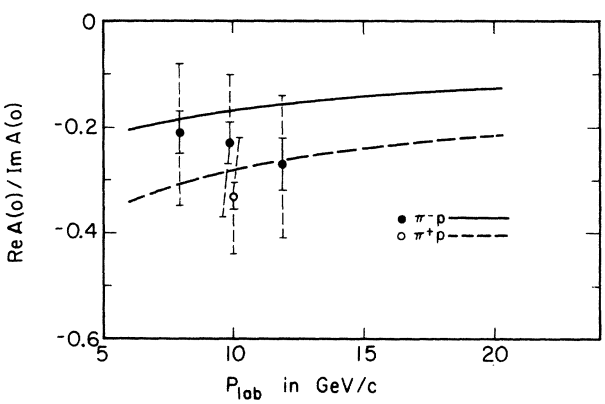

FIG. 1. The ratio of the real to the imaginary part of the forward amplitude for π^+p and π^-p scattering. The curves are Regge-pole predictions. The data are from reference 1: The inner error flags are statistical; the outer ones are estimated limits of systematic error.

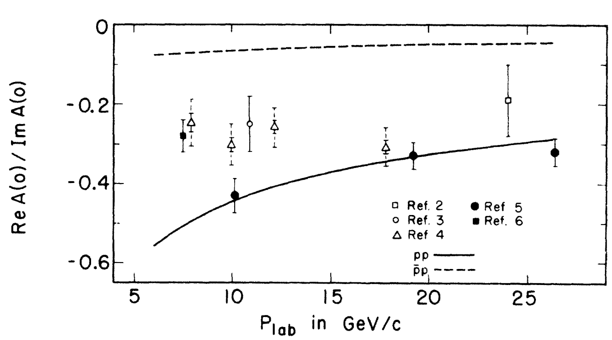

FIG. 2. The ratio of the real to the imaginary part of the forward spin-averaged amplitude, for pp and $\bar{p}p$ scattering. The curves are Regge-pole predictions. The data refer only to pp scattering. Where double error flags are shown, the inner ones are statistical and the outer ones are estimated limits of systematic error.

and is therefore directly related to the energy dependence.

For πN scattering, at least three Regge poles are needed. The Pomeranchuk pole P gives the asymptotic limit; a second vacuum pole P' and the ρ pole give the differences of the $\pi^{\pm}$-p amplitudes from the asymptotic limit and from each other. The signature $\pm$ in Eq. (2) is + for P and P', − for ρ. Let us take the coefficients B_i above to refer to π^-p elastic scattering (for π^+p, B_ρ changes sign). Since the fit to data is not very sensitive to the precise values of the α_i, we fix them at suitable[14] values $\alpha_P = 1$, $\alpha_{P'} = 0.5$, $\alpha_\rho = 0.6$. Then by a least-squares fit to the total cross sections,[10] the B_i are determined: $B_P = 19.9 \pm 0.1$ mb, $B_{P'} = 18.1 \pm 0.2$ mb, and $B_\rho = 2.4 \pm 0.4$ mb.

The predicted ratios $\mathrm{Re}A(0)/\mathrm{Im}A(0)$ for $\pi^{\pm}$-p scattering are shown in Fig. 1. They agree with the experimental determination in sign, in magnitude, and in giving a larger value for π^+p than for π^-p; however, the experimental uncertainties are rather large. The dispersion calculation of reference 9 agrees closely with our π^-p curve but gives a π^+p curve displaced upwards.

For NN and $\bar{N}N$ scattering, at least two more poles are usually invoked. One is the negative-signature ω pole (which is supposed to include any contribution from the φ pole, lying near, with the same quantum numbers). The other is the R pole, proposed by Pignotti.[15] However, the experimental uncertainties in the data we use are such that the R contribution that is determined is not significantly different from zero, and we ignore this term. Let us take the coefficients B_i in Eq. (2) to refer to $\bar{p}p$ elastic scattering; for $\bar{n}p$ the ρ term changes sign; for pp the ω and ρ terms change sign; for np the ω term changes sign. We fix $\alpha_\omega = 0.5$, with α_P, $\alpha_{P'}$, and α_ρ as before; then by a least-squares fit to the total cross sections,[5,10] we determine $B_P = 36.2 \pm 0.2$ mb, $B_{P'} = 33.8 \pm 0.6$ mb, $B_\rho = 1.0 \pm 1.2$ mb, and $B_\omega = 21.0 \pm 2.3$ mb.

The predicted ratios $\mathrm{Re}A(0)/\mathrm{Im}A(0)$ for pp and $\bar{p}p$ scattering are shown in Fig. 2. The experimental points refer only to pp. Below 10 GeV/c there is a marked divergence between prediction and experiment; the former becomes steadily more negative while the latter (in a region not illustrated) finally becomes positive below 1.5 GeV/c. Above 10 GeV/c, however, the agreement with experiment is surprisingly good. There have been several dispersion calculations which agree roughly with one another and with experiment. Söding's calculation,[7] for example, gives roughly 70% of our pp values (above 10 GeV/c); it also agrees rather closely with our $\bar{p}p$ values.

Regge-pole models are designed for high energies. As the energy is lowered, the various correction terms play more and more important roles. We may expect the real part of the forward-scattering amplitude to be especially sensitive to these corrections, since it is in a sense a correction term itself—coming wholly from the secondary trajectories. Thus a divergence of the prediction from experiment is to be expected at lower energies.

Our Regge-pole parameters B_i have been fixed by total cross sections only. A complete Regge-pole model should also fit elastic angular distributions; this seems to present no serious difficulty,[16,17] but the best fit to this wider range of data generally gives slightly different parameters. However, in the cases we have studied[17] the change is small; the curves in Figs. 1 and 2 should be little altered by a more complete fit to data.

After this work was completed we received a preprint from Bialas and Bialas,[18] who have made a closely related analysis in more general terms, without appealing explicitly to Regge poles. In terms of their analysis, the value of the Regge-pole hypothesis seems to be that it unites and makes plausible many assumptions that have to be introduced to get practical results: e.g., that the total cross section is given by a sum of powers of E, that the sum contains few terms, that there are no logarithms, that no "odd-signature" terms with α close to 1 are present, etc.

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[1]S. J. Lindenbaum, Proceedings of the International Conference on High-Energy Physics, Dubna, 1964 (to be published).

[2]E. Lohrmann, H. Meyer, and H. Winzeler, Phys. Letters 13, 78 (1964).

[3]L. Kirillova, L. Khristov, V. Nikitin, M. Shafrano-

va, L. Strunov, V. Sviridov, Z. Korbel, L. Rob, P. Markov, Kh. Tchernev, T. Todorov, and Z. Zlateva, Phys. Letters 13, 93 (1964).

[4]K. J. Foley, R. S. Gilmore, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters 14, 74 (1965).

[5]G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillethun, J. Pahl, J. P. Scanlon, J. Walters, A. M. Wetherell, and P. Zanella, Phys. Letters 14, 164 (1965).

[6]A. E. Taylor, A. Ashmore, W. S. Chapman, D. F. Falla, W. H. Range, D. B. Scott, A. Astbury, F. Capocci, and T. G. Walker, Phys. Letters 14, 54 (1965).

[7]P. Soding, Phys. Letters 8, 285 (1964).

[8]I. I. Levintov and G. M. Adelson-Velsky, Phys. Letters 13, 185 (1964).

[9]V. S. Barashenkov and V. I. Dedyu, to be published.

[10]W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontić, R. H. Phillips, A. L. Reed, and R. Rubinstein, Proceedings of the International Conference on High-Energy Physics, Dubna, 1964 (to be published).

[11]S. Mandelstam, Nuovo Cimento 30, 1127, 1148 (1963).

[12]G. F. Chew and V. L. Teplitz, Phys. Rev. 136, B1154 (1964).

[13]See, for instance, S. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. 126, 2204 (1962).

[14]These values are determined within a few percent by Regge-pole fits to a wide range of data: T. O. Binford and B. R. Desai, "High Energy Elastic Scattering at Low Momentum Transfers" (to be published); and R. J. N. Phillips and W. Rarita, unpublished calculations.

[15]A. Pignotti, Phys. Rev. 134, B630 (1964).

[16]Binford and Desai, reference 14.

[17]Phillips and Rarita, reference 14.

[18]A. Bialas and E. Bialas, "The High-Energy Behavior of the Real Part of the Elastic Scattering Amplitude" (to be published).

$\pi^{\pm}$ AND $K^{\pm}$ PRODUCTION CROSS SECTIONS FOR 12.5-BeV PROTONS ON Be†

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High-energy neutrino experiments in progress at the Argonne zero-gradient synchrotron (ZGS) employ a wide-angle focusing device[1] to enhance the neutrino or antineutrino flux. Existing data[2-4] on particle production by nuclear interactions of high-energy protons are dispersed over a range of primary energies, target materials, production angles, and secondary energies. In order that the resulting flux may be accurately known, a precise measurement of meson production by 12.5-BeV protons on Be has been completed. Pion-production cross sections have been measured over the angular range from 2° to 16°, and at pion momenta from 1 to 12 BeV/c. Data on kaon production were also obtained in a narrower momentum range. Scattered proton fluxes were measured, and are presented here for their value in pratical beam design.

A slow extracted proton beam, ~100-msec duration and 12.5-BeV kinetic energy, was focused onto a Be target of 2.8-cm diameter and 10-, 20-, or 40-cm length. For beam extraction, ordinary internal targets, rather than a jump target, were used, resulting in about 1% extraction efficiency and allowing a "parasitic" mode of operation. The beam was focused to a spot of 1.2-cm diameter and constantly monitored by a closed-circuit TV viewing a plastic scintillator placed just before the target. During this experiment, the extracted intensity was about 10^8-10^9 protons per pulse.

An absolute intensity calibration was performed by two independent foil (Al, Au) activation measurements. The relative intensity was monitored during runs by means of three triple-scintillation-counter telescopes viewing the target from 15°, 25°, and 135° laboratory angles. Internal consistency of these monitors was better than 0.1%, and the foil-activation measurements agreed to less than half their quoted error of 10%.

Figure 1 depicts the external proton-beam layout. The secondary particles produced in the target were deflected in a simple magnetic spectrometer placed 20 feet away. Two small scintillators (1, 2) defined the production angle. Two more scintillators (3, 4) and a single or tandem gas Cherenkov counter at a bending angle of 3.5° behind the magnet served to de-