British Columbia. In this case, the assumed forms for the initial velocity distribution and the elastic-scattering and annihilation cross sections, σ_c and σ_a, were as follows:

$$f(v,0)=1, \quad v \leq V_{th},$$

$$=0, \quad v > V_{th},$$

where V_{th} is the positron velocity at the threshold energy for positronium formation, viz. 8.9 eV for the case of argon;

$$\sigma_c(v)=1.32\pi a_0^{\,2}V_{th}/v,$$

and

$$\sigma_a(v)=3.80\times10^{-6}\pi a_0^{\,2}(V_{th}/v)^{1.5}.$$

A more detailed presentation of both the experimental results and the extent to which the cross sections may be determined by such experiments will appear in a later publication.

The authors wish to express their gratitude to Mr. A. Fowler of the Computing Centre at the University of British Columbia for his invaluable aid in developing the programs required for this work, and also to Dr. L. de Sobrino for many stimulating discussions.

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*This work was supported by research grants from the National Research Council and the University of British Columbia.

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MEASUREMENT OF THE MOMENTUM SPECTRUM OF POSITRONS FROM MUON DECAY*

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(Received 16 February 1965)

Over the past decade many measurements have been made of the momentum spectrum of electrons from the decay of unpolarized muons.[1-3] The shape of this spectrum can be characterized by a number, ρ, the Michel parameter.[4] The two-component neutrino theory predicts that ρ is equal to $\frac{3}{4}$.[5-7] Bubble chamber experiments, which are relatively free of systematic errors, have shown agreement with this value of ρ within their 4% accuracy. In order to make a more precise measurement of the spectrum, we have performed an experiment using a magnetic spectrometer with sonic spark chambers on-line to an IBM 1401 computer. This technique[8,9] permitted analysis of a large number of events with high momentum resolution but without most of the systematic errors usually associated with magnetic spectrometers which use scintillation counters.

The experimental setup is shown in Fig. 1. Four single-gap spark chambers and all of the counters lie inside a large air-core magnet, so that the muon decay takes place in the region of uniform magnetic field. The field, pointing in the direction perpendicular to the plane of the figure, was corrected with current coils so that the average nonuniformity along a positron trajectory was about 0.05%, while the worst trajectories (near the outer radius) would meet nonuniformities not exceeding 0.1%.

A positive pion beam from the Columbia University Nevis synchrocyclotron is incident along the field direction and stopped in the 3-in.×3-in.×$\frac{1}{8}$-in. plastic scintillator target counter. About 80% of the decay muons stop and subsequently decay in this target. Approximately 5% of the resulting positrons reach

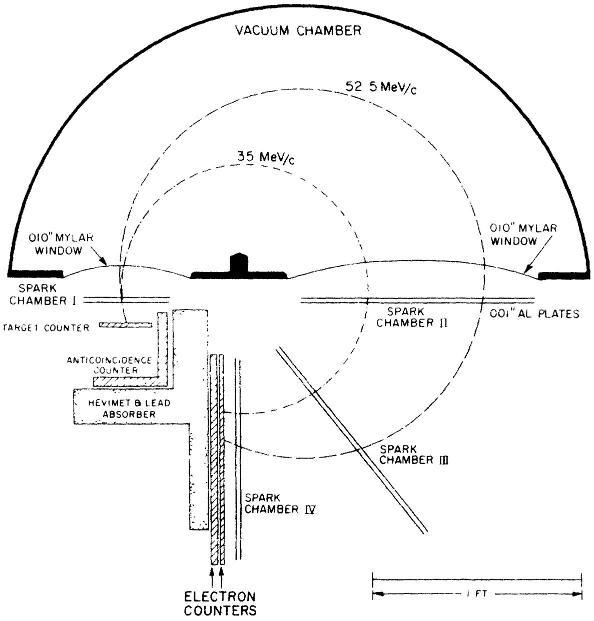

FIG. 1. Experimental arrangement. The entire setup is in a homogeneous magnetic field. The two circular trajectories have been drawn for the minimum and maximum positron momenta accepted at a field of 6.62 kG. The two 0.003-in. Mylar windows for each chamber, which are not shown, are the only other material in the path of the positrons.

the electron counters. Each stopping pion generates a 2-μsec gate delayed by 0.5 μsec. A fast coincidence between the target and electron counters (with no pulse in the anticoincidence counter) which falls within the gate triggers the four spark chambers. The pulse height from the target counter is recorded to determine the ionization energy loss of the emerging positron.

The momentum and angle of emission of the positron is determined from the positions of sparks in chambers I, II, and III. Chamber IV permits the exclusion of particles that may have been scattered by surrounding materials. The major part of the trajectory, between chambers I and II, is in vacuo. The 0.010-in.-thick Mylar windows to the vacuum tank and the thinner windows (0.003-in. Mylar) and foils (0.001-in. Al) of the first two chambers are near the 180-degree line of the spectrometer, where multiple scattering causes relatively little uncertainty in the momentum measurement.

Within each spark chamber there are four cylindrical piezoelectric microphones located near the corners but outside the plate areas,

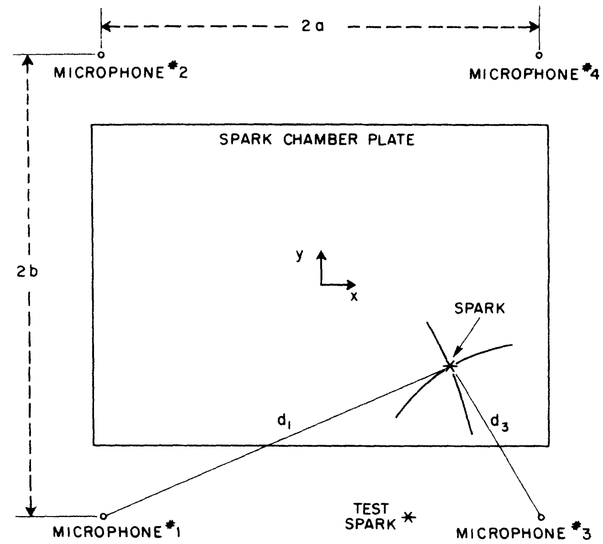

FIG. 2. Plan of a sonic spark chamber, showing the arrangement of the four microphones with respect to the plates. The construction is indicated for determining the position of a spark from the distances to two microphones.

as shown in Fig. 2. The time (~1 msec) between the spark trigger and the arrival of the sonic signal from the spark at each microphone is measured by scaling a 5-Mc/sec oscillator.

After each event the four-digit times for each of the 16 microphones and two digitized counter pulse heights are loaded serially into the memory of the IBM 1401. After 12 events a record is written on magnetic tape. The trajectory of each event is subsequently reconstructed, using an IBM 7094 computer. The limited computational facilities of the on-line 1401 were used for checks during the run and for extensive calculations during preliminary tests at low rates.

The method of obtaining spark coordinates from the transit times, t_i, of the sonic signals is indicated in Fig. 2. The times from two microphones, e.g., t_1 and t_3, are, in principle, sufficient to determine the position of a single spark. A third time is needed to indicate the presence of more than one spark, and the fourth time provides a consistency check.

The coordinates of the spark are determined by

$$x = (v^2/4a)[(t_1 + T)^2 - (t_3 + T)^2] = (v^2/4a)[(t_2 + T)^2 - (t_4 + T)^2],$$

and a similar expression for y, where v is the velocity of sound in the gas, $2a$ is the microphone separation, and T is introduced to compensate for the initially higher velocity of the shock wave ($vT \approx 5$ mm). Disagreement between two pairs of microphones in excess of 1 mm is taken to indicate the presence of more than one spark, and the event is rejected. The value of $v^2/4a$ is determined by comparing sonic measurements with spark locations measured in photographic calibrations at the beginning and end of the run. These calibrations were made for different temperatures and relative neon-helium concentrations of the gas. For each calibration the transit time was recorded for the sonic signal from a test spark originating at a pair of fixed tungsten needles in each chamber. Periodic measurements of the test spark were also made during the run to permit reference to the appropriate value of $v^2/4a$.

Including various checks, a total of 13×10^6 events were recorded, at a rate of up to 20 per second. At the two most useful magnetic field settings, 6.62 and 5.35 kG, 8×10^6 events were analyzed. Of these, 0.8×10^6 events are used in the present determination of the spectrum, since they reconstruct as events leaving the target counter within a solid angle region selected for 100% acceptance by the chamber system over a given momentum range.

The absolute accuracy and the resolution of momentum measurement can be determined from the sharp falloff of the spectrum near the endpoint. For events associated with a given range of energy loss in the target counter (i.e., pulse height), the falloff is consistent with an integrated Gaussian with standard deviation of 0.14 MeV. This effective resolution of the system is caused primarily by the statistical uncertainty in the pulse height and by multiple scattering in the thin windows and foils.

Considering the spectrum at 6.62 kG for each region of pulse height, the midpoint of the falloff can be plotted versus pulse height (Fig. 3). The straight line through such points can be extrapolated to zero pulse height, indicating the measured momentum of 52.62 ± 0.02 MeV/c for a positron starting at the upper edge of the target counter. Adding the correction of 0.19 MeV/c for momentum lost in material after the target and for the effect of the shape of the spectrum, a value of 52.81 ± 0.02 MeV/c is obtained. This should be compared with the maximum momentum of 52.83 MeV/c for

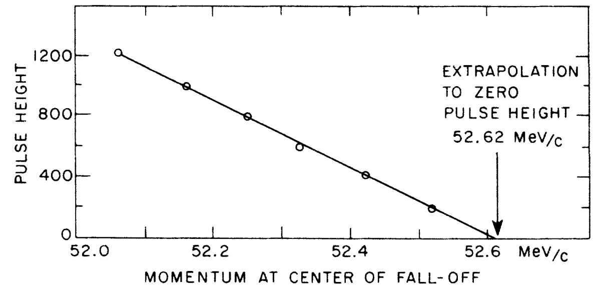

FIG. 3. The momentum of the center of the falloff at the high-energy end of the positron spectrum has been determined for each of six regions of pulse height in the target counter. A straight line extrapolated to zero pulse height indicates the maximum momentum of positrons emerging from the target counter, if we add energy losses in other materials along the trajectory giving 52.81-MeV/c end point.

the theoretical spectrum, assuming zero rest mass for the neutrinos. Alternatively, one can use the 0.02 ± 0.02 MeV/c difference to set a preliminary "upper limit" of five electron masses for the muon neutrino.[10]

Similar measurements at 7.51 and 7.95 kG yield extrapolated momenta of 52.64 and 52.63 MeV/c, respectively (to be compared with 52.62 $\pm$ 0.02), giving a good check on the momentum measurements in different parts of the spectrometer.

Figure 4(a) shows the combined data for magnetic field settings of 5.35 and 6.62 kG. The data in the overlap region agree within the statistical accuracy. The experimental points have been adjusted for ionization energy loss in the target, and the theoretical curve for $\rho=0.75$ has been corrected for internal radiative effects,[11] for bremsstrahlung in the target counter, windows, and foils, and for ionization loss in the windows and foils. A least-squares fit to the data gives a best value of $\rho=0.747$ [Fig. 4(b)].

Several of the runs to check systematic errors have not yet been analyzed, e.g., a spectrum with extra material above the target to check bremsstrahlung calculations, and a measurement of the $\pi\to e$ decays. In addition, calculations have not yet been completed for energy loss from the small fraction of delta rays which are not bent back into the counter and for higher order effects from multiple scattering. Although the statistical error is ±0.002, the uncertainty of the estimate of the remaining corrections combined with requirements of internal consistency of the data over various momentum and angular regions lead to larger uncertainty and a preliminary result of $\rho = 0.747 \pm 0.005$.

We wish to thank Dr. G. Sutter for help in the early phases of the experiment; F. Sippach for the design of the electronic system; G. Doremus for fabricating the chambers; J. Williams for computer programming; and C. Carlson, S. Herzka, B. Palatnick, and S. Stein for general assistance in the experiment.

*Work supported in part by the U. S. Office of Naval Research under Contract No. Nonr-266(72).

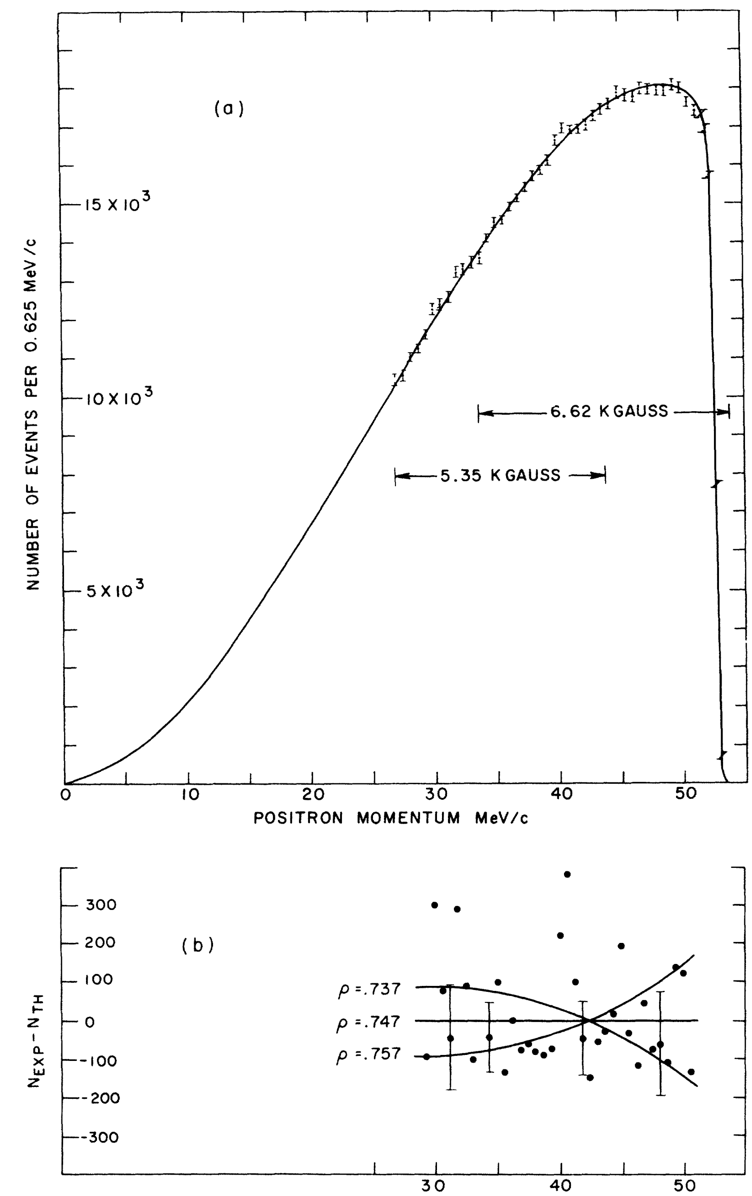

FIG. 4. (a) Experimental points for magnetic-field settings, normalized to the overlap region. The solid line is the theoretical spectrum for $\rho = 0.75$. The Michel spectrum,[4]

$$\rho(x)dx = \tfrac{1}{2}\{12x^2 - 12x^3 + \rho[(32/3)x^3 - 8x^2]\}dx,$$

where x is the positron momentum divided by its maximum value, has been corrected for internal radiation, bremsstrahlung, and ionization loss. (b) The deviation of experimental points from the best-fit theoretical curve for $\rho = 0.747$, showing typical experimental errors for four points. Curves for $\rho = 0.737$ and 0.757 are shown for comparison.

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MESON-MESON COUPLINGS IN AN SU(6)-INVARIANT THEORY*

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(Received 18 January 1965)

The SU(6) theory of Gürsey and Radicati[1] has been successfully applied to assigning particles to supermultiplets,[1,2] calculating mass formulas,[1,3] the electromagnetic[4] and weak[5] interactions, and the meson-baryon interaction.[1,6,7] In this note we shall be concerned with the application of SU(6) to the interactions of the bosons among themselves. Since the pseudoscalar meson octet (P), the vector-meson octet (V), and the vector-meson singlet (φ) are joined in the 35-dimensional representation of SU(6), we expect to be able to derive relations between the SU(3)-independent VVV, VVP, VPP, and $V\varphi P$ couplings, and thus to correlate processes such as φ, ω, and ρ decay.

Due to the nonrelativistic nature of SU(6) and the difficulty of providing a clear relativistic foundation,[8] we must resort to models in order to calculate results involving physical particles. Thus, for example, Bég and Pais[6] introduce the notion of Lorentz completion and use it to discuss the meson-baryon coupling, while Mahanthappa and Sudarshan[9] consider SU(6)⊗O(3) to obtain the correct, parity-conserving, interaction. For the boson interactions we shall assume[6] that we must search for an SU(6) structure in the static limit of zero momentum transfer. Since all the usual boson trilinear interactions vanish in this limit it would appear necessary, as in the meson-baryon case, to use Lorentz completion to go to the next order in v/c. This procedure is not as simple here, though, since the Lorentz-complete meson matrix M given by Bég and Pais is a function of the parameter f^T/f^V, which is not determined by SU(6), and the meson couplings depend on this parameter. In fact, for $f^T = 0$ even the boosted trilinear meson couplings vanish. This ratio may be fixed by appealing to SU(12) invariance, but in the light of these uncertainties it is interesting to explore alternative methods to find the coupling.

For bosons, however, we do have at our disposal an alternative method to that of Lorentz completion. We may utilize the fact that in a theory involving vector and pseudoscalar mesons the requirement that the former be coupled to a conserved current leads to quartic and higher order terms in the interaction Lagrangian whose coupling strengths are not independent of the trilinear ones.[10] Furthermore, the quartic terms persist in the zero-energy limit and we shall require that they have an SU(6)-invariant structure. In this way we ob-