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$$G_{uv} = \partial_u V_v - \partial_v V_u, \quad g_{uv} = \partial_u \varphi_v - \partial_v \varphi_u.$$

[12]This problem is clearly related to the absence of a $\rho\pi\pi$ coupling in an SU(6)-invariant theory.

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Σ-Λ MASS DIFFERENCE IN THE SU(6)-SYMMETRIC BOOTSTRAP MODEL

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A natural generalization of the Gell-Mann–Okubo mass formula to SU(6) may be obtained from the hypothesis that the SU(3)-octet terms in the meson and baryon mass (or mass-squared) operators correspond to the 35-fold representation of SU(6).[1] The experimental data concerning meson masses are consistent with this generalization.[1] In the SU(6)-symmetric baryon bootstrap model, the baryons are μB (meson-baryon) compounds.[2] One might suppose that if the baryon mass splitting is assumed to originate from the meson mass splitting in this model, the "35 rule" for the mesons would lead to a corresponding 35 rule for the baryons. However, the baryon 35 rule predicts a zero Σ-Λ mass difference, and is therefore in strong disagreement with experiment.[1] The purpose of this note is to examine the strong (isospin-preserving) baryon mass splitting in the SU(6)-symmetric bootstrap model, with particular reference to the Σ-Λ mass difference.

We regard the components of the mass-squared matrix for the baryon supermultiplet as components of a vector, following the technique introduced by Glashow and discussed in detail by the author.[3,4] There are three components of the baryon strong mass-splitting vector that transform as SU(3) octets; these are

$$A_{1/2} = \tfrac{1}{2}(\Xi - N), \tag{1a}$$

$$B_{1/2} = \tfrac{1}{5}(N + \Xi + \Lambda - 3\Sigma), \tag{1b}$$

$$A_{3/2} = \tfrac{1}{5}(\Omega + \Xi^* - 2N^*), \tag{1c}$$

where the baryon symbol denotes the average over the appropriate isotopic spin multiplet of the diagonal elements of the mass-squared matrix. Since there is no strong mixing for the baryon supermultiplet, these diagonal elements may be identified with the corresponding experimental square masses. The normalization is such that for pure octet-type mass splitting, $\Lambda - \Sigma = 2B_{1/2}$ and $\Omega = N^* = 3A_{3/2}$.

A 35-fold meson multiplet of SU(6) consists of a spin-zero octet, a spin-one octet, and a spin-one singlet. Again there are three octet-type mass-splitting components. Two of them, denoted by b_0 and b_1, refer to mass splitting within the spin-0 and -1 octets; the equations for these components may be obtained by replacing the baryon symbols in Eq. (1b) by the appropriate meson symbols. The third component may be written

$$b_{\mathrm{mix}} = (2)^{-1/2}[(\omega\varphi) + (\varphi\omega)], \tag{2}$$

where $(\omega\varphi) = (\varphi\omega)$ denotes the off-diagonal term in the mass-squared matrix that connects the SU(3) singlet with the isoscalar member of the spin-one octet.

In the SU(6) bootstrap model, the wave function of the baryon j may be written in the form $\Psi_j = \Psi(B_j) = \sum_{kl} C_{jkl} B_k \mu_l$, where the C_{jkl} are Clebsch-Gordan coefficients of SU(6). In the "probability matrix approximation," derived in previous references, the baryon mass splitting is assumed to be given by a simple function of the meson mass splitting and the coefficients of the $\Psi(B_j)$.[5] Particle-mixing effects may be included in a straightforward generalization of this approximation. The baryon and meson mass-squared matrices are written in the respective forms $M^2=M_0^2(1+\Delta)$ and $m^2=m_0^2(1+\delta)$, where M_0^2 and m_0^2 are constants identified with the experimental average square masses within the baryon and meson multiplets. The generalized probability matrix approximation is the following linear equation for Δ:

$$\Delta_{ij}=\alpha\langle\Psi_i|\delta'|\Psi_j\rangle+(1-\alpha)\langle\Psi_i|\Delta'|\Psi_j\rangle, \qquad (3)$$

where δ' is defined to be the direct product of δ operating in the space of the mesons and the identity operator in the space of the baryons, and Δ' is defined similarly. The coefficient α is a positive number between zero and one that measures the relative importance of meson and baryon mass deviations to the position of the bound-state pole. If the Δ, δ', and Δ' are diagonal, this equation is identical to that derived in reference 5.

We define fractional octet-type components by the relations $\Delta_{A1/2}=A_{1/2}/M_0^2$, $\delta_1=b_1/m_0^2$, etc. If the Clebsch-Gordan coefficients of SU(6) are used,[2] application of Eq. (3) to the octet components yields the results

$$\Delta_{A1/2}=\alpha[(4/9)\delta_1-(2/45)\delta_{\text{mix}}]+(1-\alpha)[(13/45)\Delta_{A1/2}-(2/9)\Delta_{B1/2}+(4/9)\Delta_{3/2}], \qquad (4a)$$

$$\Delta_{B1/2}=\alpha[(1/15)\delta_0-(7/45)\delta_1-(2/45)\delta_{\text{mix}}]+(1-\alpha)[-(8/45)\Delta_{A1/2}+(1/9)\Delta_{B1/2}+(8/45)\Delta_{3/2}], \qquad (4b)$$

$$\Delta_{3/2}=\alpha[(1/10)\delta_0+(19/90)\delta_1-(1/9)\delta_{\text{mix}}]+(1-\alpha)[(4/45)\Delta_{A1/2}+(2/45)\Delta_{B1/2}+(29/45)\Delta_{3/2}]. \qquad (4c)$$

The condition that the octet-type mass splitting corresponds to the 35-fold SU(6) representation implies the relations[6,7]

$$\delta_0=\delta_1, \qquad (5a)$$

$$\delta_{\text{mix}}=-2\delta_1, \qquad (5b)$$

$$\Delta_{3/2}=\Delta_{A1/2}, \qquad (6a)$$

$$\Delta_{B1/2}=0. \qquad (6b)$$

It can be seen from Eqs. (4a)-(4c) that the validity of the above two conditions for the δ implies the validity of the two Δ conditions. Since the Σ-Λ mass difference is not small, Eq. (6b) is not satisfied experimentally.

In the SU(6)-symmetric baryon bootstrap model, the effective spins of the pseudoscalar and vector mesons are reversed.[2] [We associate the X_0 particle with the SU(3) singlet, for the reasons discussed in reference 2.] The component δ_{mix} refers to $X^0\eta$ mixing, rather than $\omega\varphi$ mixing. The value of neither $(X^0\eta)$ nor $(\omega\varphi)$ is known experimentally. However, there is no simple and reasonable symmetry argument leading to a predicted nonzero value of $(X^0\eta)$, so we set δ_{mix} equal to zero. The components δ_0 and δ_1 are associated with the vector and pseudoscalar mesons, respectively. It is seen from Eq. (4b) that the condition $\delta_{\text{mix}}=0$, together with the experimentally observed condition, $\delta_0\cong\delta_1>0$, leads to a predicted negative $\Delta_{B1/2}$, and hence a positive Σ-Λ mass difference. In fact, if the α terms dominate the right side of Eqs. (4a) and (4b), the predicted ratio $(\Sigma-\Lambda)/(\Xi-N)$ is $\frac{1}{5}$, as compared to the experimental value of 0.21.

In order to make more explicit predictions of the baryon mass ratios, we abandon the linear approximation, and use instead the simple formula suggested previously by the author, i.e.,[8]

$$M_i^{-2}=\lambda\textstyle\sum_j(\Pi_{ij}\mu_j^{2}). \qquad (7)$$

The symbol M_i denotes the mass of the baryon i, μ_j is the total mass of the meson-baryon state j, Π_{ij} is the probability of the state j in the wave junction $\Psi(B_i)$ that corresponds to SU(6) symmetry, and λ is a normalization constant. The linear terms of Eq. (7) are equivalent to Eq. (3), with $\alpha=m_0/(m_0+M_0)$. The isoscalar member of the vector-meson octet is taken as the experimental φ particle. We have calculated the right-hand side of Eq. (7) from experimental meson and baryon masses; the results are compared with experiment in Table I. The agreement is satisfactory for the mass splittings within the octet and decuplet. On the other hand, the calculated decuplet-octet mass splitting is seen to be only about half as large as the experimental value. It appears that relativistic spin-dependent effects

Table I. Calculated and experimental squares of baryon mass ratios.

Ratio	Calculated value	Experimental value
Σ/Λ	1.12	1.15
N/Λ	0.72	0.71
Ξ/Λ	1.38	1.40
N^*/Y^*	0.79	0.80
Ξ^*/Y^*	1.22	1.22
Ω/Y^*	1.46	1.47
Y^*/Λ	1.28	1.53

may play a large role in determining this mass difference.

The eigenvalues of the baryon probability matrix corresponding to the $(1-\alpha)$ terms of Eqs. (4a) through (4c) are 11/15, 17/45, and $-1/15$. These may be identified with the SU(6) representations of dimensions 35, 405, and 2695, respectively. The fact that 11/15 is close to one leads to enhancement of both the strong and electromagnetic mass splitting that corresponds to the representation $\underline{35}$.

It has been shown in previous references that rough agreement between calculated and experimental baryon mass-splitting values occurs also in the standard reciprocal bootstrap model, in which the vector mesons are not involved.[5,9] For this reason one cannot argue that the observed baryon mass splitting is strong evidence for SU(6). Our principal conclusion is that the Σ-Λ mass difference, which is somewhat of a mystery in a quark model of SU(6),[7] arises naturally in the SU(6)-symmetric bootstrap model.

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LORENTZ-COVARIANT SU(6), PARTICLE-ANTIPARTICLE ALGEBRAS, AND SUPERMULTIPLET STRUCTURE

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With the remarkable success of SU(3),[1] culminating in the prediction and discovery of Ω^- resonance, a considerable effort is being made to search for a higher symmetry to explain certain empirical facts which are not encompassed by SU(3). The effort has been roughly in three directions: The first has been to search for a bigger internal-symmetry group; the examples are SU(4),[2] W(3),[3] and R(8).[4] The second approach consists in looking for the algebra which the currents satisfy; the examples are $SU^+(3) \otimes SU^-(3)$, or $\overline{W}(3)$, which leads to the parity doubling of mesons,[5] and further enlargement of the same to the F- and D-type currents.[6] Already in this case parity, which is normally considered as a property of space, has been