value zero. The observation of very few events of this type would be sufficient to clarify this point.

As a final point let us remark that any increase in the statistics for the three best known reactions (6) would decrease the uncertainties in the parameters determined from them and thus in our numerical predictions. This would therefore allow more stringent comparisons between the predicted and the experimental branching ratios.

We would like to acknowledge helpful discussions with Professor L. M. Brown and Professor R. H. Capps. One of us (V.D.S.) is grateful to Dr. R. R. Silbar for a private communication on this subject.

†Work supported in part by the National Science Foundation.

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[1]V. De Santis, Phys. Rev. Letters 13, 646 (1964).

[2]The notation is slightly different from that of reference 1.

[3]While Eqs. (8.1) and (8.2) determine x^2 unambiguously, the value of y^2 depends upon the sign of x. We choose the positive sign because it leads to a set of predictions in better agreement with the experiments. Since only y^2 appears in the branching ratios, the sign of y is unimportant.

[4]A. H. Rosenfeld et al., Rev. Mod. Phys. 36, 977 (1964).

[5]H. K. Ticho, Proceedings of the International Conference on Fundamental Aspects of Weak Interactions (Brookhaven National Laboratory, Upton, New York, 1964), p. 410.

CP-NONCONSERVING DECAY $K_1{}^0 \to \pi^+ + \pi^- + \pi^0$ †

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(Received 12 February 1965)

In our paper[1] on the absolute decay rate $\Gamma_2(+-0)$ for $K_2{}^0 \to \pi^+ + \pi^- + \pi^0$, we made the observation that the time distribution of our 16 $\pi^+\pi^-\pi^0$ events is completely compatible with $\Gamma_1(+-0) = 0$, where $\Gamma_1(+-0)$ is the rate for $K_1{}^0 \to \pi^+ + \pi^- + \pi^0$. Thus our results are consistent with CP invariance.[2] In reference 1 we imposed the constraint $\Gamma_1(+-0) = 0$ in obtaining the result $\Gamma_2(+-0) = (2.90 \pm 0.72) \times 10^6\ \mathrm{sec}^{-1}$.

We have discovered that two good events were inadvertently omitted from that paper.[3] Adding these two events to the sample of reference 1, we find that $\Gamma_1(+-0)$ is still consistent with zero. Our corrected result is $\Gamma_2(+-0) = (3.26 \pm 0.77) \times 10^6\ \mathrm{sec}^{-1}$, still in good agreement with the prediction $\Gamma_2(+-0) = (2.87 \pm 0.23) \times 10^6\ \mathrm{sec}^{-1}$ of the $\Delta I = \frac{1}{2}$ rule.

The discovery[4] that CP invariance may not hold in neutral kaon decay admits the possibility that $\Gamma_1(+-0)$ is of the same order of magnitude as $\Gamma_2(+-0)$.[5] In this paper we reanalyze our 18 events without the assumption that $\Gamma_1(+-0)$ is zero, and thus without the assumption of CP invariance.

Let a_1 and a_2 denote the complex amplitudes for $K_1{}^0$ and $K_2{}^0$ decay into $\pi^+ + \pi^- + \pi^0$, where $K_1{}^0$ and $K_2{}^0$ refer to the short- and long-lived decay eigenstates; let x and y denote the real and imaginary parts of $a_1/a_2 = x + iy$. Then for K^0 produced at time $t = 0$ via the reaction $\pi^- + p \to \Lambda + K^0$, the total decay rate into $\pi^+ + \pi^- + \pi^0$ has the form[6]

$$\Gamma(+-0) = \tfrac{1}{2}|a_2|^2 |1 + (x + iy)\exp(-t/2\tau_1 + imt)|^2, \quad (1)$$

where $|a_2|^2 = \Gamma_2(+-0)$, $|a_1|^2 = \Gamma_1(+-0)$, $m = m_2 - m_1$, and where we can (for our experiment) take the $K_2{}^0$ lifetime to be effectively infinite as far as the time dependence of (1) is concerned. For each event we construct an a priori decay probability p_i based on Eq. (1)[7] and normalized to unity for decay between $t = 0$ and $t = T_i$, where T_i is the potential time for the event.[8] We then construct the likelihood function $L(x, y) = \prod_i p_i$. From a contour plot of $L(x, y)$ we obtain the results[9-11]

$$x = +0.25 \pm 0.65, \quad y = +1.00 \pm 0.65. \quad (2)$$

Figure 1 shows a comparison of the data with the time distribution corresponding to the result (2).[12]

In the above analysis we made use of only the time distribution of the 18 events. We now

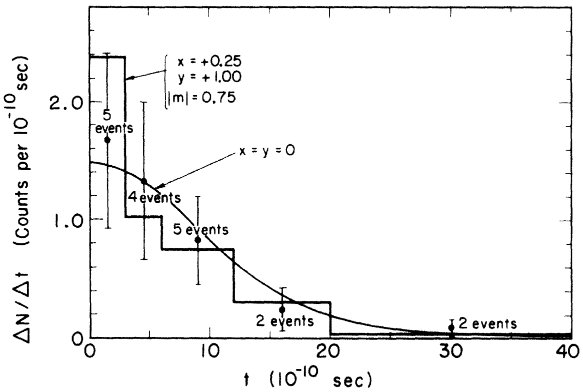

FIG. 1. Time distribution. The smooth curve is the geometrical detection efficiency $\epsilon(t)$, normalized so that it represents dN/dt for the 18 events, if they are due to K_2^0 only, i.e., $x=y=0$. The histogram corresponds to dN/dt predicted by the maximum-likelihood result (2). The points with error flags are the observed events.

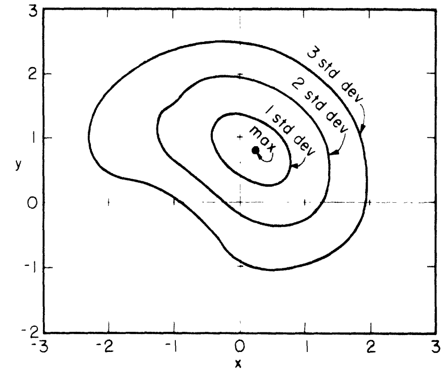

FIG. 2. Contours of equal likelihood for $x=\mathrm{Re}(a_1/a_2)$ and $y=\mathrm{Im}(a_1/a_2)$, where a_1 and a_2 are the amplitudes for K_1^0 and K_2^0 decay into $\pi^+\pi^-\pi^0$. The contours labeled 1, 2, and 3 std dev correspond to a decrease in the likelihood function $L_1(x,y)$ by factors $e^{-1/2}$, $e^{-4/2}$, and $e^{-9/2}$ from $L_1(\max)$.

reanalyze these events with the additional hypothesis that $\Gamma_2(+-0)$ satisfies the $\Delta I=\frac{1}{2}$ rule, which predicts $\Gamma_2(+-0)=(2.87\pm0.23)\times10^6\ \mathrm{sec}^{-1}$.[13] We construct a likelihood function $L_1(x,y)$ by multiplying the likelihood $L(x,y)$ by the Poisson probability $e^{-\bar{n}}\bar{n}^n/n!$; here $n=18$ is our observed total number of events, and $\bar{n}=\bar{n}(x,y)$ is the total predicted number of events calculated by combining the $\Delta I=\frac{1}{2}$ rule, the size of our sample of K^0, the time distribution (1), and our geometrical detection efficiency $\epsilon(t)$, which is the smooth curve plotted in Fig. 1. In Fig. 2 we show a contour plot of $L_1(x,y)$. From this plot we obtain the results

$$x=+0.25\pm0.55,\quad y=+0.80\pm0.55. \qquad (3)$$

The most likely value for $x^2+y^2\equiv\Gamma_1(+-0)/\Gamma_2(+-0)$ is 0.70. If we integrate over the relative phase of a_1 and a_2 in the likelihood function we obtain a probability distribution for $\Gamma_1(+-0)/\Gamma_2(+-0)$.

We conclude that the odds are 9 to 1 that $\Gamma_1(+-0)/\Gamma_2(+-0)$ is less than 5. Our best estimate for the amplitude ratio $a_1(+-0)/a_2(+-0)\equiv x+iy$ is given by Eq. (3). We cannot rule out $a_1(+-0)/a_2(+-0)=0$.

We are grateful to Sheldon L. Glashow for stimulating discussions, and to Luis W. Alvarez for his interest and support and for valuable comments.

†Work performed under the auspices of the U. S. Atomic Energy Commission.

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[1]D. Stern, T. O. Binford, V. G. Lind, J. A. Anderson, F. S. Crawford, Jr., and R. L. Golden, Phys. Rev. Letters 12, 459 (1964).

[2]In $K(\text{neutral})\to\pi^++\pi^-+\pi^0$, pion angular-momentum states higher than S states are strongly suppressed by angular-momentum barrier-penetration factors. If the pions are in S states, $\pi^+\pi^-\pi^0$ has $CP=-1$; hence $K_1^0\to\pi^++\pi^-+\pi^0$ is forbidden by CP invariance.

[3]In the notation of Table I of reference 1, they are event 1 845 161: $\chi^2(\text{prod})=3.4$, $\chi^2(\text{dec})=1.7$, $p_{K^0}(\text{lab})=590\pm9$, $t_{K^0}=5.31$, $T_{K^0}=14.7$; event 1 849 320: 1.1, 1.1, 628±8, 21.1, 31.1.

[4]J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964); see also A. Abashian, R. J. Abrams, D. W. Carpenter, G. P. Fisher, B. M. K. Nefkens, and J. H. Smith, Phys. Rev. Letters 13, 243 (1964).

[5]See, for example, S. L. Glashow, Phys. Rev. Letters 14, 35 (1965).

[6]Equation (1) is not exact; it is based on the approximation $a=1$ and $b=0$ in

$$K^0=a(|K_1\rangle+|K_2\rangle)/\sqrt{2}+b(|K_1\rangle-|K_2\rangle)/\sqrt{2},$$

whereas actually $a-1$ and b are each of order 10^{-3} according to reference 4. For the experiment reported here this contributes a negligible correction to Eq. (1), because we can determine a_1/a_2 only to about ±1, not to $\pm10^{-3}$.

[7]We use $\tau_1=0.89\times10^{-10}$ sec, and $|m|=0.75\times10^{10}\ \mathrm{sec}^{-1}$ (which is $0.67/\tau_1$). The choice 0.75 is our weighted average of the values summarized in Table I of T. Fujii, J. V. Jovanovich, F. Turkot, and G. T. Zorn, Phys. Rev. Letters 13, 253 (1964). Our result (2) is, however, quite insensitive to the precise value we choose for $|m|$, for $|m|$ between $(0.4\text{ and }1.1)\times10^{+10}$

$\sec^{-1}$. For example, for $|m|=0.50$, we obtain $x=+0.6\pm0.7$, $y=+1.1\pm0.7$; for $|m|=1.00$ we find $x=0.1\pm0.7$, $y=+0.9\pm0.7$.

[8]The decay times t_i are listed in Table I of reference 1. The potential times T_i for the 18 events are as follows (in the order of that Table, and in units of 10^{-10} sec): 11.88, 24.24, 15.65, 8.12, 7.72, 4.13, 6.92, 17.62, 13.06, 11.76, 9.83, 8.59, 14.20, 3.99, 153.0, 22.4, 14.7, and 31.1.

[9]The quoted errors correspond to a decrease of the likelihood function $L(x,y)$ by a factor $e^{-1/2}$ from its maximum value. We prefer to give our results in terms of x and y rather than in terms of $\Gamma_1/\Gamma_2=x^2+y^2$ and the phase $\varphi=\arg(a_1/a_2)$, because the likelihood function $L(x,y)$ is to a fair approximation given by $L=f(x)f(y)$, where $f(x)$ and $f(y)$ are nearly Gaussian in shape. The probability distribution for Γ_1/Γ_2 is, on the contrary, very non-Gaussian.

[10]The sign of x is determined (in principle) by this experiment, but the sign of y is not separable from that of m_2-m_1. Thus our result (2) for y is actually $[(m_2-m_1)/|m_2-m_1|]y=+1.00\pm0.65$. In writing (2) we take m_2-m_1 to be positive.

[11]If the result (2) were known to be exact, we would have to assign 18 % of the observed counts to $K_1{}^0$ decay. Then our measured value of $\Gamma_2(+-0)$ would be corrected by a factor of 0.82 to $\Gamma_2(+-0)=0.82\times(3.26\pm0.77)=(2.65\pm0.63)\times10^6\ \sec^{-1}$.

[12]Inspection of Fig. 1 suggests that (within the large statistical uncertainties) $x=y=0$ fits the data slightly better than the maximum-likelihood result (2). This slight apparent inconsistency is mainly due to the fact that in $L(x,y)$ we make use of the individual decay times t_i and potential times T_i of the 18 events; each t_i is correlated with its own T_i in the factor p_i. The function $\epsilon(t)$ in Fig. 1 is, on the contrary, based on a smoothed distribution of potential times obtained from several thousand associated production events.

[13]The prediction $\Gamma_2(+-0)=2.87\times10^6\ \sec^{-1}$ is based on a weighted average of results for $\Gamma_+(+00)$ compiled in Table I of G. Alexander and F. S. Crawford, Jr., Phys. Rev. Letters 9, 68 (1962).