However, it must be recognized that for such a theory the Lorentz invariance is clear only when it is looked upon as an unquantized C-number theory.

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Note added in proof.—After this note had been written we received a preprint of a paper by K. T. Mahanthappa and E. C. G. Sudarshan, in which these authors have also employed the operator $\vec{\Sigma}$ for constructing a U(6) algebra.

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DECAY $Y_1^*(1660) \to Y_0^*(1405) + \pi$†

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Thus far, particles of the same known spin and parity have been successfully assigned into SU(3) multiplets by using the Gell-Mann–Okubo mass formula.[1] Attempts to classify particles whose spins and parities are not well established into particular multiplets on the basis of the mass formula may lead to contradictory assignments, as in the case of the $Y_1^*(1660)$.[2,3] Additional information on the multiplet assignment of a particle may be derived from its decay modes. Where the SU(3)-breaking interactions can be neglected, SU(3) gives definite predictions of branching ratios and selection rules for the decay of a member of a given multiplet into members of other multiplets.[4] We report here experimental evidence for the decay $Y_1^*(1660) \to Y_0^*(1405) + \pi$, which can be used as evidence that the $Y_1^*(1660)$ is a member of an octet if $Y_0^*(1405)$ is assumed to be a unitary singlet.

The data on the $Y_1^*(1660)$ or[5] $\Sigma(1660)$ decay modes were obtained from an analysis of the following reactions:

$$K^- + p \to \Sigma^+ + \pi^+ + \pi^- + \pi^-, \tag{1}$$

$$K^- + p \to \Sigma^- + \pi^+ + \pi^+ + \pi^-, \tag{2}$$

$$K^- + p \to \Lambda + \pi^+ + \pi^0 + \pi^-. \tag{3}$$

Reactions (1) and (2) give information on the decay $\Sigma(1660) \to \Lambda(1405) + \pi$, and Reaction (3) is used to give an upper limit on the amount of $\Sigma(1660) \to \Sigma(1385) + \pi$.

The reactions were studied on film from a recent exposure of the Berkeley 72-inch hydrogen bubble chamber to 2.45-, 2.65-, and 2.70-BeV/c K^- beams.[6] The sample of film obtained at each momentum has a K^- path length corresponding to one event expected for a cross section of 0.5, 0.15, and 0.3 μb, respectively. The above reactions were analyzed using the Alvarez-group program system.[7] Events that fit more than one hypothesis were designated as ambiguous if the ratio of the χ^2 probabilities for the hypotheses was less than three to one. For these events the higher probability hypothesis was chosen, and these events are included in our data. The location of the ambiguous events has been omitted on Fig. 1 but is shown in Fig. 2, in order to demonstrate that they are not responsible for the effects which are discussed below. No ambiguous events happen to fall in the sample used in Figs. 3 and 4.

The results reported here are based on an analysis of the distributions which include all events, corresponding to no cutoff in the Σ track length. In the analysis of Reactions (1) and (2), a minimum-length cutoff of 0.5 cm has been considered for the projected length of the sigma track in a plane perpendicular to the optical axes. Events having the projected length greater than 0.5 cm were corrected by an approximate weighting factor to compensate for

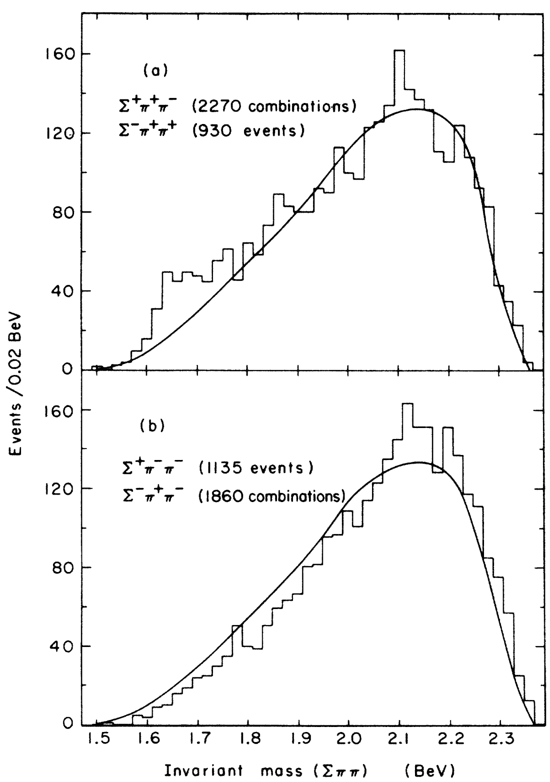

FIG. 1. Distribution of $\Sigma\pi\pi$ invariant mass with an estimate of non-Σ(1660) events. (a) Net charge positive; (b) net charge negative.

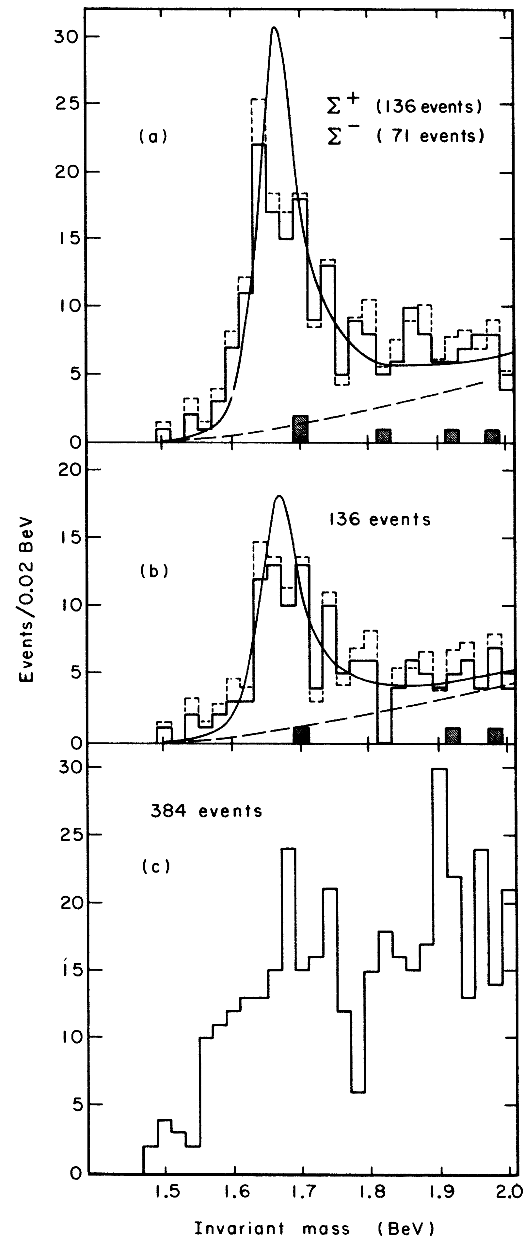

FIG. 2. Distribution of $Y\pi\pi$ invariant mass for $\cos\theta^* < -0.9$. On (a) and (b) the dashed histogram represents the weighted events with a projected length greater than 0.5 cm for the Σ. The shaded events are ambiguous. The continuous curve represents our fit; the dashed curve is the contribution of background estimated from the fit. (a) All $(\Sigma\pi\pi)^+$ events; (b) $\Sigma^+\pi^+\pi^-$ events only; (c) $\Lambda^0\pi^+\pi^0$ events.

the effect of the cutoff. The various histograms containing only these weighted events were then compared with the corresponding histograms containing all events (i.e., assuming no cutoff in Σ track length, and hence all events having a weight of one). The histograms were found to be statistically equivalent, indicating that there is no bias from the scanning efficiency for short Σ's. To illustrate, we also show some of the weighted distributions corresponding to the 0.5-cm cutoff [Figs. 2(a), 2(b), 4(a), and 4(b)].

It is out of the question that pionic contamination has affected the results presented below. Actually, to understand the effect of the $\simeq 20\%$ π^- contamination in the beam,[6] we have studied interactions obtained when the bubble chamber was exposed to pure π^- beams of about the same momenta. For a number of π^- comparable to that present as contamination in our total K^- exposure, we found only 153 events of the same topology as Reactions (1) and (2).

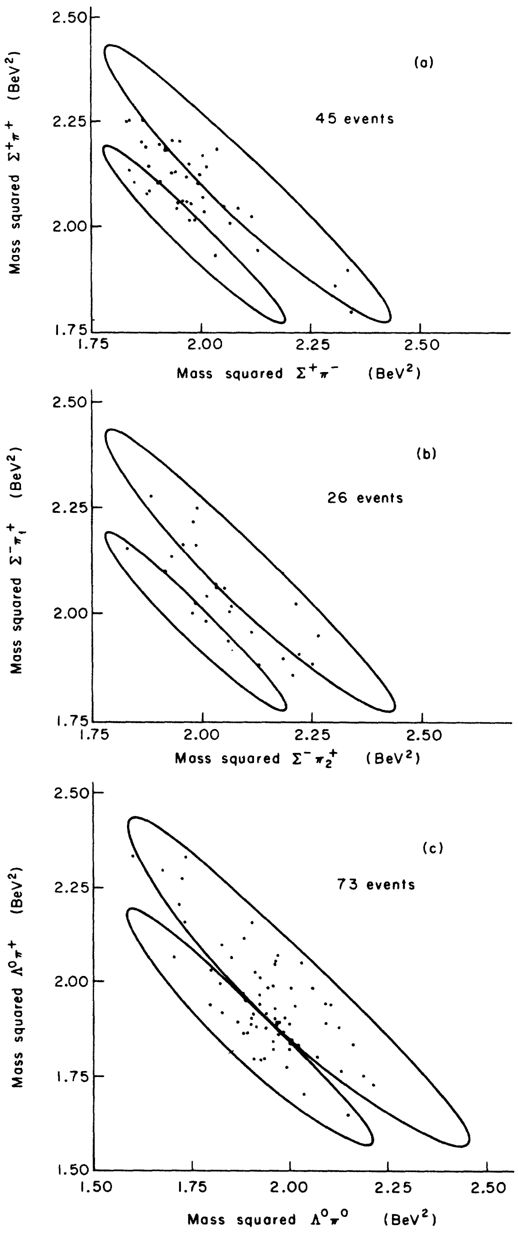

FIG. 3. Dalitz plot for the sample of events selected for the study of $\Sigma(1660)$ decay. (a) $\Sigma^+\pi^+\pi^-$ events; (b) $\Sigma^-\pi^+\pi^+$ events; (c) $\Lambda^0\pi^+\pi^0$ events.

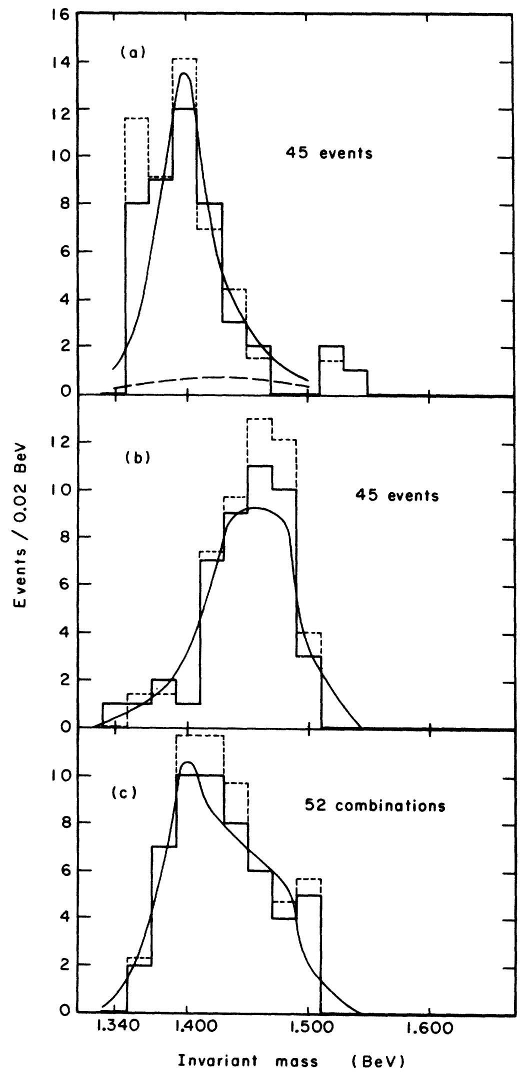

FIG. 4. Distribution of $\Sigma\pi$ invariant mass of the events appearing in Fig. 3. (a) $\Sigma^+\pi^-$ invariant mass; (b) $\Sigma^+\pi^+$ invariant mass; (c) $\Sigma^-\pi^+$ invariant mass. On (a), the continuous curve represents our best fit, described in the text; the dashed curve is the estimated contribution of non-$\Lambda(1405)$ events. On (b) and (c), the curves represent the expected distribution if all events are due to the $\Lambda(1405)$ resonance.

These events were measured and then analyzed as if they were K-induced events. Of the 153 events, none fitted hypothesis (2) and only two fitted (1). The calculated invariant masses of the $\Sigma^+\pi^+\pi^-$ combinations all fell above 2 BeV in these two cases.

Figure 1(a) shows the invariant-mass distribution for the $\Sigma\pi\pi$ particle combinations from Reactions (1) and (2), with an over-all charge of +1. For the events from Reaction (1), two combinations per event are plotted. The curve shown represents the combined effect, averaged over the three incident beam momenta, of phase space plus the effects of the $\Lambda(1405)$ and $\Lambda(1520)$ resonances produced in the $(\Sigma^\mp\pi^\pm)$ system of particles.[8] A definite excess of events is seen about 1660 MeV. The corresponding mass spectrum for the $\Sigma\pi\pi$ system with an over-all charge of -1 [Fig. 1(b)] does not show the same feature. That is, there is some indication of $\Sigma(1660)$ production in the $(\Sigma\pi\pi)^+$ system and none in the $(\Sigma\pi\pi)^-$ system, suggesting that the $\Sigma(1660)$ may be produced peripherally.

The center-of-mass production angular distribution with respect to the incident K^- for the $(\Sigma\pi\pi)^+$ particle combinations lying in the mass range 1620 to 1700 MeV has been examined. The production angle, θ^*, is defined by $\cos\theta^* = -K^-\cdot\pi^-$, where K^- and π^- are unit vectors along the direction of the incident K^- and the π^- not included in the mass combination, respectively, in the over-all c.m. system. From the angular distribution we conclude that the $\Sigma(1660)$ is produced at very low momentum transfers (-0.08 to $+0.06$ BeV^2).

Figure 2(a) shows the $(\Sigma\pi\pi)^+$ mass distribution for those events having $\cos\theta^* \leq -0.9$. The presence of the $\Sigma(1660)$ is clearly seen. The $\Sigma^+\pi^+\pi^+\pi^-$ events are each represented twice on Fig. 1(a), but there are only five events where both $\Sigma^+\pi^+\pi^-$ combinations have $\cos\theta^* < -0.9$. The lowest mass combination for these events lies above 1860 MeV, so that these events cannot perturb the analysis of the $\Sigma(1660)$.

The solid curve of Fig. 2(a) represents a best fit to the data for a distribution of the following form: $a\times$[modified phase space including effects of $\Lambda(1405)$ and $\Lambda(1520)$ resonances in the $(\Sigma\pi)^0$ system] $+b\times$[Breit-Wigner form for $\Sigma(1660)$].[9] From this fit we determine the phase-space background in the region $1620 < M(\Sigma\pi\pi) < 1700$ MeV to be $6\pm1\%$. Thus the angular selection $\cos\theta^* < -0.9$ and this mass criterion make a relatively clean sample of $\Sigma(1660)$. It will be used for the study of $\Sigma(1660)$ decay. The subset of events in Fig. 2(a) which are produced in Reaction (1) are plotted separately in Fig. 2(b). These are of particular importance for the decay analysis, since there is only one neutral Σ-π combination.

The histogram analogous to those in Figs. 2(a) and 2(b) for Reaction (3) is presented in Fig. 2(c) for approximately the same bubble-chamber exposure. In this case the cutoff $\cos\theta^* < -0.9$ does not separate $\Sigma(1660)$ clearly from the background. Those events with mass of $\Lambda\pi^+\pi^0$ between 1620 and 1700 MeV represent an upper limit on the production of $\Sigma(1660)$ in our $\cos\theta^*$ interval.

For our sample of $\Sigma(1660)$ events, as defined above, we form Dalitz scatter plots of $\Sigma^+\pi^+\pi^-$ in Fig. 3(a), $\Sigma^-\pi^+\pi^+$ in 3(b), and $\Lambda\pi^+\pi^0$ in 3(c). The closed curves represent the boundary defined by the upper and lower mass limits imposed for the $\Sigma\pi\pi$ system. Figures 4(a) and (b) show the number of events versus the invariant mass (rather than mass squared) for the $\Sigma^+\pi^+\pi^-$ events of Fig. 3(a). The histogram of the $\Sigma^+\pi^-$ mass [Fig. 4(a)] shows a pronounced peak near 1405 MeV. If we exclude the three events that lie in the $\Lambda(1520)$ region and which could belong to the background, a fit to this histogram using a two-parameter distribution of the form [Breit-Wigner $\Sigma(1660)$]$\times[a+b\times$(Breit-Wigner terms for $\Lambda(1405)$)] gives a branching ratio

$$\frac{\Sigma(1660)\to\Lambda(1405)+\pi}{\Sigma(1660)\to\text{all }\Sigma\pi\pi} = 90_{-18}^{+10}\%.$$

The continuous curve of Fig. 4(a) represents this best fit; the dashed curve represents the "phase-space" contribution to it. This result is, of course, consistent with the hypothesis that all events belong to the $\Lambda(1405)$ resonance.

The $\Sigma^+\pi^+$ mass distribution [Fig. 4(b)] shows an enhancement around 1450 MeV. The solid curve represents the expected distribution if all decays were via $\Lambda(1405)+\pi^+$. This result then constitutes a check of consistency for the hypothesis $\Sigma(1660)\to\Lambda(1405)+\pi$. The fact that such a decay would reflect itself at 1450 MeV in the $\Sigma^+\pi^+$ mass was first noticed by Alston et al.[10]

We can rule out the reciprocal hypothesis that we see no $\Lambda(1405)$ but a $\Sigma^+\pi^+$ resonance of mass 1450 MeV which is reflected at 1405 MeV in the $\Sigma^+\pi^-$ distribution, by computing the expected ratio for $\Sigma^-\pi^+\pi^+$ to $\Sigma^+\pi^+\pi^-$ decay modes under the assumption of $I=1$ for the $\Sigma^*(1660)$ and $I=2$ for the Σ-π system. This ratio would be less than 2/49 for a maximum contribution of interference effects in the $\Sigma\pi\pi$ system. The observed decay branching ratio of 26/45 is totally in disagreement with this number.

The projected $\Sigma^-\pi^+$ mass distributions are shown in Fig. 4(c) with each event from Fig. 3(a) plotted twice since there are two $\Sigma\pi$ combinations of zero total charge. The solid curve represents the expected shape of the distribution under the assumption that all events proceed through $\Lambda(1405)+\pi^+$, and that interference effects may be ignored. The curve and histogram appear to be compatible.

It is worth while pointing out that decay of $\Sigma(1660)$ into $\Lambda(1405)+\pi^+$ does not necessarily imply a branching ratio of one for the Σ^- to Σ^+ decay modes. We would expect that $\Sigma^-\pi^+$ interference effects of the type discussed by Dalitz and Miller[11] will enhance or suppress that decay channel relative to the $\Sigma^+\pi^-$.

We have investigated the possibility that the 1660-MeV enhancement is merely a result of a reflection caused by strongly peripheral production in the reaction

$$K^- + p \rightarrow \Lambda(1405) + \rho^0,$$

together with the angular selection of the π^-. We find that only seven events of Fig. 3(a) have both a $(\Sigma^+\pi^-)$ combination between 1350 and 1460 MeV, and $(\pi^+\pi^-)$ in the ρ-meson mass range (700 to 800 MeV).

There remains the possibility that we are actually observing only the $\Sigma\pi$ decay mode of the $\Sigma(1385)$, which is known to be as high as 9% of the total decay.[12] If such a phenomenon were happening, it would mean that our sample of 71 events comes from 710 $\Sigma(1660)^+$ decays into $\Sigma(1385)^0+\pi^+$. Isospin conservation would permit us to expect 710 events of the type $\Sigma(1660)^+ \rightarrow \Sigma(1385)^+ + \pi^0$. Taking into account the correction for neutral decay of the Λ, we should see at least $1420\times0.9\times\frac{2}{3}=850$ $\Lambda^0\pi^0\pi^+$ events in our sample selected from Fig. 2(c) between 1620 and 1700 MeV. The important result to be learned from Fig. 3(c)—the Dalitz plot for those events from Fig. 2(c) that lie in the $\Sigma(1660)$ region—is that one finds only a comparable number and not 12 times the number of events plotted in Figs. 3(a) and 3(b). From the 73 events on Fig. 3(c), we can estimate the maximum possible contribution to the $\Sigma(1660)$ decay rate from the mode $\Sigma(1385)+\pi$. Comparing that number to the 71 events found in the $\Sigma^\pm\pi^\mp$ and applying corrections for neutral decays and the branching ratio of the $\Sigma(1385)$, we can set a lower limit of 0.8 for the branching ratio $\Sigma(1660)\rightarrow\Lambda(1405)+\pi$ to $\Sigma(1660)\rightarrow\Sigma(1385)+\pi$. The uncertainty of this ratio is attributed to (a) the amount of $\Sigma(1660)$ events actually occurring in the $\Lambda\pi\pi$ channel and (b) the differences in rejection efficiency of events in each channel. The latter do not exceed 25%.

In summary, we conclude that the $\Sigma\pi\pi$ decay of the $\Sigma(1660)$ is dominated by the intermediate state $\Lambda(1405)+\pi^+$, and that it is at least comparable to the decay $\Sigma(1385)+\pi$.

Under the assumption that the $\Lambda(1405)$ is the member of a unitary singlet,[13] the decay mode $\Sigma(1660)\rightarrow\Lambda(1405)+\pi$ is forbidden by SU(3) interactions if the $\Sigma(1660)$ belongs to any multiplet other than an octet. Certainly if that decay mode proceeds via SU(3)-breaking interactions, it is difficult to understand why a forbidden process would have a rate higher than or comparable to $\Sigma(1660)\rightarrow\Sigma(1385)+\pi$, which is allowed by SU(3) interactions up to a high order of multiplet assignment for the $\Sigma(1660)$. Therefore, unless we invoke some mixing,[14] we conclude that either the $\Sigma(1660)$ is a member of an octet, or the $\Lambda(1405)$ is not a unitary singlet.

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JOHNSON-TREIMAN RELATIONS FROM VECTOR-MESON EXCHANGE*

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It is perhaps worth pointing out that the relations in the SU(6) theory between Kp and πp total cross sections, derived by Johnson and Treiman,[1] follow also from more conventional alternative assumptions:

(I) If exchanges of octet vector-meson Regge trajectories give rise to the cross-section differences involved in the relations, then both Johnson-Treiman relations,

$$\tfrac{1}{2}[\sigma(K^+p)-\sigma(K^-p)]=[\sigma(K^0p)-\sigma(\bar{K}^0p)] \qquad (1a)$$

and

$$\tfrac{1}{2}[\sigma(K^+p)-\sigma(K^-p)]=\sigma(\pi^+p)-\sigma(\pi^-p), \qquad (1b)$$

are obtained, provided only that the vector-meson trajectories have pure F-type (conserved-current) coupling to the baryons.

(II) Exchange of the unitary-singlet vector-meson state makes no contribution to the differences (1). Furthermore, φ-ω mixing leaves the relations unchanged in the zeroth-order approximation. The possible octet of even-signature boson Regge trajectories suggested by Pignotti[2] also makes no contributions to the differences in (1), the neutral members being even under charge conjugation.[3,4]

Assertion (I) may immediately be verified by using the Clebsch-Gordan coefficients from SU(3) to relate coupling constants.[5] The total cross-section predictions arise from the usual application of the optical theorem to the Regge-exchange amplitude.

Alternatively, we may directly see the equivalence of the SU(6) theory and the vector-meson-exchange model for these predictions in the following way: The meson-baryon scattering-matrix elements are written in terms of the forms $\bar{B}(1)B(2)M(3)M(4)$, where $\bar{B}$, B, and M are the standard tensors for the baryon (56) and meson (35) states in the SU(6) space.[6-9] Of the four possible SU(6)-invariant forms, three are symmetrical under the interchange of the [spin, SU(3)] indices of $M(3)$ with those of $M(4)$. Such symmetrical terms give equal contributions to the forward-scattering amplitudes of $m+b\to m+b$ and of $\bar{m}+b\to\bar{m}+b$, where b is any particular baryon and m is any particular pseudoscalar meson. Thus only the single antisymmetrical form contributes to the differences in (1),

$$A(m+b\to m+b)-A(\bar{m}+b\to\bar{m}+b)$$
$$\sim\bar{B}^{ABC}(1)B_{ABD}(2)$$
$$\times[M_C{}^E(3)M_E{}^D(4)-M_E{}^D(3)M_C{}^E(4)]. \qquad (2)$$

By inserting Sakita's expression for $\bar{B}^{ABC}B_{ABD}$ [reference 9, Eq. (3)], it is easily shown that the form for the coupling of the baryon octet, N, to the pseudoscalar octet, Π, is (all space