tional Conference on High Energy Physics, Dubna, 1964 (to be published).

[10]R. K. Logan, following Letter [Phys. Rev. Letters 14, 414 (1965).

[11]K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 10, 543 (1963).

[12]K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 11, 425 (1963).

[13]J. D. Jackson and H. Pilkuhn, Nuovo Cimento 33, 906 (1964); N. Armenise, B. Ghidini, S. Mongelli, A. Romano, P. Waloschek, J. Laberrigue-Frolow, Nguyen Huu Khanh, C. Ouannes, M. Sené, and L. Vigneron, Phys. Letters 13, 341 (1964), and references quoted there.

SINGLE REGGE-POLE ANALYSIS OF π^-p CHARGE-EXCHANGE SCATTERING*

Robert K. Logan
Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 25 January 1965)

The measurement of the π^-p charge-exchange differential cross section described in the preceding Letter by Mannelli et al.[1] provides an excellent test of the Regge-pole hypothesis. The particle or resonance exchanged in the crossed channel must have isotopic spin $\geqslant 1$, zero baryon number, positive G parity, and parity $(-1)^J$, where J is the spin of the particle. The only presently known candidate is the ρ meson. However, if the presently accepted spin and parity assignments for the B meson are not correct, the B could conceivably be exchanged as well.

We assume in our analysis that only the ρ trajectory is exchanged. Our expression for the differential cross section, including both the spin-flip and spin-nonflip contributions, is thus given by[2]

$$\frac{d\sigma_{\text{cex}}}{dt}=\frac{B^2(t)}{16\pi}\left|\frac{1-e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}\right|^2\left(\frac{s}{2m_\pi m_N}\right)^{2\alpha(t)-2}, \quad (1)$$

where m_π and m_N are the pion and nucleon masses, $\alpha(t)$ is the trajectory of the ρ, s and t are the usual Mandelstam variables, and

$$B^2(t)=|b^{(1)}(t)|^2-\frac{t}{4m_N^2}|b^{(1)}(t)-\alpha(t)b^{(2)}(t)|^2.$$

The first term in the expression for $B^2(t)$ is the spin-nonflip contribution whereas the second arises from pure spin flip. From this it follows that $b^{(1)}$ and $b^{(2)}$ are related to the ρ coupling constants as follows:

$$b^{(1)}(m_\rho^2)=2\pi\epsilon\gamma_{\rho\pi\pi}\gamma_{\rho NN}$$

$$b^{(1)}(m_\rho^2)-\alpha(m_\rho^2)b^{(2)}(m_\rho^2)=4\pi\epsilon\gamma_{\rho\pi\pi}m_N\mu_{\rho NN}, \quad (2)$$

where

$$\epsilon=\operatorname{Re}\frac{d\alpha(t)}{dt}\bigg|_{t=m_\rho^2}.$$

Making use of the high-energy approximation $s\approx 2m_NE$, where E equals the total energy of the incident pion in the laboratory system and simplifying our expression for $d\sigma_{\text{cex}}/dt$, we obtain

$$\frac{d\sigma_{\text{cex}}}{dt}=\frac{B^2(t)}{16\pi}\{1+\tan^2[\tfrac{1}{2}\pi\alpha(t)]\}\left(\frac{E}{m_\pi}\right)^{2\alpha(t)-2}. \quad (3)$$

This equation predicts that a plot of $\ln(d\sigma/dt)$ versus $\ln E$ at constant t will yield a straight line with slope $2\alpha(t)-2$. In order to verify the above prediction and to determine the slope we performed a least-squares fit of $\ln(d\sigma/dt)(E,t)$ to the form

$$\sum_{n=0}^{N} m_n(t)(\ln E)^n$$

at constant t using the data of Mannelli et al.[1] at $P_{\text{lab}}=6$, 8, 10, 12, 14, and 16 GeV/c. We used t intervals of 0.04 (GeV/c)^2 from $t=0$ to $t=-0.32$ (GeV/c)^2. For all values of t, reasonable straight-line fits to the data were obtained (see Fig. 1). At certain values of t we obtained slightly better fits for $N=2$ and 3. We can exclude these fits, however, since their extrapolation to 3.8 GeV/c gives results which are in contradiction with existing experimental data.[3]

From $m_1(t)$, the slope of the straight-line fit, we obtain the trajectory $\alpha(t)$ which is listed in Table I. Fitting these values of $\alpha(t)$, with the constraint that $\alpha(t=m_\rho^2)=1$, to polynomials

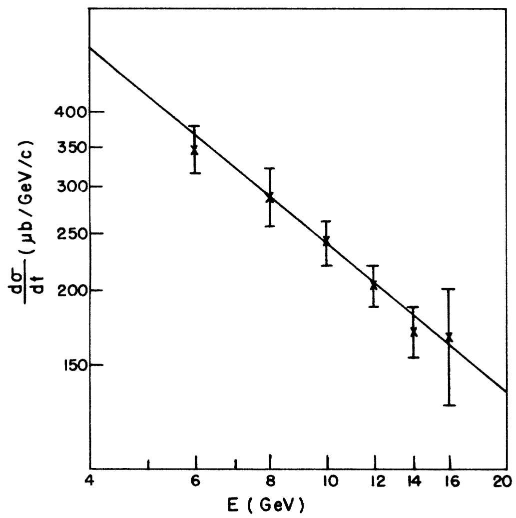

FIG. 1. $\ln(d\sigma_{\rm cex}/dt)$ versus $\ln E$ at $t=-0.06$ $(\mathrm{GeV}/c)^2$.

of the form

$$\sum_{n=0}^{N} b_n t^n,$$

we find that $N=1$ gives the best fit, namely,

$$\alpha(t)=0.64\pm 0.02+(0.64\pm 0.04)t. \qquad (4)$$

Removing the constraint that $\alpha=1$ at $t=m_\rho^2$, $\alpha(t)$ is still consistent with a straight-line fit, but with a slope of only 0.40 ± 0.40 $(\mathrm{GeV}/c)^{-2}$.

Also listed in Table I is $B(t)$ obtained from $m_0(t)$ and $\alpha(t)$, using Eq. (3). Notice that $B(t)$ does not seem to display any marked t dependence. In fact, as demonstrated by a least-squares fit, $B(t)$ is consistent with being a constant, with a value of 17.9 ± 2.4 $(\mathrm{GeV}/c)^{-2}$. This is in sharp contrast to the exponential t dependence of the residue of the Pomeranchuk trajectory as evaluated from πp and pp elastic scattering data.[4] It should be remarked, however, that a perfectly constant $B(t)$ would not be able to explain the flattening of $d\sigma_{\rm cex}/dt$ in the forward direction as reported by Mannelli et al.[1]

Table I. Values of $\alpha(t)$ and $B(t)$ obtained from least-squares fits.

t	$\alpha(t)$	$B(t)$ $[(\mathrm{GeV}/c)^{-2}]$
−0.02	0.58 ± 0.06	23.2 ± 6.8
−0.06	0.60 ± 0.06	21.1 ± 5.9
−0.10	0.58 ± 0.06	21.0 ± 5.3
−0.14	0.63 ± 0.07	15.5 ± 5.2
−0.18	0.48 ± 0.10	18.5 ± 7.9
−0.22	0.40 ± 0.11	33.4 ± 13.8
−0.26	0.64 ± 0.14	8.2 ± 5.4
−0.30	0.45 ± 0.24	45.4 ± 39.1

Near $t=0$ the residue function $B(t)$ is given by $b^{(1)}(t)$, since the factor $-t/4m_N^2$ approaches zero. In order to compare the values of $b^{(1)}(t=0)$ with $b^{(1)}(t=m_\rho^2)$, we evaluate Eq. (2) by taking $\gamma^2{}_{\rho\pi\pi}/4\pi=0.5$, $\gamma^2{}_{\rho NN}/4\pi=0.5$,[5,6] and $\epsilon=0.64$.[7] This gives $b^{(1)}(m_\rho^2)=25$ $(\mathrm{GeV}/c)^{-2}$, which is almost identical to $b^{(1)}(-0.02)$.

The above analysis has shown the consistency of the data with the $E^{\alpha(t)}$ energy dependence of the charge-exchange amplitude, $A_{\rm cex}$, defined by

$$\frac{d\sigma_{\rm cex}}{dt}(E,t)=\frac{16\pi}{s^2}[\mathrm{Re}A_{\rm cex}{}^2(E,t)+\mathrm{Im}A_{\rm cex}{}^2(E,t)] \qquad (5)$$

and $s\approx 2m_N E$. Dispersion relations now predict that

$$R(t)=\frac{\mathrm{Re}A_{\rm cex}}{\mathrm{Im}A_{\rm cex}}=\tan\frac{\pi}{2}\alpha(t), \qquad (6)$$

since

$$\mathrm{Re}A_{\rm cex}(E,t)=\int_0^\infty \mathrm{Im}A_{\rm cex}(E',t)\left(\frac{1}{E'-E}-\frac{1}{E'+E}\right)dE'$$

$$=\tan\frac{\pi}{2}\alpha(t)E^{\alpha(t)} \qquad (7)$$

for $\mathrm{Im}A_{\rm cex}=E^{\alpha(t)}$. At $t=0$ we find $R(0)=1.5\pm 0.2$ using Eq. (4) to determine $\alpha(0)$.

This value obtained using dispersion relations may be checked experimentally by making use of the optical theorem:

$$\Delta\sigma\equiv\sigma^- - \sigma^+ = \frac{16(2\pi)^{1/2}}{s}\mathrm{Im}A_{\rm cex}(t=0), \qquad (8)$$

where $\sigma^\pm$ are the $\pi^\pm p$ total cross sections. We obtained values of $\Delta\sigma(E)$ from the recent measurements of Galbraith et al.[8] in the momentum range 6 to 20 (GeV/c). Since we have found $\mathrm{Im}A_{\rm cex}(t=0)\propto E^{\alpha(0)}$, Eq. (8) predicts that $\Delta\sigma$ has an $E^{\alpha(0)-1}$ energy dependence which we have verified within the experimental errors using a least-square fitting analysis similar to the one performed for $d\sigma_{\rm cex}/dt$. It was found that $\alpha(0)=0.56\pm 0.15$ which is in good agreement with the values of $\alpha(t)$ obtained from the charge-exchange data.

To determine $R(0)$ using unitarity and the experimentally measured quantities, $\Delta\sigma$ and $(d\sigma_{\text{cex}}/dt)_{t=0}$, we combine Eq. (5) and Eq. (8)

$$R(0)=\frac{\mathrm{Re}A(t=0)}{\mathrm{Im}A(t=0)}=\frac{[32\pi(d\sigma/dt)_{t=0}-(\Delta\sigma)^2]^{1/2}}{\Delta\sigma}. \quad (9)$$

Having shown that $d\sigma/dt$ and $\Delta\sigma$ have the same energy dependence within experimental errors, we see from Eq. (9) that $R(0)$ is energy independent. Approximating $(d\sigma/dt)_{t=0}$ by $(d\sigma/dt)_{t=-0.02}$ in Eq. (9), we find $R(0)=1.15\pm0.35$. This agreement with the value of $R(0)$ obtained using dispersion relations establishes the consistency of the two experiments with unitarity, dispersion relations, and the $E^{\alpha(t)}$ energy dependence of A_{cex}.

The author wishes to thank Professor F. E. Low, Dr. M. Wahlig, and Dr. J. Bronzan for their helpful discussions.

*This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract No. AT(30-1)2098.

[1]I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters 14, 408 (1965).

[2]S. C. Frautschi, M. Gell-Mann, and F. Zacharaisen, Phys. Rev. 126, 2204 (1962).

[3]M. Wahlig, private communication.

[4]B. Desai, Phys. Rev. Letters 11, 59 (1963).

[5]J. J. Sakurai, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 176.

[6]It should be noted that the coupling constants used in this Letter are related to those defined by Sakurai in reference 5 by the relation $\gamma^2=\frac{1}{4}f^2$.

[7]This value of ϵ is obtained from Eq. (4).

[8]W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontić, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. (to be published).

SHELL MODEL FOR BARYON RESONANCES

Maurice M. Miller
Lockheed-Georgia Company, Marietta, Georgia
(Received 21 December 1964; revised manuscript received 25 February 1965)

It has recently been proposed[1] that the SU(6)-symmetry scheme, involving the spin and unitary-spin independence of forces between SU(3) triplets such as quarks,[2] can be applied to the physics of strong interactions. Subsequently,[3] shell models involving both LS-coupling and jj-coupling approximations have been investigated treating the triplets as spin-$\frac{1}{2}$ fermions. Alternately, LS-coupling models have been proposed[4] which treat quarks as para-Fermi particles; these models require complete symmetry of the three-quark wave function in the space, spin, and unitary-spin variables. The purpose of this note is to explore the characteristics of the symmetrical jj-coupling shell model for baryon resonances and apply the results to interpretation of current data concerning baryon resonances and "shoulders."

We start with a spin-$\frac{1}{2}$ SU(3) triplet such as the quark triplet, although more sophisticated structures[3,5] could be introduced. Since the quark mass appears very large and since the binding energies of the baryon resonances are presumably large,[3] we adopt the jj-coupling approximation. It is reasonable to assume that the three quarks which make up the 56-dimensional symmetric baryon representation of SU(6) are in s states[4] to form the basis of a shell model; we therefore require symmetric quark wave functions and thus require the same Young diagram $[\lambda]$ for the jj-coupled basis functions as for the unitary-spin basis functions. The latter requirement thus distinguishes the current model from the one proposed in reference 3.

We thus propose that there exist supermultiplets of baryon resonances which transform according to the three-particle symmetric representation of $SU(6j+3)$ for a shell of given j, and further propose the following chain of symmetry breaking:

$$SU(6j+3)\rightarrow SU(3)\otimes SU(2j+1), \quad (1)$$

$$SU(2j+1)\rightarrow Sp(2j+1)\rightarrow O^+(3), \quad (2)$$

$$SU(3)\rightarrow \text{broken } SU(3). \quad (3)$$

The SU(3) applies to the unitary spin, the $SU(2j+1)$ applies to j spin, Sp refers to the symplectic symmetry group, and $O^+(3)$ refers to the rotation group.[6] We use the formalism and notation of Flowers[7] to describe (2) in terms of the partition number (σ) and the seniority s. In addition, we propose an irreducible representation $[\lambda']$ of SU(3) applicable to a state ψ of seniority s; this representation is to be interpret-