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[18]Use is made of the following relation between decay width Γ and transition amplitude A: $\Gamma = |A|^2 m_B p^3 / m_{B^*}$, where p stands for the final state momentum and m_{B^*} and m_B for the masses of the initial baryon resonance and the final baryon, respectively.

VELOCITY-DEPENDENT NUCLEON-NUCLEON POTENTIALS

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In a recent work Bryan and Scott[1] cast the nucleon-nucleon potentials in the form $V = V^{(0)} + \tau_1 \cdot \tau_2 V^{(1)}$, where both $V^{(0)}$ and $V^{(1)}$ have the forms

$$V^{(i)}(r) = V_C^{(i)}(r) + V_{\sigma\sigma}^{(i)}(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_\tau^{(i)}(r)S_{12} + V_{LS}^{(i)}(r)\vec{\mathrm{L}} \cdot \vec{\mathrm{S}}, \tag{1}$$

where the $V^{(0)}$ terms arise from the exchange of isoscalar mesons and $V^{(1)}$ arise from isovector mesons. They present a set of eight one-boson-exchange potentials derived from pseudoscalar, vector, and scalar meson theory. These are compared with corresponding potentials extracted from a phenomenological model due to Lassila, Hull, Ruppel, McDonald, and Breit[2] and a phenomenological model due to Hamada and Johnston.[3] The results show good agreement, particularly in view of the fact that they use only nine adjustable parameters. Purely phenomenological studies of the nucleon-nucleon interaction have gone higher than thirty adjustable parameters. The success of Bryan and Scott's work has led us to re-examine a closely related study which was reported[4] at the June, 1949 meeting of the American Physical Society in Cambridge.

In this study of two Dirac particles coupled five-vectorially (scalar plus four-vector), one obtains the chief explicit interaction

$$V = (1 - \beta_1\beta_2 - \vec{\alpha}_1 \cdot \vec{\alpha}_2)J. \tag{2}$$

In the usual form of meson theory, J is the Yukawa potential. Following Breit's[5] method, the small components of the Dirac wave function were eliminated to obtain the Schrödinger-Pauli interaction

$$V = -(\hbar/Mc)^2\{\nabla^2 J/4 + 2J\nabla^2 + 2\nabla J \cdot \nabla - (\nabla^2 J/6)\vec{\sigma}_1 \cdot \vec{\sigma}_2 - 2r^{-1}(dJ/dr)\vec{\mathrm{L}} \cdot \vec{\mathrm{S}} + (\tfrac{1}{12})[rd(r^{-1}dJ/dr)/dr]S_{12}\}. \tag{3}$$

The major burden of the earlier study was to show that a plausible interpretation could be given to some of the properties of the deuteron with this five-vector interaction. In this connection particular use was made of a two-parameter trial wave function,

$$\psi = C[\exp(-\kappa_a r) - \exp(-\kappa_b r)]/r, \tag{4}$$

where

$$4\pi C^2 = 2(\kappa_a + \kappa_b)\kappa_a\kappa_b/(\kappa_a - \kappa_b)^2 \text{ and } \kappa_a < \kappa_b.$$

This spherically symmetric wave function was assumed to represent approximately the ground state of the deuteron, and the expectation values of various terms in the Schrödinger interaction were assessed in terms of the coupling constant and the meson mass parameter. In addition the Diracian pseudoscalar-pseudoscalar interaction

$$V = \beta_1\beta_2\gamma_{51}\gamma_{52}J \tag{5}$$

was considered. This explicit interaction also arises out of a five-dimensionally invariant interaction between a spinorial and tensorial field as discussed by Watanabe.[6] If this is reduced to large components one finds

$$V = (\hbar/\mathrm{M}c)^2\{(\nabla^2 J/12)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + (\tfrac{1}{12})[rd(r^{-1}dJ/dr)/dr]S_{12}\}. \tag{6}$$

The recent discovery, mass measurement, and classification of heavy strongly interacting mesons along with the extensive phenomenological studies of the nucleon-nucleon interaction provide the opportunity to test the interactions described by Eqs. (2) or (3) and (5) or (6). For this initial study we choose as a universal coupling constant the pi-meson nucleon coupling constant, $g^2 = 14.7$ as deduced by Hamilton and Woolcock[7] from pion-scattering experiments. Figures 1(a), 1(b), and 1(c) show the tensor, spin-spin, and spin-orbit isoscalar potentials generated by an ω meson (M_ω

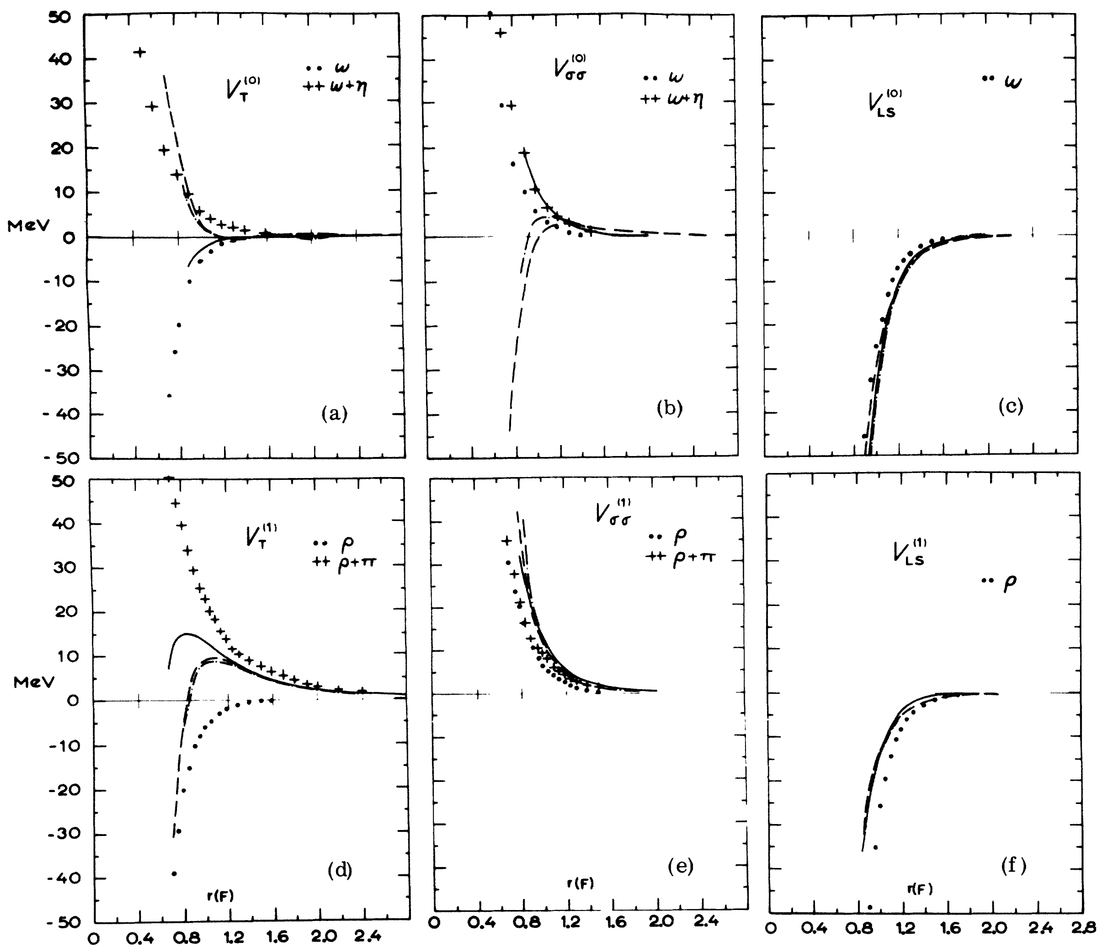

FIG. 1. (a) $V_T^{(0)}(r)$, (b) $V_{\sigma\sigma}^{(0)}(r)$, (c) $V_{LS}^{(0)}(r)$, (d) $V_T^{(1)}(r)$, (e) $V_{\sigma\sigma}^{(1)}(r)$, and (f) $V_{LS}^{(1)}(r)$.

=783 MeV), assuming that it is a particle manifestation of the five-vector field. In addition, we show the tensor and spin-spin potentials generated under the assumption that the η meson (m_η = 549 MeV) contributes to the isoscalar nucleon potential with the same coupling constant. Figures 1(d), 1(e), and 1(f) show the corresponding isovector potentials based upon a five-vector ρ meson (m_ρ = 763 MeV) and a pseudoscalar pi meson (m_π = 137 MeV). The fact that we come close to each of these six potentials without availing ourselves of any adjustable constants is encouraging.

Because of the velocity-dependent terms in Eq. (2) it is more difficult to compare our central potentials with those reported by Bryan and Scott. One method, however, would be to choose as a basis of comparison

$$V = -(\hbar/Mc)^2[\nabla^2 J/4 + 2J\psi^{-1}\nabla^2\psi + 2\psi^{-1}\nabla J\cdot\nabla\psi]. \quad (7)$$

Then one must utilize a wave function which closely represents the state of the system. For this purpose we have used a Born-approximation wave function for 30, 100, and 320 MeV for $l = 0$ and $l = 3$, and, in addition, for $l = 0$ we used Eq. (4) as an approximate one for the ground state of the deuteron. Figures 2(a) and 2(b) show the equivalent potentials as compared with the isoscalar central potentials given by Bryan and Scott. Note that at low energies for $l = 0$ the potential is attractive, whereas at high energies it becomes strongly repulsive. This is precisely what is needed to influence the phase shifts in the manner usually assigned to a static potential with an outer attractive region and an inner repulsive core.

Since it is difficult to compare in detail the velocity-dependent terms arising out of the five-vector Dirac form with the static mesonic

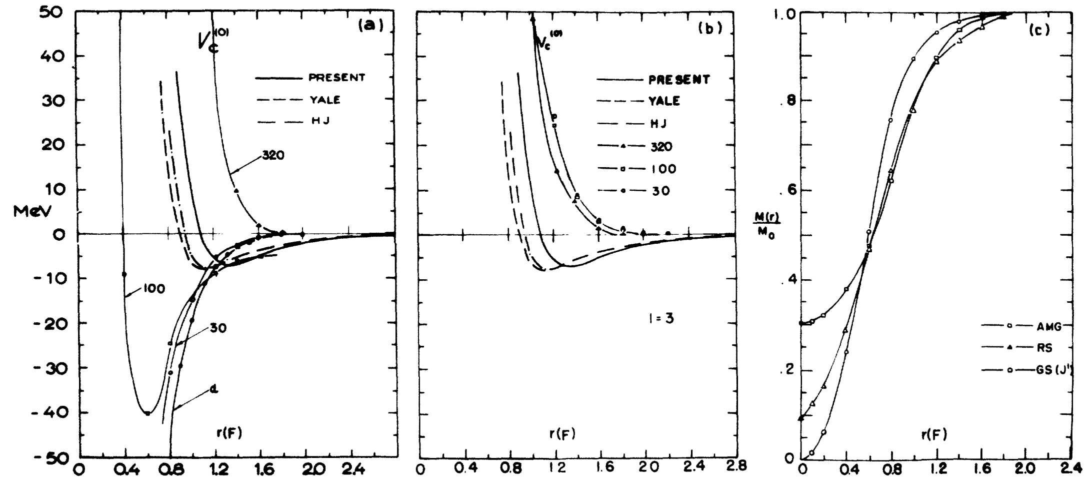

FIG. 2. (a) $V_C^{(0)}(r)$ for $l=0$ versus velocity-dependent potentials for deuteron, and $E=30$, 100, and 320 MeV. (b) $V_C^{(0)}(r)$ versus velocity-dependent potentials for $l=3$ and $E=30$, 100, and 320 MeV. (c) Reduced mass function.

potentials, let us compare these terms with velocity-dependent interactions which have been proposed in recent phenomenological studies of the nucleon-nucleon force.[8,9] All of the proposed velocity-dependent interactions including the first three terms of Eq. (2) may be viewed to be equivalent to a choice of Hamiltonian which may be placed in the form

$$H(r,p)=V_S(r)+M(r)^{-1}p^2+\vec{p}M(r)^{-1}\cdot\vec{p}, \qquad (8)$$

where $M(r)$ is a radially varying effective mass. In Fig. 2(c) we compare our $M(r)/M$ with those derived from the velocity-dependent potentials of Green[8] and the work of Rojo and Simmons.[9] It should be clear that the five-vector interaction generates a reduced mass function quite close to that inferred from a phenomenological analysis. The result suggests that a major source of the reduced masses encountered in phenomenological nucleon-nuclear potentials[10-12] arises from the explicit velocity dependence of the nucleon-nucleon potential itself. The $V_S(r)$ used in phenomenological theories also compare favorably with that arising out of the five-vector theory. By breaking slightly the degeneracy of coupling constants or masses between the four-vector and scalar fields we can augment or subtract from our static potential to any reasonable degree required.

We might also call attention to the close relationship between our work and that of Duerr[13] and Rozsnyai[14] on velocity-dependent nucleon-nuclear potentials.

We come finally to the problem of dealing directly with a Schrödinger's equation containing the r^{-3} tensor and spin-orbit singularities to calculate the nuclear phase shifts. This problem is normally dealt with by introducing a cutoff at a small radius. We have explored the phenomenological consequences of introducing higher mass pseudoscalar and five-vector mesons which act subtractively with respect to the π, η, ρ, and ω mesons which dominate the long-range interaction. We are considering higher mass mesons whose relative couplings b_i satisfy[15,16]

$$\sum_{i=1}^{N} b_i=0,\quad \sum_{i=1}^{N} b_i\xi_i^{\,2}=0,\text{ and }\sum_{i=1}^{N} b_i\xi_i^{\,4}=0. \qquad (10)$$

While this work is in a preliminary stage we can report that their introduction not only makes it possible to improve the fits to the phenomenological potentials of Lassila et al.[2] and Hamada and Johnston[3] but also serves to bring our radial mass function closer to the phenomenological forms obtained from the work of Green[5] and Rojo and Simmons.[9]

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TWO-PION DECAY OF THE K_2^0 MESON*

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The existence of the decay $K_2^0 \to \pi^+ + \pi^-$ was first observed by Christenson et al.[1] They determined the branching ratio, R, of this mode compared to all charged decay modes of the K_2^0 as $(2.0 \pm 0.4) \times 10^{-3}$ at a K_2^0 momentum of 1.1 GeV/c. This Letter reports the preliminary results of an experiment at Nimrod in which the two-pion decay mode is clearly distinguished from the background of three-body decays of the K_2^0, confirming the earlier observation.[1] The branching ratio R is found to be $(2.08 \pm 0.35) \times 10^{-3}$ averaged over the K_2^0 momentum spectrum which extends from 1.5 to 5.0 GeV/c. A comparison of this result with that of Christenson et al.[1] rules out a variation of the branching ratio with the square of the K_2^0 total energy. Such a variation has been suggested on the basis of an interaction of the K_2^0 meson with a long-range vector field.[2,3]

A neutral beam taken at zero degrees from an internal target (6 in. of copper) emerges from the shield wall. A lead converter 2 in. thick and a sweeping magnet remove γ rays and their charged products from the beam. The resulting neutral beam is 17 cm wide and 8 cm high (base widths) at entry to a vacuum chamber 40 m from the machine target. The minimum inner dimensions of the vacuum chamber are 45 cm wide and 20 cm high, its length is 5 m, and the entrance and exit windows are of Mylar (0.01 in. thick).

The charged products of decay of K_2^0's within the vacuum chamber are momentum analyzed by two spectrometer systems each comprising four sonic spark chambers and a bending magnet. All the spark chambers are triggered by a six-fold coincidence of scintillation counters. The sonic chamber data, consisting of scalar readings from which the coordinates of the sparks can be determined, are recorded on magnetic tape and later analyzed by an Orion computer. The spark chambers determine particle positions to an accuracy of ±0.3 mm.[4]

Each event is analyzed to find the decay point (x_0, y_0, z_0), the mass M_0, the momentum P_0, and the direction of motion θ_0 (with respect to the beam direction) of the decaying particle. The assumption is made that the decay is to two bodies only, both being π mesons. Only decays occurring within a fiducial volume 16 cm wide, 10 cm high, and 460 cm in length, were accepted, thus ensuring that all decays took place well within the vacuum chamber. The apparatus has an estimated mass resolution $\delta M_0 = \pm 3$