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INTRANUCLEAR CASCADE AND FERMI-MODEL BREAKUP CALCULATIONS ON THE PRODUCTION OF Li, Be, AND B ISOTOPES IN C^{12} BY 156-MeV PROTONS

E. Gradsztajn, F. Yiou, R. Klapisch, and R. Bernas
Laboratoire Joliot-Curie, Orsay, Seine-et-Oise, France
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A complete calculation of the cross sections for the production of He^6, Li^6, Li^7, Li^8, Li^9, Be^9, Be^{10}, B^{10}, B^{11}, C^{10}, and C^{11} from 156-MeV proton-irradiated carbon has been performed using the hypothesis of a two-step mechanism of which the first step is the intranuclear cascade initiated by the incident proton and leading to a particular excited residual nucleus, and the second step is the breakup of the residual nucleus into various particles.

In all the following the term "residual nuclei" will refer to excited nuclei produced by the cascade, and "final nuclei" refer to nuclei left at the end of the two-step process.

I. The cascade.—We shall not describe here the Monte-Carlo cascade calculations on C^{12} which were recently discussed by Gradsztajn.[1] Let us only recall that the method used involved the possibility for any nucleon moving in the nucleus to have a probability P of colliding with an alpha cluster. In the present work 10 000 cascades have been calculated both for $P = 0$ (no clusters) and for $P = 0.40$. The frequency of occurrence of residual nuclei after the cascade is given in Tables I and II for $P = 0$ and $P = 0.40$, respectively. In Figs. 1 and 2 are shown the excitation energy spectra of these.

II. Breakup of highly excited residual nuclei.—The evaporation of particles, which is usually considered as a likely process of de-excitation in the case of heavy nuclei, cannot be used here due to the small number of nucleons in the residual nuclei. So we have applied to this problem a different statistical approach which

Table I. Frequency of occurrence of various residual nuclei as calculated from the cascade interaction of 156-MeV protons on C^{12} (taking $P = 0$). Number of incident protons: 10 000.

Type of cascade	Residual nucleus	Number[a]	Type of cascade	Residual nucleus	Number[a]
p	N^{13}	0	$p, 2p2n$	B^9	158
p, n	N^{12}	253	$p, 2p3n$	B^8	10
$p, 2n$	N^{11}	121	$p, 2p4n$	B^7	1
$p, 3n$	N^{10}	15	$p, 3p$	Be^{10}	231
p, p	C^{12}[b]	604	$p, 3pn$	Be^9	117
p, pn	C^{11}	1550	$p, 3p2n$	Be^8	31
$p, p2n$	C^{10}	348	$p, 3p3n$	Be^7	0
$p, p3n$	C^9	36	$p, 4p$	Li^9	27
$p, 2p$	B^{11}	1.253	$p, 4pn$	Li^8	6
$p, 2pn$	B^{10}	709	$p, 4p2n$	Li^7	1

[a]Total, including the transparencies: 9777. The missing residual nuclei are produced by cascades with more than six emitted nucleons and are not useful for the breakup calculation.
[b]Not including 4306 transparencies.

Table II. Frequency of occurrence of various residual nuclei as calculated from the cascade interaction of 156-MeV protons on C^{12} (taking $P = 0.4$). Number of incident protons: 10 000.

Type of cascade	Residual nucleus	Number[a]	Type of cascade	Residual nucleus	Number[a]
p	N^{13}	0	$p, 2p4n$ $p, \alpha 2n$	B^7	21
p, n	N^{12}	174	$p, 3p$	Be^{10}	32
$p, 2n$	N^{11}	21	$p, 3pn$	Be^9	6
$p, 3n$	N^{10}	3	$p, 3p2n$ $p, \alpha p$	Be^8	1980
p, p	C^{12}[b]	499	$p, 3p3n$ $p, \alpha pn$	Be^7	382
p, pn	C^{11}	577	$p, 4p$	Li^9	3
$p, p2n$	C^{10}	48	$p, 4pn$	Li^8	0
$p, p3n$	C^9	3	$p, 4p2n$ $p, \alpha 2p$	Li^7	311
$p, 2p$	B^{11}	546	$p, \alpha 2pn$	Li^6	96
$p, 2pn$	B^{10}	112	$p, \alpha 3p$	He^6	24
$p, 2p2n$ p, α	B^9	11			
$p, 2p3n$ $p, \alpha n$	B^8	126			

[a]Total, including the transparencies: 9011. The missing residual nuclei are produced by cascades with more than six emitted nucleons or more than two α particles and therefore are not useful for the breakup calculation.
[b]Not including 4036 transparencies.

has been pointed out recently by Zdanov and Fedotov[2] and was first introduced by Fermi[3] in order to deal with the high-energy collisions of protons with multiple production of particles. More precisely, if a large amount of energy is suddenly released in a small region of space, it will be rapidly distributed among the various degrees of freedom present in the volume, according to statistical laws. (This is valid even for a small number of particles in the volume.) When the energy concentration is released by the breakup of the excited nucleus into many particles, all possible states will occur with frequencies proportional to their statistical weights.

The statistical weight W for a given reaction channel has been established by Fermi,[3] Lepore and Stuart,[4] Rosenthal,[5] and more recently by Ericson.[6] In the nonrelativistic case, with due account of energy and momentum conservation

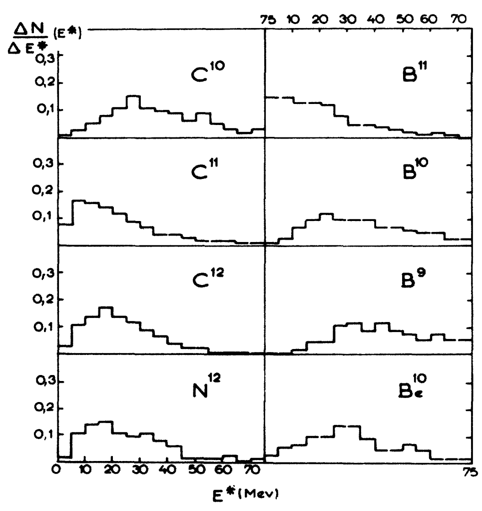

FIG. 1. Spectra of the excitation energy of residual nuclei from 10 000 cascades with $P = 0$. The ordinates are expressed in fraction of the total number of the corresponding nuclei. Transparencies are not included in the case of C^{12}.

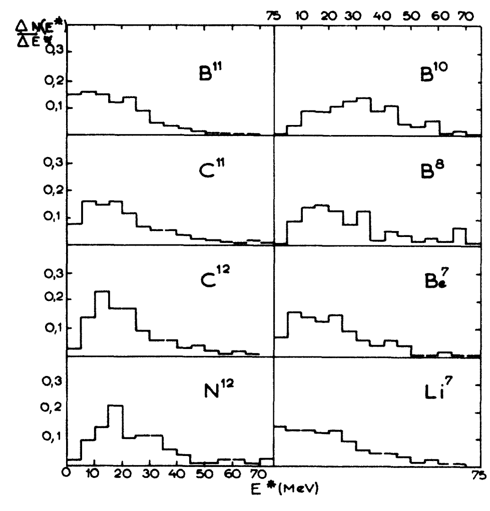

FIG. 2. Spectra of the excitation energy of residual nuclei from 10 000 cascades with $P = 0.4$. The ordinates are expressed in fraction of the total number of the corresponding nuclei. Transparencies are not included in the case of C^{12}.

(but with no account of angular momentum), one finds

$$W=\frac{S}{G}\left(\frac{V}{\hbar^3}\right)^{n-1}\frac{1}{(2\pi)^{3(n-1)/2}}\frac{T^{(3n-5)/2}}{\Gamma(3(n-1)/2)}\times\left(\prod_{j=1}^{n}M_j\Big/\sum_{j=1}^{n}M_j\right)^{3/2}. \qquad (1)$$

V is the volume of the interaction and we have taken it to be equal to the nuclear volume with $r_0=1.3$ F; T is the kinetic energy released by the breakup, $T=E^*-Q$, where E^* is the excitation energy of the residual nucleus and Q the Q value corresponding to a given disintegration; n is the number of emitted particles;

$$S=\prod_{j=1}^{n}(2I_j+1);$$

I_j and M_j are the spin and mass of the jth particle; $G=\prod n_j!$ takes into account the indistinguishability of the n_j identical particles j; Γ is the Γ function.

If we start with the residual nuclei produced by the cascade and take account of their excitation energy spectra, we can then compute the relative probability of occurrence of all the energetically possible channels.

About 1000 breakup channels have been computed for the 18 residual nuclei and 12 final nuclei. We have neglected B^7, C^9, N^{10}, and N^{11}, the masses of which are unknown or not well known; but it is clear from Tables I and II that the number of these residual nuclei is very small. We have also neglected the channels leading to Li^5 and He^5.

The calculations are performed as follows: For each final nucleus j the statistical weight

Table III. Comparison of calculated and experimental absolute cross sections.

Final nucleus	σ calculated $P=0.4$ (mb)[a]		σ experimental (mb)	σ calculated $P=0$ (mb)
Be^7 reference	12.1		12.1 ±10 %[b]	12.1
	Run 1	Run 2		
Be^{10}	1.3	1.1	1.8 ±0.6[c]	1.6
Li^9	0.007	0.004	d	0.008 or 0.038[e]
Li^8	1.1	0.8	<1[f]	1.2
Li^7	8.6	6.8	7.8 ±1.6[g]	7.8
Li^6	9.9	9.4	h	12.6
He^6	0.7	0.5	<0.5[i]	0.8
Li^6+He^6	10.6	9.9	9.8 ±1.8[g]	13.4
Li^7+Be^7	20.7	18.9	19.9 ±4.0[g]	19.9
$R=\sigma 7/\sigma 6$	2.0	1.9	2.0 ±0.1	1.5
C^{10}	1.1	0.9	2.6 ±0.3[j]	0.8
C^{11}	8.0	7.1	45 ±2	13.3
Be^9	1.6	1.4	d	1.3
B^{10}	15.8	14.3	d	12.9
B^{11}	8.5	9.7	d	17.8

[a]Runs 1 and 2: two separate 10 000-cascade runs. The distribution given in Table II and Fig. 2 refers to Run 2.
[b]J. B. Cumming, Ann. Rev. Nucl. Sci. 13, 261 (1963).
[c]M. Honda and D. Lal, Nucl. Phys. 51, 363 (1964).
[d]Not yet available.
[e]0.008 mb is the cross section given by the breakup of all other nuclei. In the 10 000 cascades we found one Li^9 with $0<E^*<5$ MeV; as the levels of Li^9 are not known there is a possibility that this Li^9 will undergo only its usual decay. The cross section must then be increased by 0.030 mb. Better statistics are required.
[f]S. Katcoff, Phys. Rev. 114, 905 (1959); O. V. Lozhkin, N. A. Perfilov, A. A. Rimski-Korsakov, and J. Fremlin, Zh. Eksperim. i Teor. Fiz. 38, 1388 (1960) [translation: Soviet Phys.—JETP 11, 1001 (1960)].
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is given by

$$R_j = \sum_{i=1}^{18} \sum_{k=1}^{n_i} \sum_{l=1}^{15} W_l n_l,$$

where W_l is given by Formula (1); i, k are, respectively, the indices of the residual nuclei and channels, n_i the number of the possible channels for a given i. The index l corresponds to the various 5-MeV intervals into which the excitation energy spectra (Figs. 1 and 2) are subdivided, and n_l is taken directly from these.

The value of R_j, for each of the final nuclei, is calculated for all levels which do not decay by particle emission, and the corresponding spin value I_j introduced into Formula (1). Spin values are taken from Nuclear Data Sheets[7] or after James, Fergusson, and Johnson.[8] The levels for which the spins have not been measured have been attributed values of the corresponding mirror nuclei, and a spin of $\frac{3}{2}$ was attributed to Li^9 in accordance with the spin-orbit coupling model and the rule for odd-Z, even-N nuclei.

Absolute cross sections can be calculated by this method only if all possible decays of residual nuclei can be taken into account. As this was not possible due to lack of experimental data, we have taken as a reference the experimentally well-determined Be^7 cross section at 156 MeV and have computed the other cross sections relative to it, using the following relation:

$$\sigma_j = \frac{R_j}{R(\mathrm{Be}^7)}\sigma(\mathrm{Be}^7) + \left[p_j - \frac{R_j}{R(\mathrm{Be}^7)}p(\mathrm{Be}^7)\right]\sigma_G,$$

where p_j is the probability that the residual nucleus j will be produced by the cascade in a sufficiently low excited state that it will be stable with respect to particle emission. p_j is taken from the spectra of Figs. 1 or 2. σ_G is the geometrical cross section of C^{12} (300 mb).

All the calculations have been performed on the Remington 1107 Univac at Orsay. The results are summarized in Table III.

We can see from the comparison with the experimental results of various workers that the agreement is quite satisfactory. The exception concerning the C^{11} values which, as a product of a p, pn reaction would not be expected to abide too well to this type of calculation, is not surprising.

Thus we feel that this method of treating the interaction of medium-energy protons on light nuclei which introduces only relatively simple assumptions may be very useful to predict cross sections of many nuclear reactions. In particular, the values given in Table III for Be^9, B^{10}, and B^{11} will be of interest in connection with the theories on the nucleosynthesis of the light elements.

Calculations are presently in progress on O^{16} for 156-MeV and higher energy incident protons.

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