INTERACTION OF LIGHT WITH LIGHT IN A PLASMA

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The interaction of light with light in a plasma has been treated both quantum mechanically[1,2] and classically.[3] However, the results obtained in references 1 and 2 differ from those in reference 3. This difference becomes very large at high temperatures. There have been controversies over the source of disagreement.

We first note that these authors have considered somewhat different phenomena. In references 1 and 2 the quantity of concern is the cross section for scattering of light by light in a plasma, while in reference 3 it is the amount of scattering of a light beam incident on a plasma which is excited by two light beams. In the former case two correlated outgoing beams are observed, while in the latter case only one light beam is observed. There are more processes contributing to the latter in addition to light-light scattering. This will be elucidated later.

We further note that in light-light scattering, one should distinguish the case of strong beams from that of weak beams. In references 1 and 2, the light-light scattering cross section is calculated only to the lowest order in the perturbation expansion and is only valid in the weak-beam case. When the intensities of the beams are strong enough so that light-light scattering predominates over the lower order single-beam incoherent scattering, one has to take all the higher order terms into account. Also, we show that there is an inelastic-scattering process not considered previously[1,2] which is of the same order of magnitude as the elastic process. On the other hand, in the classical treatment,[3] approximating $\langle|n(\vec{k},\omega)|^2\rangle$ by $|\langle n^{(1)}(\vec{k},\omega)\rangle|^2$ to obtain the scattering cross section [$n^{(1)}(k,\omega)$ being the Fourier component of the electron density to the lowest order in the external fields] cannot be justified a priori. We show below that with neglect of an "intrinsic" four-particle correlation [Eq. (9)], the above approximation is valid.

The transition probability for photons 1 and 2 going into 3 and 4 (see Fig. 1) in the lowest order, after summing over all final plasma states, is given by

$$P=\frac{(2\pi)^4e^8}{m^4V^2}\frac{(\hat{e}_1\cdot\hat{e}_3)^2(\hat{e}_2\cdot\hat{e}_4)^2}{\omega_1\omega_2\omega_3\omega_4}\tau[\delta_{\vec{k},\vec{k}'}2\pi\delta(\omega-\omega')|S_T(\vec{k}',\omega')|^2+\delta_{\vec{k},-\vec{k}'}2\pi\delta(\omega+\omega')|S(\vec{k}',\omega')|^2]. \quad (1)$$

In (1), $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$, and $\hat{e}_4$ are the unit polarization vectors of photons 1, 2, 3, and 4, respectively; $\omega'=\omega_1-\omega_3$, $\omega=\omega_4-\omega_2$, $\vec{k}'=\vec{k}_1-\vec{k}_3$, $\vec{k}=\vec{k}_4-\vec{k}_2$; V is the volume of interaction of the light beams in the plasma; τ is the total time of interaction; $S_T(\vec{k},\omega)$ is the Fourier transform of the time-ordered density-density correlation function; $S(\vec{k},\omega)$ is the same correlation function as $S_T(\vec{k},\omega)$ but without the time ordering. As is well known, $S_T(\vec{k},\omega)$ and $S(\vec{k},\omega)$ can be expressed in terms of the longitudinal plasma dielectric function $\epsilon(\vec{k},\omega)$:

$$S(\vec{k},\omega)=\frac{-2\hbar}{1-e^{-\beta\omega\hbar}}\frac{k^2}{4\pi e^2}\operatorname{Im}\frac{1}{\epsilon(\vec{k},\omega)}, \quad (2)$$

$$S_T(\vec{k},\omega)=\frac{k^2\hbar}{4\pi e^2}\left[-\coth\frac{\beta\hbar\omega}{2}\operatorname{Im}\frac{1}{\epsilon(\vec{k},\omega)}+i\operatorname{Re}\left(\frac{1}{\epsilon(\vec{k},\omega)}-1\right)\right]. \quad (3)$$

At a zero of $\operatorname{Re}\epsilon(\vec{k},\omega)$, a resonance in the scattering occurs. The first term in the right-hand side of (1) corresponds to elastic scattering, with the plasma state unchanged. This term has been obtained in references 1 and 2. The second term in the right-hand side of (1) corresponds to inelastic scattering, each photon receiving an energy transfer ω and momentum transfer $\vec{k}$ from the plasma. At $\omega\approx\omega_p$, this

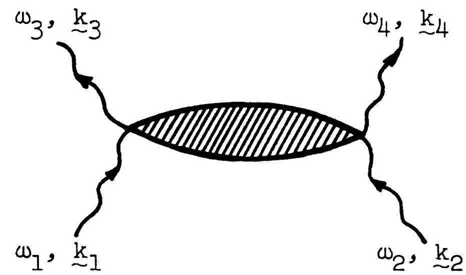

FIG. 1. The scattering of photons 1 and 2 into 3 and 4 via the plasma (the shaded part) in the lowest order.

latter term is of the same order of magnitude as the elastic-scattering term.

Next, we consider the case when two strong laser beams 2 and 4 are applied to the plasma and only the scattering of beam 1 into beam 3 is observed. Then we have to include all processes as long as they give rise to scattering of beam 1 into beam 3. For instance, we have to take into account not only the scattering of beams 1 and 2 into beams 3 and 4, but also that of beams 1 and 4 into beams 3 and 2. To the lowest order their contributions are in fact proportional to the first and the second term of the right-hand side of (1), respectively. There are also other contributing inelastic processes in which the plasma receives $\pm\vec{k}$, $\pm\omega$.

The Hamiltonian of the plasma-photon system is given by

$$H = H_0 + H_1 + H_2, \tag{4}$$

where H_0 is the Hamiltonian of the plasma and the free radiation field; H_1 represents the interaction of beams 2 and 4 with the plasma; and H_2 is responsible for the scattering of beam 1 into beam 3 in the plasma. In H_1 and H_2, the $\vec{A}\cdot\vec{j}$ term will be neglected.[2]

We remark that although the matrix elements of H_2 are small, the matrix elements of H_1 are greatly enhanced by the fact that both laser beams 2 and 4 are of high intensity.

The probability of transition of photon 1 to photon 3, after summing over the final states of the plasma and those of photons 2 and 4, and after averaging statistically over the initial plasma states, is given by

$$P \approx \langle (M^\dagger M)_{n_2 n_4, n_2 n_4} \rangle, \tag{5}$$

where

$$M \equiv \frac{C_{13}}{\hbar}\int_{-\infty}^{\infty} \exp(-i\omega' t_2) \times T\left\{\exp\left[-\frac{i}{\hbar}\int_{-\infty}^{\infty} H_1(t_1)dt_1\right]\rho_{-\vec{k}'}(t_2)\right\} dt_2.$$

In (5), n_2 and n_4 are the number of photons initially present in beams 2 and 4, respectively; $\langle\rangle$ denotes the ensemble average over the initial plasma states; $\rho_{-\vec{k}'}(t) = \exp(iH_0 t/\hbar)\rho_{-\vec{k}'} \times \exp(-iH_0 t/\hbar)$, $\rho_{-\vec{k}'}$ being the Fourier component of the electron density operator; $C_{ij} = 2\pi\hbar e^2(m^2 V\omega_i\omega_j)^{-1/2}\hat{e}_i\cdot\hat{e}_j$; T is the time-ordering operator. Notice that M is the amplitude for the scattering of photon 1 into photon 3 which is linear in H_2 but to all orders in H_1. Equation (5) is just the result of the usual S-matrix theory, immediately giving rise to Feynman diagrams which correspond to the various physical processes mentioned above. However, it is now more convenient to rewrite the amplitude M in the following form:

$$M = \frac{C_{13}}{\hbar} T \exp\left[-\frac{i}{\hbar}\int_{-\infty}^{\infty} H_1(t_1)dt_1\right] N_{-\vec{k}', -\omega'}, \tag{6}$$

where

$$N_{-\vec{k}', -\omega'} \equiv \int_{-\infty}^{\infty}\left\{T\exp\left[-\frac{i}{\hbar}\int_{-\infty}^{t} H_1(t')dt'\right]\right\}^{-1}\rho_{-\vec{k}'}(t)\left\{T\exp\left[-\frac{i}{\hbar}\int_{-\infty}^{t} H_1(t')dt'\right]\right\}e^{-i\omega' t}dt$$
$$= \int_{-\infty}^{\infty}\left\{\rho_{-\vec{k}'}(t) + \frac{i}{\hbar}\int_{-\infty}^{t}[H_1(t_1), \rho_{-\vec{k}'}(t)]dt_1 + \left(\frac{i}{\hbar}\right)^2\int_{-\infty}^{t}dt_1\int_{-\infty}^{t_1}dt_2\left[H_1(t_2), [H_1(t_1), \rho_{-\vec{k}'}(t)]\right] + \cdots\right\}e^{-i\omega' t}dt. \tag{7}$$

Notice that $N_{-\vec{k}', -\omega'}$ is the Fourier transform of the electron density operator in the interaction picture defined by $H_0 + H_1$, and its expectation value is the Fourier transform of the electron density of the plasma under the influence of laser beams 2 and 4. The probability of transition of photon 1 to photon 3 can therefore also be written alternatively as

$$P \approx (C_{13}{}^2/\hbar^2 \langle (N_{-\vec{k}', -\omega'}{}^\dagger N_{-\vec{k}', -\omega'})_{n_2 n_4, n_2 n_4} \rangle. \quad (8)$$

Equation (8) is the quantum analog of the classical formula[4] $P \sim \langle |n_{\vec{k}, \omega}|^2 \rangle$. To evaluate (8) we shall make the following approximation:

$$\langle \rho_{\vec{k}_1}(t_1)\rho_{\vec{k}_2}(t_2)\rho_{\vec{k}_3}(t_3)\rho_{\vec{k}_4}(t_4)\rangle = \langle \rho_{\vec{k}_1}(t_1)\rho_{\vec{k}_2}(t_2)\rangle\langle \rho_{\vec{k}_3}(t_3)\rho_{\vec{k}_4}(t_4)\rangle + \langle \rho_{\vec{k}_1}(t_1)\rho_{\vec{k}_3}(t_3)\rangle\langle \rho_{\vec{k}_2}(t_2)\rho_{\vec{k}_4}(t_4)\rangle + \langle \rho_{\vec{k}_1}(t_1)\rho_{\vec{k}_4}(t_4)\rangle\langle \rho_{\vec{k}_2}(t_2)\rho_{\vec{k}_3}(t_3)\rangle. \quad (9)$$

In doing so we have neglected the diagrams in which all four vertices are connected, which correspond to an "intrinsic" four-particle correlation.

By making use of Wick's theorem and the multiple-commutator nature of the expansion of $N_{-\vec{k}', -\omega'}$ given by (7), it is not too difficult to show that within the approximation (9) one can write

$$\langle N_{-\vec{k}', -\omega'}{}^\dagger N_{-\vec{k}', -\omega'}\rangle = \langle N_{-\vec{k}', -\omega'}{}^\dagger\rangle\langle N_{-\vec{k}', -\omega'}\rangle + \langle \rho_{-\vec{k}', -\omega'}{}^\dagger \rho_{-\vec{k}', -\omega'}\rangle, \quad (10)$$

where the suffices $n_2 n_4$ and $n_2 n_4$ have been omitted for clarity. Similarly one can also show, within the approximation (9), that in the perturbation expansion of $\langle N_{-\vec{k}', -\omega'}\rangle$ according to (7), all higher order terms in H_1 beyond the first order vanish. As a result, the probability of transition of photon 1 to photon 3 is just given by expanding (8) up to the second order in H_1. This is of course equal to the P given by (5) when expanded to the same order, although it is difficult to visualize the cancellation of the higher order terms directly from the perturbation expansion of (5).

We obtain for the counting rate of photon 3 per unit interaction volume

$$\frac{d\Gamma}{d\Omega_3} = d_1 d_2 d_4 \frac{r_0{}^2\hbar^2 c}{4m^2} \frac{k^4}{\omega_1\omega_2} \frac{\omega_3}{\omega_4} (\hat{e}_1 \cdot \hat{e}_3)^2 (\hat{e}_2 \cdot \hat{e}_4)^2 V \delta_{\vec{k}, \vec{k}'} \delta_{\omega_1 + \omega_2 - \omega_4, c|\vec{k}_1 + \vec{k}_2 - \vec{k}_4|} \left| \frac{1 - \epsilon(\vec{k}, \omega)}{\epsilon(\vec{k}, \omega)} \right|^2, \quad (11)$$

where d_1, d_2, and d_4 are the number of photons per unit volume for beams 1, 2, and 4. In the presence of beams 2 and 4, the second Kronecker δ function makes the direction and frequency of the probing beam 1 no longer completely arbitrary. The factor V on the right-hand side of (11) indicates that the counting rate per unit volume is proportional to the total volume of interaction. We remark that since the roles of beams 2 and 4 can be interchanged, there is also a term for $d\Gamma/d\Omega_3$ exactly the same as the right-hand side of (11), with ω and $\vec{k}$ replaced by $-\omega$ and $-\vec{k}$, respectively.

One can show that Eq. (11), which has taken all higher order terms into account, agrees with the result in reference 3. This is because $\langle N^\dagger N\rangle$ does factorize into $\langle N^\dagger\rangle\langle N\rangle$ and all the higher order terms vanish in the approximations (9). However, as we have shown above, this cross section includes contributions not only from the process of elastic light-light scattering, but also from the process of inelastic light-light scattering (plasma receiving $\pm 2\omega_p$) and from other inelastic processes in which the plasma receives $\pm\omega_p$. Furthermore, the cross section in reference 1 is larger than that of (11) because interfering higher order terms were not considered.

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