to the hard-core values for polycrystalline sources.

To demonstrate experimentally the effect of a random magnetic interaction on the angular correlation we performed a time-differential measurement with a sample of In^{111} dissolved in Ni (less than 1 part in 10^{10}). The result is shown in Fig. 2. The large anisotropy of the 172- to 247-keV cascade in Cd^{111} confirms clearly the prediction of two frequencies for $k=2$ [Eq. (7)]. A least-squares fit of the data yields a Larmor frequency $\omega_L=(0.995\pm0.010)\times10^8$ $\sec^{-1}$ which gives, with a g factor of $g=-0.318\pm0.007$,[4] a magnetic field for Cd dissolved in Ni of

$$|H|=65.3\pm1.6 \text{ kG}.$$

The accuracy of this value is limited by the uncertainty of the time calibration (1%) and by the fact that the g factor is only known within 2%.

Several features of this method are worth pointing out: (1) The presence of a low-frequency (ω_L) component allows the measurement of fields twice as large as would otherwise be possible, with a given instrumental time resolution. (2) Very small fields are also accessible. In the present experiment any field between 5 and 500 kG could have been observed. (3) Fields are measured throughout the sample, not just in domain walls. (4) Measurements may be made at any temperature and pressure, provided that the spin-correlation time is long compared with $1/\omega_L$. (5) Induced fields in antiferromagnets may also be measured. (6) Polarization in an external field may be followed independently of frequency shifts by observing the disappearance of the low-frequency component in the correlation function.

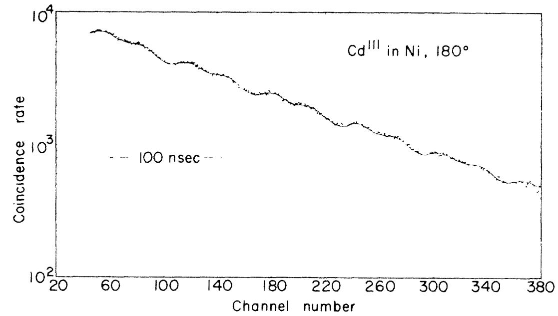

FIG. 2. Time-differential measurement of a random magnetic interaction with a source of In^{111} dissolved in Ni. The solid curve represents the best fit of the points to the function $F(t)=Ne^{-\lambda t}\times\{1+a[1+2\cos(\omega_L t+\varphi)+2\cos2(\omega_L t+\varphi)]\}+C$.

Aside from the famous case of Cd^{111} it appears that there are quite a few isotopes available which would allow the investigation of internal magnetic fields with the aid of this method.

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INTRINSICALLY BROKEN U(6)⊗U(6) SYMMETRY FOR STRONG INTERACTIONS. II

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Recently the idea that strong interactions might have the approximate symmetry group $U(6)\otimes U(6)\equiv W(6)$ was proposed by the present authors,[1] and independently, by Feynman, Gell-Mann, and Zweig.[2] In I this symmetry was discussed in terms of the full six-dimensional linear group GL(6), which does not in itself contain W(6), but whose connection to W(6) is made through the "unitary trick" of Weyl.[3] Extending the considerations of I, we shall show in this note that the group W(6) arises naturally when one enlarges the group GL(6) to a 144-parameter, noncompact group which will be denoted by $M(12)$. It will then be shown that W(6) is the maximal compact subgroup of $M(12)$. While the group $M(12)$ and all of its noncompact subgroups—for instance GL(6)—may be considered as intrinsically broken symmetries of a classical Lagrangian (the intrinsic breaking mechanism is provided by the kinetic energy of fields[1]), one encounters a difficulty when one tries to transplant a noncompact symmetry of this type to the quantum domain. Namely, the canonical commutation relations are not invariant under the noncompact group. Furthermore, the noncompact part of the group cannot be generated in the usual fashion by the use of currents in the second quantized theory.[4] We shall therefore concentrate on the compact subgroup W(6) in applications, as only this part of $M(12)$ is meaningful in quantized theories.

There is a natural way of proceeding from an essentially nonrelativistic invariance of the U(6) type to a fully relativistic theory, which automatically generates higher symmetries. We discuss this procedure in a quark model, as in I. Introduce the 12-component Dirac spinor ψ:

$$\psi=\begin{pmatrix}L\\R\end{pmatrix},\quad \begin{matrix}L\\R\end{matrix}=\frac{1\pm\gamma_5}{2}\psi, \tag{1}$$

and U(6) matrices $D(\alpha)=\exp(i\alpha_i J_i)$, $i=0,1,\cdots,35$, where $J_i=\{\sigma_k\otimes\lambda_j\}$, $k=0,1,\cdots,3$; $j=0,1,\cdots,8$. The Hermitian forms $\bar\psi\psi=L^\dagger R+R^\dagger L$; $i\bar\psi\gamma_5\psi=i(L^\dagger R-R^\dagger L)$ are invariant under $L\to D(\alpha)L$, $R\to[D(\alpha)]^{\dagger-1}R$ [$=D(\alpha)R$ for real α !]. These forms are invariant even if the parameters α are complex. The matrices $D(\alpha)$ then form a representation of GL(6) as do the matrices $[D(\alpha)]^{\dagger-1}=D(\alpha^*)$. They contain a subgroup which is isomorphic to the homogeneous Lorentz group.[5] Thus $\bar\psi\psi$ and $\bar\psi\gamma_5\psi$ are not only U(6)-, but also GL(6)-invariant. They also have additional symmetries that mix R and L, which we shall presently identify. $\bar\psi\psi$ is invariant under a three-parameter group whose generators are[6]

$$\rho_1,\quad i\rho_2,\quad i\rho_3, \tag{2a}$$

whereas $\bar\psi\gamma_5\psi$ is invariant under a group with the three generators

$$\rho_2,\quad i\rho_1,\quad i\rho_3. \tag{2b}$$

Here, ρ_i are Pauli matrices acting on a two-dimensional space labeled by R and L. The generators of GL(6), considered as a 72-real-parameter group, can be written in the same Pauli matrix basis as

$$\begin{pmatrix}J_i&0\\0&J_i\end{pmatrix}=\rho_0\otimes J_i\equiv K_i,\quad \begin{pmatrix}iJ_i&0\\0&-iJ_i\end{pmatrix}=i\rho_3\otimes J_i\equiv L_i, \tag{3}$$

ρ_0 being the unit matrix in the RL space.

The fact that $\bar\psi\psi$ [$\bar\psi\gamma_5\psi$] is invariant under the transformations generated by the matrices of Eq. (2a) [Eq. (2b)] as well as under those generated by K_i and L_i of (3) implies[7] that it must be invariant under a larger group whose generators are obtained by completing the commutation relations between K_i, L_i, and ρ_j. It is easy to see that in this way we obtain a noncompact group[1] $M(12)$ with 144 generators:

$$\begin{pmatrix}J_i&\\&J_i\end{pmatrix}=\rho_0\otimes J_i,\quad \begin{pmatrix}iJ_i&\\&-iJ_i\end{pmatrix}=i\rho_3\otimes J_i,$$

$$\begin{pmatrix}0&J_i\\J_i&0\end{pmatrix}=\rho_1\otimes J_i,\quad \begin{pmatrix}0&-J_i\\J_i&0\end{pmatrix}=i\rho_2\otimes J_i, \tag{4a}$$

which leaves $\bar\psi\psi$ invariant, or

$$\rho_0\otimes J_i,\quad \rho_2\otimes J_i,\quad i\rho_1\otimes J_i,\quad i\rho_3\otimes J_i, \tag{4b}$$

which leaves $\bar\psi\gamma_5\psi$ invariant. The groups generated by (4a) and (4b) are isomorphic. However an essential difference of these two possibilities arises when we identify $\rho_1\otimes 1$ with the parity operation. The maximal compact subgroup of $M(12)$ is W(6). For the case (4a) it is spanned by the generators $M_i^\pm=(\rho_0\pm\rho_1)J_i$, whereas for (4b) by $N_i^\pm=(\rho_0\pm\rho_2)J_i$.

The group W(6) generated by the algebra $N=\{N_i^+,N_i^-\}$ is isomorphic to, though not identical with, that discussed in references 1 and 2. With respect to parity $\mathcal{P}$, $\mathcal{P}N_i^\pm\mathcal{P}^{-1}=N_i^\mp$. The mass term $m\bar\psi\psi$ is not invariant under this group. A four-quark interaction Lagrangian invariant under this group is

$$\mathcal{L}_{\text{int}}=g(\bar\psi\gamma_5\psi)^2,$$

which is equivalent to

$$\mathcal{L}_{\text{int}}=\tfrac{1}{8}g[(S+P+T)-(V+A)]$$

under Fierz transformation. [The terms $(S+P+T)$ and $(V+A)$ are, respectively, $\mathcal{L}_I$ and $\mathcal{L}_I'$ of I.[8] It was noted there that they are separately invariant under U(6).] Physical implications of this symmetry have already been studied in references 1 and 2; a characteristic of this symmetry is that a boson supermultiplet exhibits a parity doubling structure, i.e., the supermultiplet $(\underline{6},\underline{6}^*)\oplus(\underline{6}^*,\underline{6})$ for instance contains nonets of vector, axial vector, scalar, and pseudoscalar mesons. The difference of this W(6) from the one discussed in references 1 and 2 appears in the transformation proper-

Table I. Transformation properties of bilinear covariants under the algebras M and N.

Covariant	Parity	Transformation under M	Transformation under N
$\bar{\psi}(\gamma_i-\delta_{i0}\lambda_0)\psi$	+	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$
$\bar{\psi}\lambda_0\psi$	+	$(\underline{1},\underline{1})$	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$
$\bar{\psi}\gamma_5(\lambda_i-\delta_{i0}\lambda_0)\psi$	−	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$
$\bar{\psi}\gamma_5\lambda_0\psi$	−	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$	$(\underline{1},\underline{1})$
$\bar{\psi}\gamma_0(\lambda_i-\delta_{i0}\lambda_0)\psi$	+	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$
$\bar{\psi}\gamma_0\lambda_0\psi$	+	$(\underline{1},\underline{1})$	$(\underline{1},\underline{1})$
$\bar{\psi}\gamma_a\lambda_i\psi$	−	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$
$\bar{\psi}\gamma_0\gamma_5\lambda_i\psi$	−	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$
$\bar{\psi}\gamma_a\gamma_5\lambda_i\psi$	+	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$
$\bar{\psi}\sigma_{ab}\lambda_i\psi$	+	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$
$\bar{\psi}\sigma_{0a}\lambda_i\psi$	−	$(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$	$(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$

ties of the bilinear covariants. The new transformation properties are listed in Table I. The structure of meson supermultiplets also follows from this table. The kinetic energy and the mass term in the Lagrangian both contribute to an intrinsic breaking of this W(6) (in the sense of I) that transforms like a component of the representation $(\underline{6},\underline{6}^*)\oplus(\underline{6}^*,\underline{6})$.

The group W(6) generated by the algebra $M=\{M_i^+, M_i^-\}$ is also not identical with the one discussed in references 1 and 2.

A four-quark interaction Lagrangian invariant under this group is evidently

$$\mathcal{L}_{\text{int}}=g(\bar{\psi}\psi)^2,$$

which, upon Fierz transformation, becomes

$$\mathcal{L}_{\text{int}}=\tfrac{1}{8}g[(S+P+T)+(V+A)].$$

The mass term $m\bar{\psi}\psi$ is invariant under this group. The algebra M is parity preserving, in the sense that $[M_i^\pm,\mathcal{P}]=0$. In the representation of the Dirac spinor in which "big" and "small" components, B and S, are separated, γ_0 is diagonal:

$$\begin{pmatrix}B\\S\end{pmatrix}=\frac{1\pm\gamma_0}{2}\psi.$$

It can be easily shown that the two commuting algebras $\{K_i^+\}$ and $\{M_i^-\}$ generate independent U(6) transformations in S and B. In Table I, we show the transformation properties of bilinear Dirac covariants under M. The meson supermultiplet $(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$ with $L=0$ consists of particles of even parity, including octets of normal and abnormal scalar mesons and two nonets of axial vector mesons. The supermultiplet $(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$ with $L=0$ contain particles of odd parity: two nonets of pseudoscalar mesons and two nonets of vector mesons. The kinetic part of the Hamiltonian, which breaks W(6) symmetry, transforms according to $(\underline{6},\underline{6}^*)\oplus(\underline{6}^*,\underline{6})$ whereas the mass term does not break this W(6).[9]

Taking together the results of I and of the present paper we see that there are "three roads" to W(6) symmetry. The first "road" passes through the unitary trick of Weyl and it is not clear whether it leads to a destination within the realm of quantum theory. The other two are routed via the large noncompact group $M(12)$,[10] and they indeed both stay within the boundaries of quantum theory. They differ in the parity structure of their supermultiplets. It is an important experimental question to see which (if any) of these roads is actually traveled on by nature.

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[1]K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 13, 698 (1964); in the following this paper will be referred to as I. Please note that $\mathcal{L}_M$ of I, though not W(6) invariant, is nevertheless GL(6) invariant.

[2]R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964).

[3]H. Weyl, Classical Groups (Princeton University

Press, Princeton, New Jersey, 1946), p. 265. The unitary trick is intimately connected with the Euclidean formulation of quantum field theories. See, e.g., J. Schwinger, Phys. Rev. 115, 721 (1959).

[4]We wish to thank Professor M. Gell-Mann for an interesting discussion on this point.

[5]It cannot be stressed too strongly that this subgroup is not identical with the physical Lorentz group. The generators of this subgroup are the "spin" part S_{ij} (as opposed to the "orbital" part L_{ij}) of the generators $M_{ij}=L_{ij}+S_{ij}$ of the Lorentz group. To the extent that S_{ij} is approximately conserved [which is implied by the approximate U(6) invariance], so is $L_{ij}=M_{ij}-S_{ij}$ (M. Gell-Mann, private communication). In a state in which $\langle L_{ij}\rangle=0$, we may utilize the transformation properties under U(6) to deduce the spin of the state.

[6]From here on, whenever we write a set of generators G_i we always refer to a group of transformations $\exp(ia_iG_i)$ with real parameters a_i.

[7]This is a consequence of the Baker-Hausdorff theorem; see, for example, D. Finkelstein, Commun. Pure Appl. Math. 8, 245 (1955). In a more pedestrian way the invariance of $\bar{\psi}\psi$ $[\bar{\psi}\gamma_5\psi]$ under the transformations (4a) [(4b)] can be checked by direct calculation.

[8]We follow here the customary notations from the Fermi theory of beta decay.

[9]All W(6) and GL(6) groups considered in our work contain (as has been emphasized in I and in this paper) a U(6) subgroup with generators $\rho^0\otimes J_i$. This subgroup leaves the mass term invariant. The fact that the mass term does not violate U(6) invariance has been noted independently by M. A. B. Bég and A. Pais, to be published.

[10]Upon application of the unitary trick $M(12)$ leads to U(12). This symmetry could also play a role only in Euclidean field theory. Whether this has anything to do with physics in our Minkowskian world is an open question.

SU(6) AND SEMILEPTONIC INTERACTIONS

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In this note we study some consequences of the assumptions that the effective strongly interacting particle currents of both vector (V) and axial vector (A) kinds associated with the semileptonic processes each transform according to the adjoint representation of the group SU(6). We discuss furthermore the implications of more restrictive dynamical assumptions which relate to each other the adjoint representation for the V current and the one for the A current. In this latter context we also encounter the question of the SU(6) completion of the Goldberger-Treiman relations. In accordance with the general considerations[1] about the interpretation of SU(6), we restrict ourselves for the present to the low-frequency limit of these effective currents, taking into account only effects up to and including the first order in v/c.

(i) Vector current. Up to this order we must consider two kinds of terms: (a) the weak charge term proportional to the Fermi constant G_V which gives the allowed Fermi transitions, and (b) the weak magnetism term.[2] In the spirit of the proportionality assumptions between these terms and the corresponding electromagnetic ones, our assumptions will here be the straight transcriptions from those made earlier[3] for the electromagnetic case. Thus we postulate that the weak charge operator transforms like an $(\underline{8},\underline{1})$ member of a $\underline{35}$ and the weak magnetic moment operator like an $(\underline{8},\underline{3})$ member of a $\underline{35}$. Again, as for electromagnetism, we do not assume that the same $\underline{35}$ representation appears in both cases. The meaning of this last proviso will be discussed in more detail elsewhere.[4]

(ii) Axial vector current. At low frequencies we have here only the Gamow-Teller transition term proportional to G_A. This term is now assumed to transform like an $(\underline{8},\underline{3})$ member of still another $\underline{35}$.

With these specifications, we can now write down the effective low-frequency four-point vertex for the interaction between leptons and strongly interacting particles. We consider specifically the interaction with the baryons of the $\underline{56}$ representation[5] of SU(6), which may be written as[6]

$$3B_{\alpha\beta\gamma}{}^{\dagger}(p_2)B^{\alpha\beta\delta}(p_1)C_{\delta}{}^{\gamma}(q), \tag{1}$$

$$C_{\delta}{}^{\gamma}(q)=\frac{G_V}{\sqrt{2}}[\delta_i^{\,j}(L_0)_A{}^B+i\mu_W(\vec{\sigma}\cdot\vec{q}\times\vec{L}_A{}^B)_i{}^j]+\frac{3G_A}{5\sqrt{2}}(\vec{\sigma}\cdot\vec{L}_A{}^B)_i{}^j, \tag{2}$$