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THEORY OF THE INTERACTION OF IONS AND QUANTIZED VORTICES IN ROTATING HELIUM II *

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Careri, McCormick, and Scaramuzzi[1] have shown that negative ions are absorbed in steadily rotating helium II when propagated perpendicular to the axis of rotation. No effect was reported for positive ions. Further experiments by Tanner, Springett, and Donnelly[2] have shown that the current is absorbed according to the equation

$$I = I_0 \exp(-2\Omega m \sigma h^{-1} y), \tag{1}$$

where Ω is the angular velocity of the container, m is the mass of the helium atom, y is the distance along the direction of the electric field, and σ is a capture diameter of order 10^{-5} cm. σ has been observed to decrease with increasing field $\mathcal{E}$, and to increase with increasing temperature to about 1.6°K, where it drops rapidly to a small value.

What force is responsible for the attraction between ions and vortices? The rotating superfluid in a vortex gives rise to a radial pressure gradient given by $dp/dr = \rho_s v_s^2/r$. This pressure gradient acting on a small object of volume V gives rise to a force directed toward the core of the vortex which, on applying the quantization restriction to the circulation, is of magnitude

$$\rho_s V(\hbar/m)^2 r^{-3}. \tag{2}$$

Let us consider, for purposes of a concrete example, that the positive and negative ions have radii 6.3 Å and 12.1 Å, and equal masses $m_i = 100\,m$.[3] Then the ions may be considered as Brownian particles in equilibrium with the quasiparticle excitations in the helium and moving in the combined electric and vortex fields. The problem breaks up into two parts: the calculation of the capture cross section σ and the calculation of the escape probability P. We follow here the review article by Chandrasekhar.[4]

Solving the steady-state Smoluchowski equation [cf. Chandrasekhar,[4] equation (420)] first in the presence of the electric field, then in the presence of the vortex term (2), we find we can match the incoming probability density and flux to the absorbed probability density and flux at a radius which satisfies the relation

$$\frac{2\pi kT}{e\mathcal{E}} = Q^{1/2}\chi(x), \tag{3}$$

where $Q = \rho_s V(\hbar/m)^2/2kT$ is the square of the length at which the force (2) dominates thermal fluctuations and $kT/e\mathcal{E}$ is the characteristic length for diffusion of ions in an electric field. The dimensionless parameter $x = Q/r^2$ and

$$\chi(x) = x^{-1/2} e^{x}[\mathrm{Ei}(x) - \mathrm{Ei}(1)], \tag{4}$$

where $\mathrm{Ei}(x)$ is the exponential integral. The cross section is taken to be $\sigma = 2(Q/x)^{1/2}$, where x is the value satisfying (3) at the temperature and field of interest. The results of calculations of σ for positive and negative ions are shown in Fig. 1(a). We see that the two ion species are trapped with comparable probability at all temperatures. The field dependence of the negative species at 1.4°K is shown in Fig. 1(b). We see that $\sigma \rightarrow \infty$ as $\mathcal{E} \rightarrow 0$ because the force (2) acts to infinity. Figure 1(b) shows that measurements of σ in an ion beam with some degree of space-charge limitation will be difficult to interpret, since $\mathcal{E}$ is a function of y, reaching zero at the source in the case of complete space-charge limitation.

Consider now the probability of escape. We follow here the Kramers method of computing the escape over a barrier [cf. Chandrasekhar,[4] Chap. 7]. We require first the shape of the potential well $U(r)$. To do this we integrate the electric and vortex forces along the y direction up to the point where the ion touches the core of the vortex (in the classical sense). At this point, the superfluid circulation must locally encircle the ion, which lowers the energy of the vortex line. This contribution has been estimated by considering the ion centrally located on the vortex and computing the change of total energy resulting from the substitution of the ion for some of the rotating superfluid. Adopting a strictly classical view of this situation results in a substitution energy $\Delta E = 2\rho_S \pi \times (\hbar/m)^2 r_i \ln(r_i/a_0) = 4.78 \times 10^{-14} \rho_S$ erg for a cylindrical ion of length $2r_i$ and some 12% less for a spherical ion ($a_0 = 1$ Å).[5] It is sufficient to approximate the shape of the well by a harmonic potential $U \cong \frac{1}{2} m_i \omega_A{}^2 r^2$ at the bottom ($r \sim 0$) and $U \cong \Delta U - \frac{1}{2} m_i \omega_C{}^2 (r - r_C)^2$ at the lip ($r \sim r_C$), where ΔU is the height of the barrier. The probability for escape may be written as

$$P = (\omega_A / 2\pi\omega_C)([\tfrac{1}{4}\beta^2 + \omega_C{}^2]^{1/2} - \tfrac{1}{2}\beta) e^{-\Delta \nu/kT}, \quad (5)$$

where $\beta = e/\mu m_i$. The results of such calculations are shown in Fig. 2, where the observed

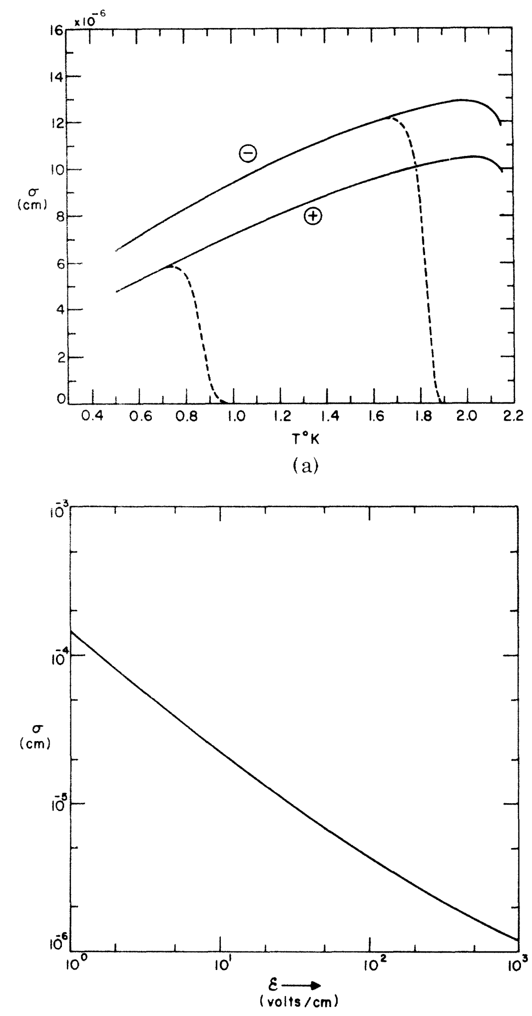

FIG. 1. (a) Trapping diameter σ as a function of temperature under an electric field $\mathcal{E} = 25$ volts/cm. Curves are for positive and negative ions 6.3 and 12.1 Å in radius, respectively, with $m_i = 100\ m$. The dashed curves show the quantity σe^{-Pt} for $t = 1$ sec, using the data of Fig. 2. (b) Field dependence of σ for negative ions at $T = 1.40$°K.

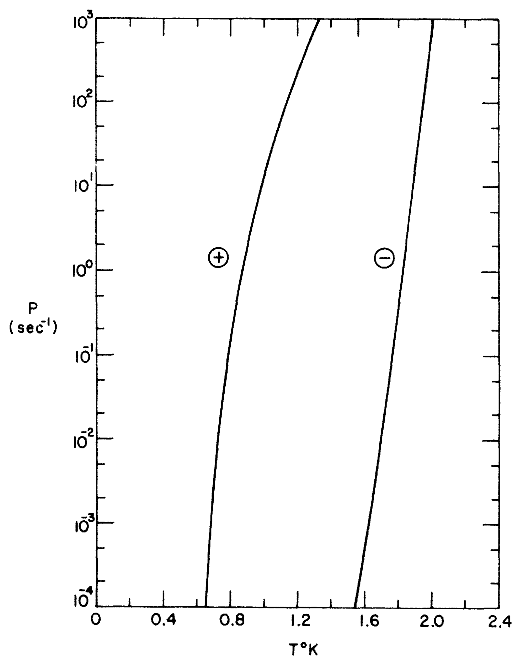

FIG. 2. The probability of escape for trapped ions in an external field $\mathcal{E} = 25$ volts/cm. At 0.5°K, $\Delta U/kT = 39.4$ and at 1.0°K, $\Delta U/kT = 19.6$ for the positive ion. At 1°K, $\Delta U/kT = 50.7$ and at 1.8°K, $\Delta U/kT = 19.3$ for the negative ion.

mobilities of Reif and Meyer[6] have been used for each ion species.

The effective cross section may now be computed as σe^{-Pt}, where t is some characteristic time (such as the length of the collector divided by the velocity of the ions in the vortex lines). For $t = 1$ sec, the positive-ion cross section will be cut off rapidly at ~0.85°K and the negative-ion cross section at ~1.8°K as shown by the dashed lines in Fig. 1(a). Thus, in experiments above 1°K, we should expect to see only negative-ion trapping, and the cross section should rise to ~1.8°K and drop off rapidly. This is in qualitative accord with our observations described above. Below 0.8°K, we should find positive and negative ions trapped with almost equal probability. While such an experiment has yet to be done with rotating helium II, support for this conclusion comes from the vortex-ring experiments of Rayfield and Reif,[5] who found approximately equal quantities of each sign of ion trapped in vortex rings. The trapping probability on creating vortex rings should be very large indeed because the vortex is presumably created as a sort of quantum mechanical "wake" behind the accelerating ion; hence there is no relative velocity between the ion and the ring and $\sigma \to \infty$.

The mean lifetime of an ion trapped in a vortex will be P^{-1} sec. Some measurements communicated to the author by Douglass[7] are in remarkable agreement with the calculations considering the uncertainties in the properties of the ions. The mean life should be sensitive to changes in electric field and, if one grants the "bubble model" of the negative ion, to changes in pressure.

Above the cutoff temperature, the vortex lines still constitute centers for ion scattering caused by the absorption and prompt emission of ions. This contribution to the mobility is of order

$$\mu = \frac{eh}{m_i^{3/2}\Omega\sigma(3kT)^{1/2}}. \tag{6}$$

This effect is normally too small to be observed unless σ is made large by a space-charge-limited beam. This has been done by Modena, Savoia, and Scaramuzzi,[8] who show experimental evidence that $\mu\Omega$ is a constant which is only weakly temperature dependent.

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