†John Tyndall Fellow (1962-1964).

[1]$\rho = 0.780 \pm 0.025$: R. Plano, Phys. Rev. 119, 1400 (1960). This report contains references to earlier work.

[2]$\rho = 0.751 \pm 0.034$: M. Block, E. Fiorini, E. Kikuchi, G. Giacomelli, and S. Ratti, Nuovo Cimento 23, 1114 (1962).

[3]$\rho = 0.661 \pm 0.016$: J. Barlow, P. Booth, L. Carroll, G. Court, J. Davies, D. Edwards, R. Johnson, and J. Wormold, Proc. Phys. Soc. (London) 84, 239 (1964).

[4]L. Michel, Proc. Phys. Soc. (London) A63, 514 (1950).

[5]T. D. Lee and C. N. Yang, Phys. Rev. 105, 1671 (1957); 108, 1611 (1957); 119, 1410 (1960). The second and third articles investigated nonlocal effects. In particular, if a vector boson mediated weak interactions, then ρ would be increased by $\frac{1}{3}(m_\mu/M_W)^2$. This experiment would thus not be sensitive to the effects of a boson mass, M_W, greater than 1 BeV.

[6]A. Salam, Nuovo Cimento 5, 299 (1957).

[7]L. Landau, Nucl. Phys. 3, 127 (1957).

[8]The earliest systems which made use of the sound wave associated with the spark discharge were described by B. Maglić and F. Kirsten, Nucl. Instr. Methods 17, 49 (1962); and H. Fulbright and D. Kohler, University of Rochester Report No. NYO-9540, 1961 (unpublished).

[9]Previous reports of the detailed operation of the system used in this experiment have been given in M. Bardon, J. Lee, J. Peoples, A. M. Sachs, and G. Sutter, Bull. Am. Phys. Soc. 8, 389 (1963); and M. Bardon, J. Lee, P. Norton, J. Peoples, and A. M. Sachs, Proceedings of the International Conference on Filmless Spark Chamber Techniques, Geneva, Switzerland, March 1964, 64-30/P41 (to be published).

[10]An experiment measuring the kinetic energy of the muon in pion decay [Walter H. Barkas, Wallace Birnbaum, and Frances M. Smith, Phys. Rev. 101, 778 (1956)] is the best previous measurement. It sets an upper limit of $7M_e$ on the muon neutrino mass.

[11]T. Kinoshita and A. Sirlin, Phys. Rev. 113, 1652 (1959), which contains references to earlier work.

MESON-MESON COUPLINGS IN AN SU(6)-INVARIANT THEORY*

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The SU(6) theory of Gürsey and Radicati[1] has been successfully applied to assigning particles to supermultiplets,[1,2] calculating mass formulas,[1,3] the electromagnetic[4] and weak[5] interactions, and the meson-baryon interaction.[1,6,7] In this note we shall be concerned with the application of SU(6) to the interactions of the bosons among themselves. Since the pseudoscalar meson octet (P), the vector-meson octet (V), and the vector-meson singlet (φ) are joined in the 35-dimensional representation of SU(6), we expect to be able to derive relations between the SU(3)-independent VVV, VVP, VPP, and $V\varphi P$ couplings, and thus to correlate processes such as φ, ω, and ρ decay.

Due to the nonrelativistic nature of SU(6) and the difficulty of providing a clear relativistic foundation,[8] we must resort to models in order to calculate results involving physical particles. Thus, for example, Bég and Pais[6] introduce the notion of Lorentz completion and use it to discuss the meson-baryon coupling, while Mahanthappa and Sudarshan[9] consider SU(6)⊗O(3) to obtain the correct, parity-conserving, interaction. For the boson interactions we shall assume[6] that we must search for an SU(6) structure in the static limit of zero momentum transfer. Since all the usual boson trilinear interactions vanish in this limit it would appear necessary, as in the meson-baryon case, to use Lorentz completion to go to the next order in v/c. This procedure is not as simple here, though, since the Lorentz-complete meson matrix M given by Bég and Pais is a function of the parameter f^T/f^V, which is not determined by SU(6), and the meson couplings depend on this parameter. In fact, for $f^T = 0$ even the boosted trilinear meson couplings vanish. This ratio may be fixed by appealing to SU(12) invariance, but in the light of these uncertainties it is interesting to explore alternative methods to find the coupling.

For bosons, however, we do have at our disposal an alternative method to that of Lorentz completion. We may utilize the fact that in a theory involving vector and pseudoscalar mesons the requirement that the former be coupled to a conserved current leads to quartic and higher order terms in the interaction Lagrangian whose coupling strengths are not independent of the trilinear ones.[10] Furthermore, the quartic terms persist in the zero-energy limit and we shall require that they have an SU(6)-invariant structure. In this way we obtain conditions relating the coupling constants of the independent quartic terms, and since the same coupling constants appear in the trilinear terms, we will have relations between the various trilinear couplings.

We consider the SU(3)- and Lorentz-invariant interaction Lagrangian[10,11]

$$\mathcal{L}_I = -g_1[\tfrac{1}{2}G_{\mu\nu}\cdot(V_\mu \times V_\nu) + \tfrac{1}{4}(V_\mu \times V_\nu)\cdot(V_\mu \times V_\nu) - V_\mu \cdot(P \times \partial_\mu P) - \tfrac{1}{2}(V_\mu \times P)\cdot(V_\mu \times P)]$$
$$+\tfrac{1}{4}(g_2/m)[\epsilon_{\alpha\beta\gamma\delta}\{(G_{\alpha\beta}{}^*G_{\gamma\delta})\cdot P + 2(V_\alpha \times V_\beta)\cdot(G_{\gamma\delta}{}^*P) + (V_\alpha \times V_\beta)^*(V_\gamma \times V_\delta)\cdot P\}]$$
$$+\tfrac{1}{2}(g_3/m)[\epsilon_{\alpha\beta\gamma\delta}\{(G_{\alpha\beta}\cdot P)g_{\gamma\delta} + (V_\alpha \times V_\beta)\cdot P g_{\gamma\delta}\}] + g_4(P\cdot P)^2, \tag{1}$$

which, together with the usual free Lagrangian, maintains the conditions

$$\partial_\mu V_\mu = \partial_\mu \varphi_\mu = 0 \tag{2}$$

as a consequence of the equations of motions. Taking the Fourier transform of (1) and setting all momenta equal to zero, we obtain

$$-g_1[\tfrac{1}{8}f_{ab'c'}\epsilon_{ij'k'}f_{ab''c''}\epsilon_{ij''k''}V_{j'}{}^{b'}V_{k'}{}^{c'}V_{j''}{}^{b''}V_{k''}{}^{c''} + \tfrac{1}{2}f_{ab'c'}f_{ab''c''}P^{b'}V_i{}^{c'}P^{b''}V_i{}^{c''}]$$
$$+g_2[f_{ab'c'}\epsilon_{ij'k'}d_{ab''c''}V_{j'}{}^{b'}V_{k'}{}^{c'}V_i{}^{b''}P^{c''}] + g_3[f_{ab'c'}\epsilon_{ij'k'}V_{j'}{}^{b'}V_{k'}{}^{c'}\Phi_i P^a] + g_4(P^aP^a)^2, \tag{3}$$

which we expect to exhibit an SU(6) structure. [We have anticipated this requirement in (3) by taking $q^0/m = 1$.] There are two independent, invariant, quartic couplings of a single SU(6) 35-plet, namely,

$$M_A[ABC]M_B M_{A'}[A'B'C]M_{B'} \tag{4a}$$

and

$$(M_A M_A)^2. \tag{4b}$$

We are using Cartesian coordinates for the 35-plet where

$$M_i = \varphi_i, \quad M_a = P^a, \quad M_{ai} = V_i{}^a. \tag{5}$$

The notation is explained in Table I where $[ABC]$, the symmetric vector-coupling coefficient [analogous to d_{abc} of SU(3)], as well as the structure constants $\{ABC\}$, are tabulated.

Two difficuties now arise in the attempt to construct (3) from a linear combination of (4a) and (4b). The first is that the $PVPV$ term of (3) is antisymmetric if we simultaneously exchange the SU(6) labels of a P and V. But (4a) and (4b) are symmetric under this interchange and so cannot give rise to this term. Indeed, Table I shows that we would expect this term to arise from an f-type coupling of the form

$$M_A\{ABC\}M_B M_{A'}\{A'B'C\}M_{B'}, \tag{6}$$

which, however, vanishes in the present case of identical boson multiplets. Thus in the SU(6)-invariant theory we cannot obtain the terms from the pseudoscalar-meson contribution to the conserved unitary-spin current. However, we do know the magnitude of this term relative to the vector-meson contribution to the unitary current and shall assume that it does arise properly in the relativistic theory.[12] Furthermore, although the coupling (4a) does give rise to all the terms of (3) except $PVPV$, it contains additional couplings as well. This is not surprising since the conditions (2) are clearly rela-

Table I. The structure constants, $\{ABC\}$, and symmetric vector coupling coefficients $[ABC]$, of SU(6); f_{abc} and d_{abc} are defined by Gell-Mann.[a] An SU(6) index A takes on the values a, i, or ai, where a runs from one to eight and i from one to three. The SU(6) generators Λ_A are normalized to $\mathrm{Tr}\Lambda_A\Lambda_B = 2\delta_{AB}$ and satisfy $[\Lambda_A, \Lambda_B] = 2i\{ABC\}\lambda_C$, $\{\Lambda_A, \Lambda_B\} = \tfrac{2}{3}\delta_{AB} + 2[ABC]\lambda_C$.

$\{a,b,c\} = 2^{-1/2}f_{abc}$	$[a,b,c] = (\tfrac{1}{2})^{1/2}d_{abc}$
$\{i,j,k\} = 3^{-1/2}\epsilon_{ijk}$	$[a,bi,j] = (\tfrac{1}{3})^{1/2}\delta_{ab}\delta_{ij}$
$\{a,bj,ck\} = 3^{-1/2}f_{abc}\delta_{ik}$	$[a,bi,cj] = (\tfrac{1}{2})^{1/2}d_{abc}\delta_{ij}$
$\{i,bj,ck\} = (\tfrac{1}{3})^{1/2}\epsilon_{ijk}\delta_{bc}$	$[ai,bj,ck] = (\tfrac{1}{2})^{1/2}\epsilon_{ijk}f_{abc}$
$\{ia,bj,ck\} = (\tfrac{1}{2})^{1/2}\epsilon_{ijk}d_{abc}$	

tivistic in nature, and so the SU(6) static theory might be expected to give rise to terms which ultimately violate them.

We shall assume that the unknown coupling constants in (3) are correctly given by (4a). We ignore (4b) since it only reproduces the last term in (3), nor can it be used to cancel the unwanted terms of (4a). These assumptions lead us to the relations

$$g_1 = -2g_2 = -6^{1/2}g_3 = -8g_4, \tag{7}$$

obtained by comparing (3) with (4a). The connection of these to the usual coupling constants is

$$g_1 = 4\gamma_{\rho\pi\pi}, \tag{8a}$$

$$g_2/m = \tfrac{1}{2}\sqrt{3}f_{\rho\omega_0\pi}, \tag{8b}$$

$$g_3/m = \tfrac{1}{2}\times 6^{1/2}f_{\rho\varphi_0\pi}, \tag{8c}$$

$$g_4 = 4\pi\lambda, \tag{8d}$$

where $\gamma_{\rho\pi\pi}$ and $f_{\rho\omega_0\pi}$ are given by Gell-Mann, Sharp, and Wagner,[13] and λ by Oneda, Kim, and Kaplan.[14] $f_{\rho\varphi_0\omega\pi}$ is defined analogously to $f_{\rho\omega_0\pi}$.

We see that the constant $\gamma_{\rho\pi\pi}{}^2/4\pi$ is the only unknown appearing above, and it has been estimated to be $\frac{1}{2}$ by Gell-Mann, Sharp, and Wagner using the data on ρ decay; thus SU(6) allows us to relate the φ and ω decay widths and the coefficient λ to the ρ width. We will take the symmetry breaking into account in the usual way by using the correct linear combination of ω_0 and φ_0 as given by the mass formula and by using the correct phase-space factors. We take m in (8b) and (8c) to be the mass of the ρ.

As a first approximation we have

$$\omega = (\tfrac{1}{3})^{1/2}\omega_0 + (\tfrac{2}{3})^{1/2}\varphi_0, \tag{9a}$$

$$\varphi = -(\tfrac{2}{3})^{1/2}\omega_0 + (\tfrac{1}{3})^{1/2}\varphi_0, \tag{9b}$$

i.e., the particles ω_U and φ_U of Bég and Singh[3] which are known to give a very good value for the mixing angle.[15] Then, since from Eqs. (7) and (8) $f_{\varphi_0\rho\pi} = \sqrt{2}f_{\omega_0\rho\pi}$, we have

$$f_{\varphi\rho\pi} = 0,$$

and the φ does not decay into ρ and π. This fact was previously noted to be a prediction of SU(6) by Gürsey, Pais, and Radicati,[2] and by Lipkin[16] on more general grounds, and it is gratifying that our model does not violate this requirement. Now, using the model of reference 14 for ω decay, we obtain

$$\Gamma(\omega\to 3\pi) = 5\ \text{MeV},$$

to be compared with the experimental value of 10 MeV.[17] The constant λ is determined from (7) and (8) as

$$\lambda = -0.13,$$

to be compared with

$$\lambda = -0.18$$

quoted in reference 15. Thus the agreement of the theory with experiment is quite good.

In order to estimate the actual φ-decay width, we must use a better value for the mixing angle. Here we mention that the assumption $f=g$ of Bég and Singh[3] may be combined with $f\approx 0$ [i.e., $(\pi-\rho)+(K^*-K)\approx 0$], after which their Formula (30) becomes

$$(\omega-\rho)(\varphi-\rho) = \tfrac{4}{3}(K^*-\rho)(\omega+\varphi-2K^*). \tag{10}$$

This mass formula has been derived perviously[15] from other assumptions and is quite well satisfied. Using (10) to give the $\varphi-\omega$ mixing angle, we find

$$\Gamma(\varphi\to\rho\pi)\approx 0.1\text{-}0.3\ \text{MeV},$$

again in excellent agreement with experiment.[17]

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[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964); B. Sakita, Phys. Rev. 136, B1756 (1964).

[2]A. Pais, Phys. Rev. Letters 13, 175 (1964); F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

[3]T. K. Kuo and Tsu Yao, Phys. Rev. Letters 13, 415 (1964); Mirza A. Baqi Bég and Virendra Singh, Phys. Rev. Letters 13, 418 (1964).

[4]M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964); B. Sakita, Phys. Rev. Letters 13, 643 (1964).

[5]C. H. Chan and A. Q. Sarker, Phys. Rev. Letters 13, 731 (1964).

[6]M. A. B. Bég and A. Pais, to be published.

[7]Richard H. Capps, Phys. Rev. Letters 14, 31 (1964); J. G. Belinfante and R. E. Cutkosky, Phys. Rev. Letters 14, 33 (1964).

[8]R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964). K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. Lee, Phys. Rev. Letters 13, 698 (1964); 14, 48 (1965). S. Okubo and R. E. Marshak, Phys. Rev. Letters 13, 818 (1964); R. Delbourgo, Abdus Salam, and J. Strathdee, to be published.

[9]K. T. Mahanthappa and E. C. G. Sudarshan, to be

published.

[10]V. I. Ogievetskj and I. V. Polubarinov, Ann. Phys. (N.Y.) 25, 358 (1963). [The terms of the type VVP, $V\varphi P$ in Eq. (1) are easily derived using the method of this reference.]

[11]For SU(3) we follow the notation introduced by C. Dullemond, to be published: $f_{abc}\alpha^b\beta^c = \alpha\times\beta$; $d_{abc}\alpha^b\beta^c = \alpha*\beta$; $\alpha^a\beta^a = \alpha:\beta$, where α and β are octets in Cartesian coordinates, and f_{abc} and d_{abc} are tabulated by Gell-Mann [M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report No. CTSL-20, 1961 (unpublished)];

$$G_{uv} = \partial_u V_v - \partial_v V_u, \quad g_{uv} = \partial_u \varphi_v - \partial_v \varphi_u.$$

[12]This problem is clearly related to the absence of a $\rho\pi\pi$ coupling in an SU(6)-invariant theory.

[13]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1964).

[14]S. Oneda, Y. S. Kim, and L. M. Kaplan, Nuovo Cimento 34, 655 (1964).

[15]F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. 135, B467 (1964); J. Schwinger, Phys. Rev. 135, B816 (1964); I. S. Gerstein and M. L. Whippman, Phys. Rev. 137, B1722 (1965).

[16]H. J. Lipkin, Phys. Rev. Letters 13, 590 (1964).

[17]A. H. Rosenfeld et al., Rev. Mod. Phys. 36, 977 (1964).

Σ-Λ MASS DIFFERENCE IN THE SU(6)-SYMMETRIC BOOTSTRAP MODEL

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A natural generalization of the Gell-Mann–Okubo mass formula to SU(6) may be obtained from the hypothesis that the SU(3)-octet terms in the meson and baryon mass (or mass-squared) operators correspond to the 35-fold representation of SU(6).[1] The experimental data concerning meson masses are consistent with this generalization.[1] In the SU(6)-symmetric baryon bootstrap model, the baryons are μB (meson-baryon) compounds.[2] One might suppose that if the baryon mass splitting is assumed to originate from the meson mass splitting in this model, the "35 rule" for the mesons would lead to a corresponding 35 rule for the baryons. However, the baryon 35 rule predicts a zero Σ-Λ mass difference, and is therefore in strong disagreement with experiment.[1] The purpose of this note is to examine the strong (isospin-preserving) baryon mass splitting in the SU(6)-symmetric bootstrap model, with particular reference to the Σ-Λ mass difference.

We regard the components of the mass-squared matrix for the baryon supermultiplet as components of a vector, following the technique introduced by Glashow and discussed in detail by the author.[3,4] There are three components of the baryon strong mass-splitting vector that transform as SU(3) octets; these are

$$A_{1/2} = \tfrac{1}{2}(\Xi - N), \tag{1a}$$

$$B_{1/2} = \tfrac{1}{5}(N + \Xi + \Lambda - 3\Sigma), \tag{1b}$$

$$A_{3\ 2} = \tfrac{1}{5}(\Omega + \Xi^* - 2N^*), \tag{1c}$$

where the baryon symbol denotes the average over the appropriate isotopic spin multiplet of the diagonal elements of the mass-squared matrix. Since there is no strong mixing for the baryon supermultiplet, these diagonal elements may be identified with the corresponding experimental square masses. The normalization is such that for pure octet-type mass splitting, $\Lambda - \Sigma = 2B_{1/2}$ and $\Omega = N^* = 3A_{3/2}$.

A 35-fold meson multiplet of SU(6) consists of a spin-zero octet, a spin-one octet, and a spin-one singlet. Again there are three octet-type mass-splitting components. Two of them, denoted by b_0 and b_1, refer to mass splitting within the spin-0 and -1 octets; the equations for these components may be obtained by replacing the baryon symbols in Eq. (1b) by the appropriate meson symbols. The third component may be written

$$b_{\text{mix}} = (2)^{-1/2}[(\omega\varphi) + (\varphi\omega)], \tag{2}$$

where $(\omega\varphi) = (\varphi\omega)$ denotes the off-diagonal term in the mass-squared matrix that connects the SU(3) singlet with the isoscalar member of the spin-one octet.

In the SU(6) bootstrap model, the wave function of the baryon j may be written in the form $\Psi_j = \Psi(B_j) = \sum_{kl} C_{jkl} B_k \mu_l$, where the C_{jkl} are Clebsch-Gordan coefficients of SU(6). In the "probability matrix approximation," derived in previous references, the baryon mass splitting is assumed to be given by a simple function of the meson mass splitting and the coefficients of the $\Psi(B_j)$.[5] Particle-mixing effects may be included in a straightforward generalization of this approximation. The baryon and meson mass-squared