PHENOMENOLOGICAL APPROACH TO A RELATIVISTIC SU(6) THEORY*

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Recently there have been several indications that the SU(6) symmetry plays an important role in the description of strong, weak, and electromagnetic phenomena.[1] Consequently there have been several attempts[2] to extend its validity to the relativistic domain. Such extensions can be classified into two types on the basis of the structure of the symmetry group G under which the theory is invariant: (i) G is the direct product of P and Q ($G=P\otimes Q$), where P is a group isomorphic with the Poincaré group and $Q \supset \mathrm{SL}(6,c)$; (ii) G is the semidirect product of T_{36} and SL(6, c) [$G=T_{36}\times \mathrm{SL}(6,c)$], where T_{36} is the group of translations in a 36-dimensional space.[3] There are difficulties in either case. In case (i), there exist no unitary representations in which there is a finite number of states for a fixed four-momentum. This corresponds physically to an infinite number of particles in a multiplet, and therefore is very unsatisfactory. In case (ii) the physical interpretation of the 36 translation operators faces serious difficulties.[4]

The purpose of this Letter is to suggest an approach to overcome these difficulties and present the important features of the results. For this purpose, consider a covariant spinor φ of SL(6, c): $\varphi'=A\varphi$, where A is a complex 6×6 matrix with $\det A=1$. We can then define another six-dimensional representation χ by the transformation property $\chi'=(A^{-1})^\dagger\chi$, so that $\chi^*\varphi$ and $\varphi^*\chi$ are invariant. In analogy with the construction of a four-component Dirac spinor from two two-component spinors, a 12-component representation Ψ of SL(6, c) can be constructed from φ and χ:

$$\Psi=\begin{pmatrix}\varphi\\ \chi\end{pmatrix}. \tag{1}$$

The group SL(6, c) contains SL(2, c)⊗SU(3). One can identify SL(2, c) as the covering group of the homogeneous Lorentz group L and the SU(3) as the familiar internal symmetry group. A representation of SL(6, c) can be decomposed into representations of SL(2, c)⊗SU(3). As in SU(6) theory, this can be done by assigning a pair of indices $i\alpha$ for each index that characterizes the components of Ψ. The index i runs from 1 to 4 and can be identified as the index of the Dirac spinor. α takes the values from 1 to 3 and corresponds to the SU(3) index. The properties of Ψ under spatial reflections can be easily included in the formalism in the standard fashion: $\Psi'=\eta\gamma_4\Psi$, where η is the phase. We also note that if we define $\bar{\Psi}^A=\bar{\Psi}^{i\alpha}=\Psi_{i'\alpha}{}^*\times(\gamma_4)_{i'}{}^{i}$, then $\bar{\Psi}\Psi$ and $\bar{\Psi}\gamma_5\Psi$ are invariant under SL(6, c).

To derive physical consequences we would now like to make the following assumptions:

(1) The fields associated with the physical particles transform like the products of Ψ's and $\bar{\Psi}$'s,[5] and the interaction Lagrangian L_{int} is invariant under the transformations of SL(6, c) and the space reflections. A second-rank mixed spinor $\Phi_A{}^B$, which is required to satisfy

$$(\gamma_4)_j{}^{j'}(\gamma_4)_{i'}{}^{i}(\Phi_{i'\alpha}{}^{j'\beta})^*=\Phi_{j\beta}{}^{i\alpha}, \tag{2}$$

represents the meson field. The above condition ensures that the mesons and their charge-conjugate partners belong to the same multiplet. A third-rank totally symmetric spinor Ψ_{ABC} represents the baryon field. Under space reflections, Φ and Ψ transform as follows:

$$\Phi\to\Phi'=\gamma_4\Phi\gamma_4, \tag{3}$$

$$\Psi\to\Psi'=\gamma_4\otimes\gamma_4\otimes\gamma_4\Psi. \tag{4}$$

We choose to work with the specific L_{int} given by[6]

$$L_{\text{int}}=gm\Phi_A{}^B\Phi_B{}^C\Phi_C{}^A+G\bar{\Psi}^{ADC}\Phi_A{}^B\Psi_{BDC}. \tag{5}$$

(2) To quantize these fields, we need to specify the equations of motion and the commutation relations for corresponding free fields. However, it is impossible to have free-field equations of motion that are covariant with respect to SL(6, c) without encountering the difficulties mentioned at the beginning. Therefore, we consider only those equations of motion for the fields that are covariant with respect to SL(2, c)⊗SU(3) and can give rise to the desired multiplet structure for the associated particles. Thus for the meson field Φ,

$$\tfrac{1}{2}[\gamma_\mu,\partial_\mu\Phi_{\text{free}}]+m\Phi_{\text{free}}=0, \tag{6}$$

where the rectangular bracket denotes a commutator. Equation (6) is equivalent to the Duffin-Kemmer equation.[7] The assumed properties of Φ under space reflections [Eq. (3)] permits one to express a solution of this equation in terms of free fields that represent a nonet of pseudoscalar mesons $P_\alpha{}^\beta$ and a nonet of vector mesons $V_{\mu,\alpha}{}^\beta$:

$$[\Phi_{\text{free}}]_{i\alpha}{}^{j\beta} = [\gamma_5 P - (1/m)\gamma_\mu\gamma_5\partial_\mu P + \gamma_\mu V_\mu - (1/2m)\sigma_{\mu\nu}F_{\mu\nu}]_{i\alpha}{}^{j\beta}, \tag{7}$$

where $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, and

$$[\Box - m^2]P = 0; \; [\Box - m^2]V_\mu = 0, \; \partial_\mu V_\mu = 0. \tag{8}$$

For the baryonic field Ψ_{ABC}, we assume the following equation of motion[8]:

$$(\gamma\cdot\partial + M)_i^{i'}(\Psi_{\text{free}})_{i'\alpha, BC} = 0. \tag{9}$$

A solution of this equation under the restrictions (4) can be expressed in terms of Rarita-Schwinger[9] fields Ψ_μ that correspond to a decuplet of $J^P = \frac{3}{2}^+$ baryons[10] and the Dirac fields Ψ that correspond to an octet of $J^P = \frac{1}{2}^+$ baryons:

$$[\Psi_{\text{free}}]_{i\alpha, j\beta, k\gamma} = D_{ijk,\alpha\beta\gamma} + B_{ijk,\alpha\beta\gamma}, \tag{10}$$

where

$$D_{ijk;\alpha\beta\gamma} = [\tfrac{1}{2}(\gamma_\mu C)_{jk}\Psi_{\mu i} - (1/4M)(\sigma_{\mu\nu}C)_{jk}\Lambda_{\mu\nu i}]_{\alpha\beta\gamma}, \tag{11}$$

$$\Lambda_{\mu\nu} = \partial_\mu\Psi_\nu - \partial_\nu\Psi_\mu,$$

and

$$(\gamma\cdot\partial + M)\Psi_\mu = 0, \quad \gamma_\mu\Psi_\mu = 0; \tag{12}$$

$$B_{ijk,\alpha\beta\gamma} = \tfrac{1}{3}[\chi_{ijk,\alpha}{}^\delta \epsilon_{\delta\beta\gamma} + \chi_{jki,\beta}{}^\delta \epsilon_{\delta\gamma\alpha} + \chi_{kij,\gamma}{}^\delta \epsilon_{\delta\alpha\beta}],$$

where

$$\chi_{ijk,\alpha}{}^\beta = [\tfrac{1}{2}(\gamma_5 C)_{jk}\Psi_i - (1/2M)(\gamma_\mu\gamma_5 C)_{jk}\partial_\mu\Psi_i]_\alpha{}^\beta,$$

and

$$(\gamma\cdot\partial + M)\Psi = 0.$$

In the above equations C is the charge-conjugation matrix, and m and M are masses that are introduced for dimensional reasons. Their values can be assumed to be the "centers" of the meson and the baryon supermultiplets, respectively.[11]

(3) For the present discussion, we compute the effective meson-meson and meson-baryon interactions using L_{int} [Eq. (5)] and the lowest order of perturbation theory.[12] This amounts to using the solutions of the free-wave equations for the fields in L_{int}.

Discussion of the results. —The effective meson-meson interaction that relates the VPP, VVP, and VVV trilinear couplings is given by[13]

$$3g\{\tfrac{3}{2}[V_\mu P\overleftrightarrow{\partial}_\mu P] + (3/m)\epsilon_{\mu\nu\rho\sigma}[\partial_\mu V_\nu\partial_\rho V_\sigma P] + [F_{\mu\nu}V_\mu V_\nu] - (1/3m^2)[F_{\mu\nu}F_{\nu\rho}F_{\rho\mu}]\}. \tag{13}$$

As expected from the requirement that the meson fields represent self-charge-conjugate particles, the VPP and VVV couplings are pure F type and the VVP coupling is pure D type. One also obtains the interesting relation

$$g_{\omega\rho\pi}{}^2/g_{\rho\pi\pi}{}^2 = 4/m^2. \tag{14}$$

From the Gell-Mann, Sharp, and Wagner model[14] for the $\omega\to 3\pi$ decay,

$$g_{\omega\rho\pi}{}^2/g_{\rho\pi\pi}{}^2 = 0.23/m^2(m/m_\pi)^2. \tag{15}$$

With $m = 615$ MeV, Eq. (13) gives a value of $4.6/m^2$ for the ratio $g_{\omega\rho\pi}{}^2/g_{\rho\pi\pi}{}^2$.

To obtain the effective meson-baryon Yukawa-type couplings, it is convenient to define a current

$$J_A{}^B = \bar{\Psi}^{BDC}\Psi_{ADC}. \tag{16}$$

If we substitute the free-field solutions (11) and (12) for Ψ in Eq. (16), we obtain

$$J_A{}^B = J_A{}^B(\bar{D}D) + J_A{}^B(\bar{D}B) + J_A{}^B(\bar{B}D) + J_A{}^B(\bar{B}B). \tag{17}$$

Each term in Eq. (17) can be separated into the SU(3) singlet and octet parts. Further, the space-time properties of each of the currents allows the decomposition into the usual scalar, vector, tensor, axial-vector, and pseudoscalar parts:

$$J_{i\alpha}{}^{j\beta}(\;) = \tfrac{1}{4}[(1)_i{}^j J^S(\;) + (\gamma_\mu)_i{}^j J_\mu{}^V(\;) + (\sigma_{\mu\nu})_i{}^j J_{\nu\mu}{}^T(\;) + (\gamma_\mu\gamma_5)_i{}^j J_\mu{}^A(\;) + (\gamma_5)_i{}^j J^P(\;)]_\alpha{}^\beta. \tag{18}$$

The results are given in Tables I and II. Table I contains the contributions to the current $J_A{}^B(\bar{D}D)$ and $J_A{}^B(\bar{D}B)$.[15] The SU(3) decomposition into a singlet and an octet part has not been carried out for these contributions.[16] Table II contains the baryonic current separated into SU(3) parts that transform like the symmetric (D) octet, antisymmetric (F) octet, and the singlet (S). The effective $\bar{D}DV$, $\bar{D}DP$, $\bar{D}BV$, $\bar{D}BP$, $\bar{B}BV$, and $\bar{B}BP$ couplings are obtained by combining these currents with the free-field solutions for mesons [Eq. (7)]. In the nonrelativistic limit, the scalar and vector parts become Fermi coupling. The tensor and axial-vector part reduce to Gamow-Teller coupling. The terms in the present calculation that contribute in the nonrelativistic limit have separately a D/F ratio of $\frac{3}{2}$ which agrees with the corresponding result in the nonrelativistic SU(6) theory.

The baryonic vector and tensor currents are of special interest in electromagnetic phenomena. Under the assumption that the electromagnetic interaction of the baryons has the general form

$$F^V(q^2)J_\rho{}^{(V)}A_\rho - (1/m)F^T(q^2)J_{\rho\sigma}{}^{(T)}\mathfrak{F}_{\rho\sigma},$$

the contributions of these terms to the charge $[F_{\rm ch}(q^2)]$ and magnetic $[F_{\rm mag}(q^2)]$ form factors[17] are given in Table III. From these results, it follows that the ratio of the total magnetic moment of the proton to that of the neutron is $-\frac{3}{2}$, as in the nonrelativistic theory. In the low momentum transfer region, the electromagnetic structure of the proton and neutron is dominated by the vector mesons. If we therefore make the further assumption that the electromagnetic field couples to the baryons in exactly the same way as the $V_1{}^1$ component of the vector-meson octet, we can calculate the absolute values of the total magnetic moments. The results are

$$\mu_p = 1 + 2M/m \text{ and } \mu_n = -\tfrac{2}{3}\mu_p,$$

in units of $e/2M$. With $M = 1065$ MeV and m

Table I. Decuplet-decuplet and decuplet-baryon currents. For separation into SU(3) parts, see reference 16.

	$\bar{D}D$	$\bar{D}B$
S	$\bar{\Psi}_\mu\Psi_\mu - \frac{1}{2M^2}\bar{\Lambda}_{\mu\nu}\Lambda_{\mu\nu}$	
V	$\bar{\Psi}_\mu\gamma_\rho\Psi_\mu - \frac{1}{2M^2}\bar{\Lambda}_{\mu\nu}\gamma_\rho\Lambda_{\mu\nu}$	$\frac{2}{3}\bar{\Psi}_\rho\gamma_5\Psi - \frac{2}{3M^2}\bar{\Lambda}_{\mu\rho}\gamma_5\partial_\mu\Psi - \frac{2}{3M}\bar{\Psi}_\mu\gamma_\rho\gamma_5\partial_\mu\Psi$
T	$\bar{\Psi}_\mu\sigma_{\rho\sigma}\Psi_\mu - \frac{1}{2M^2}\bar{\Lambda}_{\mu\nu}\sigma_{\rho\sigma}\Lambda_{\mu\nu}$	$\frac{1}{3}(\bar{\Psi}_\sigma\gamma_\rho\gamma_5\Psi - \bar{\Psi}_\rho\gamma_\sigma\gamma_5\Psi) - \frac{1}{3M^2}(\bar{\Lambda}_{\mu\sigma}\gamma_\rho\gamma_5\partial_\mu\Psi - \bar{\Lambda}_{\mu\rho}\gamma_\sigma\gamma_5\partial_\mu\Psi)$
A	$\bar{\Psi}_\mu\gamma_\rho\gamma_5\Psi_\mu - \frac{1}{2M^2}\bar{\Lambda}_{\mu\nu}\gamma_\rho\gamma_5\Lambda_{\mu\nu}$	$\frac{2}{3}\Psi_\rho\Psi - \frac{2}{3M^2}\Lambda_{\mu\rho}\partial_\mu\Psi$
P	$\bar{\Psi}_\mu\gamma_5\Psi_\mu - \frac{1}{2M^2}\bar{\Lambda}_{\mu\nu}\gamma_5\Lambda_{\mu\nu}$	$\frac{2}{3M}\Psi_\mu\partial_\mu\Psi$

Table II. Baryon-baryon current. The expression $\bar{\Psi}\overleftrightarrow{\partial}_\rho\Psi = \bar{\Psi}\partial_\rho\Psi - (\partial_\rho\bar{\Psi})\Psi$.

	F	D	S
S	$\frac{1}{3}\left(\bar{\Psi}\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\partial_\mu\Psi\right)$	0	$\frac{1}{3}\left(\bar{\Psi}\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\partial_\mu\Psi\right)$
V	$\frac{2}{9}\left(\bar{\Psi}\gamma_\rho\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_\rho\partial_\mu\Psi\right)$ $-\frac{1}{9M}\bar{\Psi}\overleftrightarrow{\partial}_\rho\Psi$	$\frac{1}{3}\left(\bar{\Psi}\gamma_\rho\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_\rho\partial_\mu\Psi\right)$ $+\frac{1}{3M}\bar{\Psi}\overleftrightarrow{\partial}_\rho\Psi$	$\frac{1}{9}\left(\bar{\Psi}\gamma_\rho\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_\rho\partial_\mu\Psi\right)$ $-\frac{2}{9M}\bar{\Psi}\overleftrightarrow{\partial}_\rho\Psi$
T	$\frac{1}{9}\left(\bar{\Psi}\sigma_{\rho\sigma}\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\sigma_{\rho\sigma}\partial_\mu\Psi\right)$ $-\frac{1}{18M^2}(\partial_\rho\bar{\Psi}\partial_\sigma\Psi - \partial_\sigma\bar{\Psi}\partial_\rho\Psi)$	$\frac{1}{6}\left(\bar{\Psi}\sigma_{\rho\sigma}\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\sigma_{\rho\sigma}\partial_\mu\Psi\right)$ $+\frac{1}{6M^2}(\partial_\rho\bar{\Psi}\partial_\sigma\Psi - \partial_\sigma\bar{\Psi}\partial_\rho\Psi)$	$\frac{1}{18}\left(\bar{\Psi}\sigma_{\rho\sigma}\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\sigma_{\rho\sigma}\partial_\mu\Psi\right)$ $-\frac{1}{9M^2}(\partial_\rho\bar{\Psi}\partial_\sigma\Psi - \partial_\sigma\bar{\Psi}\partial_\rho\Psi)$
A	$\frac{2}{9}\left(\bar{\Psi}\gamma_5\gamma_\rho\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_5\gamma_\rho\partial_\mu\Psi\right)$	$\frac{1}{3}\left(\bar{\Psi}\gamma_5\gamma_\rho\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_5\gamma_\rho\partial_\mu\Psi\right)$	$\frac{1}{9}\left(\bar{\Psi}\gamma_5\gamma_\rho\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_5\gamma_\rho\partial_\mu\Psi\right)$
P	$\frac{2}{9}\left(\bar{\Psi}\gamma_5\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_5\partial_\mu\Psi\right)$	$\frac{1}{3}\left(\bar{\Psi}\gamma_5\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_5\partial_\mu\Psi\right)$	$\frac{1}{9}\left(\bar{\Psi}\gamma_5\Psi - \frac{1}{M^2}\partial_\mu\bar{\Psi}\gamma_5\partial_\mu\Psi\right)$

Table III. Contributions of baryonic vector and tensor currents to form factors. q^2 is the square of the momentum transfer. The form vectors $F^V(q^2)$ and $F^T(q^2)$ that multiply the vector and tensor contributions, respectively, are omitted from the Table.

	Contribution of J^V		Contribution of J^T	
	F	D	F	D
$F_{\text{ch}}(q^2)$	$\frac{2}{3}\left(1+\frac{q^2}{4M^2}\right)$	0	$-\frac{q^2}{3Mm}\left(1+\frac{q^2}{4M^2}\right)$	0
$F_{\text{mag}}(q^2)$	$\frac{1}{2M}\frac{4}{9}\left(1+\frac{q^2}{4M^2}\right)$	$\frac{1}{2M}\frac{2}{3}\left(1+\frac{q^2}{4M^2}\right)$	$\frac{4}{9m}\left(1+\frac{q^2}{4M^2}\right)$	$\frac{2}{3m}\left(1+\frac{q^2}{4M^2}\right)$

$=615$ MeV, $\mu_p = 3.9$ in units of the nuclear magneton. Since symmetry-breaking effects are not taken into account, it is reasonable to conclude that the result is in good agreement with experiments as far as the sign and order of magnitude are concerned. We would also like to note that the ratio of $F_{\text{ch}}{}^P(q^2)$ to $F_{\text{mag}}{}^P(q^2)$ for the proton is given by

$$\frac{F_{\text{ch}}{}^P(q^2)}{F_{\text{mag}}{}^P(q^2)} = 2M\frac{\{1-(q^2/2Mm)[F^T(q^2)/F^V(q^2)]\}}{\{1+(2M/m)[F^T(q^2)/F^V(q^2)]\}},$$

and if

$$\lim_{q^2\to\infty} q^2\frac{F^T(q^2)}{F^V(q^2)} = 0,$$

the ratio approaches the limit conjectured by Sachs.[18]

A more detailed discussion of the theory as well as the results will be published elsewhere. Delbourgo, Salam, and Strathdee[19] have independently considered the approach and many of the results discussed in the present work. The authors would like to thank Professor Y. Nambu and Professor A. Salam for many helpful discussions.

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[3]G must contain SL(6, c) as a subgroup. See L. Michel and B. Sakita, to be published.

[4]S. Coleman, private communication.

[5]This assumption is consistent with the model in which every particle is constructed from one or more of the fundamental triplets.

[6]Invariance under SL(6, c) and space reflections permits also the interactions of the type $g\,\mathrm{Tr}(\Phi\gamma_5\Phi\gamma_5\Phi)$. L_{int} in Eq. (4) is invariant under a larger group $M(12)$ [or $\tilde{U}(12)$]: K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 48 (1965); R. Delbourgo, A. Salam, and J. Strathdee, to be published.

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[8]We would like to note here that Eq. (5) can be derived from the free Lagrangian $-\frac{1}{2}m\Phi_A{}^{i\alpha}(\gamma\cdot\partial+m)_i{}^j \times\Phi_{j\alpha}{}^A$. However, we do not know of any simple free Lagrangian from which Eq. (9) can be derived. The quantization of Ψ field needs further investigation.

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[11]Following Bég and Pais, we take $M = 1065$ MeV, $m = 615$ MeV. See reference 2.

[12]The use of lowest order perturbation theory is unreasonable from the point of view of strong-interaction theory. Our results, however, can be obtained assuming that the vertex functions are invariant under $M(12)$.

[13]The rectangular bracket in Eq. (13) implies taking the trace with respect to SU(3) indices.

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$\pi^- + p \to \pi^0 + n$ CHARGE-EXCHANGE SCATTERING AT HIGH ENERGIES*

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We report here the differential and total cross sections for elastic charge-exchange scattering $\pi^- + p \to \pi^0 + n$ at incident lab pion momenta of 6, 8, 10, 12, 14, 16, and 18 GeV/c.[1] Measurements were made at values of the four-momentum transfer squared between 0 and −0.5 (GeV/c)^2. This represents the high-energy part of a recent spark-chamber run at the Brookhaven AGS; preliminary results of the lower energy part of this run (2.4 to 6 GeV/c) have been published previously.[2]

Only minor changes were made to the experimental apparatus as described in reference 2. The length of the liquid-hydrogen target was increased from 2 to 6 in., and the distance between it and the spark chamber was increased with increasing energy. The spark chamber was triggered each time a π^- entered the target and failed to produce an "anti" signal in any of the surrounding scintillation counters. For all of the data reported here, the counters at the rear of the spark chamber (which in reference 2 served to confirm the presence of a shower in the spark chamber) were not used.

The beam used for the 6- through 18-GeV/c run was the one originally set up by Galbraith