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MODEL OF WEAK INTERACTIONS WITH *CP* VIOLATION*

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We propose a simple model of weak interactions which allows for CP violation.[1] A single current-current coupling is introduced where vector and axial-vector currents transform differently under SU(3), yet each transforms like a member of a unitary octet.

We start from Cabibbo's elegant assumption[2] that the hadronic weak current—including both A_μ, the axial-vector current, and V_μ, the vector current—transforms under SU(3) like $\pi^+\cos\theta + K^+\sin\theta$. Then the weak current, like the electric current, transforms like a generator of SU(3). (Models with CP violation proposed by Sachs and Treiman[3] and by Sachs[4] invoke CP-odd currents with $I=\frac{3}{2}$. Such currents do not have octet transformation properties.) Furthermore, it is usually assumed that the weak currents are chosen from among a single octet of vector currents (the traceless 3×3 Hermitean matrix J_μ) and one of axial vector currents (the traceless 3×3 Hermitean matrix K_μ). These octets behave in a definite and identical fashion under CP, which may be taken to be $J_\mu(x,t)\to\tilde{J}_\mu(-x,t)$ and $K_\mu(x,t)\to\tilde{K}_\mu(-x,t)$, where tilde denotes matrix transposition. These are the 16 conserved currents of a chiral SU(3)⊗SU(3) symmetry which can hold in the limit of vanishing pseudoscalar meson masses.[5] (A model with CP violation due to Cabibbo[6] requires the introduction of other octets of currents with opposite CP properties. Matrix elements of these abnormal currents are "of the second kind"[7] and give rise to observable CP violations.)

We modify the original Cabibbo proposal by introducing neither non-octet currents nor abnormal octet currents. Rather, we let V_μ and A_μ transform like different members of a unitary octet.[8] We assume that V_μ transforms like $\pi^+\cos\theta_V + K^+\sin\theta_V$, and that A_μ transforms like $\pi^+e^{i\Phi'}\cos\theta_A + K^+e^{i\Phi}\sin\theta_A$. We lose no generality writing V_μ as a real linear combination of π^+ and K^+, since the overall phase of the weak current and the relative phase of its $Y=0$ and $Y=1$ parts are unobservable. Because nuclear β decay seems CP invariant, we put $\Phi'=0$; because of the success of Cabibbo's model in relating decay rates,[9] we put $\theta_V=\theta_A=\theta$. We are left with a two-parameter description of the weak current involving $\theta\approx15°$ (the relative strength of the $Y=0$ and $Y=1$ currents) and Φ (the degree of CP violation, which may or may not be small):

$$V_\mu = \mathrm{Tr}J_\mu C = \mathrm{Tr}J_\mu \begin{bmatrix} 0 & \cos\theta & \sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{1}$$

$$A_\mu = \mathrm{Tr} K_\mu D = \mathrm{Tr} K_\mu \begin{bmatrix} 0 & \cos\theta & e^{i\Phi}\sin\theta \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (2)$$

We give a brief description of some consequences of the model. A more detailed analysis will be forthcoming.

Leptonic decays. – The leptonic currents are assumed to be the usual ones, and these immediate conclusions about leptonic decay modes follow: All $\Delta Y = 0$ leptonic processes (nuclear β decay, muon capture, muon decay) conserve CP to lowest order in weak interactions. The Cabibbo prescription properly describes hyperon β-decay rates, but vector and axial-vector matrix elements now have relative phase Φ. (The study of CP violation in Λ β decay would give a direct determination of Φ.) The decay modes K_{l2} and K_{l3} conserve CP, because in neither case is there the possibility for interference between V and A. The degree of CP violation in K_{l4} depends on the relative contribution of V and A to these modes. The $\Delta Y = \Delta Q$ selection rule remains exact.

Nonleptonic decays. – The interaction Lagrangian responsible is $G(A_\mu + V_\mu)(A_\mu^\dagger + V_\mu^\dagger)$. In general, all nonleptonic decays will display some CP violation. Recent observations[10] of $\Lambda \to p + \pi^-$ are consistent with CP conservation, but they allow the possibility of significant CP violation. (The relative phase of S and P amplitudes may differ by as much as 30°, after corrections are made for final-state interactions.) Since we have not been able to express the amount of CP violation in Λ decay in terms of Φ, we feel that the possibility that Φ is large is still admissible.

Although the $\Delta I = \frac{1}{2}$ rule approximately characterizes all nonleptonic decays, it is not implicit in any model with only one current-current coupling. Perturbative calculations do not satisfy $\Delta I = \frac{1}{2}$ and are too small by several orders of magnitude for allowed transitions.[11] A selective enhancement of the octet channel due to the symmetric strong interactions is often invoked.[12] We assume such an enhancement in the "normal" pseudoscalar and "normal" scalar octet channels.[13] The four admissible spurions correspond to $K_{(1)}$, $K_{(2)}$, $K_{(1)}'$, and $K_{(2)}'$ tadpoles.[8,12]

The parity-conserving part of the nonleptonic weak interactions may be written

$$\begin{aligned} V_\mu V_\mu^\dagger + A_\mu A_\mu^\dagger &= (JC)(JC^\dagger) + (KD)(KD^\dagger) \\ &= -\tfrac{1}{2}(J^2)(CC^\dagger) - \tfrac{1}{2}(K^2)(DD^\dagger) + (JCJC^\dagger + KDKD^\dagger) \\ &\quad + (J^2\{C, C^\dagger\} + K^2\{D, D^\dagger\}), \end{aligned} \quad (3)$$

where parentheses denote traces, curly brackets denote anticommutators, and space-time indices have been supressed. The first two terms are SU(3) singlets, the next is mostly 27-plet; only the last term behaves like a member of a normal scalar octet and is subject to enhancement. Its two parts are independent, and they give rise to the effective spurion

$$\sin\theta\cos\theta\{(\alpha + \beta\cos\Phi)\langle K_{(1)}'\rangle + \beta\sin\Phi\langle K_{(2)}'\rangle\},$$

where α and β depend on the process being considered.

Similarly, the parity-violating interaction becomes

$$\begin{aligned} V_\mu A_\mu^\dagger + \mathrm{H.c.} &= (JC)(KD^\dagger) + \mathrm{H.c.} = -\tfrac{1}{2}(JK)(CD^\dagger) \\ &\quad + \tfrac{1}{2}(JCKD^\dagger + JD^\dagger KC) + \tfrac{1}{2}(\{J, K\}\{C, D^\dagger\}) \\ &\quad + \tfrac{1}{2}[(JC)(KD^\dagger) - (JD^\dagger)(KC)] + \mathrm{H.c.}, \end{aligned} \quad (4)$$

where the first term is a unitary singlet, the second term is mostly 27-plet, the third term transforms like a member of an abnormal pseudoscalar octet,[14] and the last term (which is antisymmetric in J and K) consists of a mixture of decuplet$\oplus$antidecuplet and normal octet $([J,K][C,D^\dagger]) + \mathrm{H.c.}$ Only the normal octet is enhanced, and it gives rise to the spurions

$$\begin{aligned} &\sin\theta\cos\theta\{(1 - \cos\Phi)\langle K_{(1)}\rangle - \sin\Phi\langle K_{(2)}\rangle\} \\ &\qquad + \sin^2\theta\sin\Phi\{\langle\pi^0\rangle + \sqrt{3}\langle\eta\rangle\}. \end{aligned} \quad (5)$$

The CP-conserving $K_{(1)}$ spurion vanishes even more rapidly than the CP-nonconserving $K_{(2)}$ spurion as $\Phi \to 0$.[15] Small Φ gives a theory in which the $K^0 \to 2\pi$ amplitude is mostly CP odd, whereas the $K^0 \to 3\pi$ amplitude is mostly CP even, and yields the experimentally unacceptable prediction that $\Gamma(K_S \to 3\pi) = \Phi^{-2}T(K_L \to 3\pi)$.[16] A large violation of CP (perhaps even $\Phi = \pi/2$) is indicated.

Gell-Mann[17] has discussed the $\Delta Y = 0$ nonleptonic weak interactions that arise from a single current-current interaction. The parity-conserving part contains $\Delta I=0,2$ terms multiplied by $\cos^2\theta$, and $\Delta I=1$ terms suppressed by $\sin^2\theta$. In our model, there is no CP nonconservation in these terms, and they are experimentally masked by electromagnetic corrections. Similarly, the parity-nonconserving terms involve $\Delta I=0,2$, and the $\Delta I=1$ terms are again suppressed by $\sin^2\theta$. However, those terms that transform like a $Y=0$ member of a normal pseudoscalar octet are subject to dynamical enhancement by a factor of ~25. From Eq. (5) it is seen that these terms are CP odd and involve $\Delta I=0,1$. Their suppression by $\sin^2\theta$ is approximately compensated by their "octet enhancement." Following an approach analogous to that of Hellesen and Bjorken,[18] we find that they give rise to the P-odd CP-odd nuclear interaction

$$\sim 10^{-6}(3\bar{N}\vec{\tau}N\vec{\pi}+\tfrac{1}{2}\bar{N}N\pi^0).$$

The $K\rightarrow 2\pi$ decays.—The remarkable feature of $K_0\rightarrow 2\pi$ which must be explained by our model is that the rate for $K_L\rightarrow 2\pi$ is so small $[\Gamma(K_L\rightarrow\pi^++\pi^-)/\Gamma(K_S\rightarrow\pi^++\pi^-)\approx 5\times10^{-6}]$. Surely, this is immediately understood if $|\Phi|\ll 1$ or $|\Phi-\pi|\ll 1$, i.e., if the degree of CP nonconservation is small. [The difficulty with small Φ discussed in the last paragraph no longer applies for very small Φ, for the suppression of the K_1 spurion by $(1-\cos\Phi)$ is SU(3) dependent and only approximate.] We prefer to exploit the experimentally more interesting possibility that Φ is large. Weinberg[19] has shown, in an idealized model (one channel, neglecting energy dependence of CP-nonconserving phase), that one linear combination of K_1 and K_2 is uncoupled to the $\pi^+\pi^-$ channel, even if CP is strongly violated. However, Sachs[4] and Sachs and Treiman[3] have pointed out three exceptions to this argument, each of which will lead to some $K_L\rightarrow 2\pi$ decays. These are, briefly, as follows: (1) The involvement of virtual 2π states, which may imply that the diagonalization of the complex decay matrix (including mass shifts) does not give the same eigenstates as the real part of the decay matrix; (2) the breakdown of the $\Delta I=\frac{1}{2}$ rule—in general the $I=0$ and $I=2$ final states need not have the same CP-violating phase; (3) the existence of other real or virtual decay modes, especially 3π. In our model, these corrections to Weinberg's argument may still leave a sufficient suppression of $K_L\rightarrow 2\pi$ to be compatible with the experimental result.

We have, in Eq. (4), decomposed the parity-nonconserving part of the weak interactions into an expression antisymmetric in the replacement $J\leftrightarrow K$, which transforms like a mixture of normal octet and $\underline{10}\oplus\underline{10}^*$, plus a term symmetric in $J\leftrightarrow K$ which transforms like a mixture of abnormal octet and $\underline{27}$. Two results are easily established:

(1) In the SU(3) limit, the terms symmetric in $J\leftrightarrow K$ do not contribute to $K_0\rightarrow 2\pi$. This is a simple generalization of a result of Cabibbo and of Gell-Mann.[15] Realistically, this suppression cannot be expected to be better than by a factor of $\sim\frac{1}{10}$ in amplitude.

(2) The $\Delta Y=1$ part of the symmetric terms bears the CP-nonconserving phase $(e^{-i\Phi}+1)$; while the $\Delta Y=1$ part of the antisymmetric terms bears the orthogonal phase $(e^{-i\Phi}-1)$.

Remembering that the normal octet is enhanced by a factor ~25 in amplitude relative to the leading $\Delta I=\frac{3}{2}$ terms (which arise from the $\underline{10}\oplus\underline{10}^*$), we find that the $\Delta I=\frac{1}{2}$ decay amplitude is dominantly characterized by the phase $e^{-i\Phi}-1$. The $\Delta I=\frac{1}{2}$ term arising from the symmetric part of the Lagrangian has the orthogonal phase, but (with $\Phi\approx\pi/2$) it is only ~1/250 the strength of the dominant term. At most, this can lead to $\Gamma(K_L\rightarrow 2\pi)/\Gamma(K_S\rightarrow 2\pi)\sim 10^{-5}$.

Now consider the $\Delta I=\frac{3}{2}$ amplitude coming from the $\underline{10}\oplus\underline{10}^*$ and $\underline{27}$. In large measure, its phase will coincide with that of the dominant $\Delta I=\frac{1}{2}$ term. A small difference in phase $(\sim\frac{1}{10})$ arises because of the suppressed contribution of the $\underline{27}$. This may lead to a contribution to $K_L\rightarrow 2\pi$ of $\sim 10^{-5}$ the $K_S\rightarrow 2\pi$-decay rate. (To obtain a more precise result, we must analyze the isotopic decomposition of the $\underline{27}$ and $\underline{10}\oplus\underline{10}^*$ terms. This is in progress.)

Finally, we consider the 3π decay modes. Another effect of large CP nonconservation is the appearance of an observable rate for the s-wave three-pion decay mode of K_S. We may relate the 2π and 3π decay modes of K_S and K_L in a simple model calculation which ignores the small effects discussed above. Diagonalization of the 2×2 $K_{(1)}$-$K_{(2)}$ mass matrix yields the short- and long-lived eigenstates, and their complex decay rates are found to satisfy

$$\Gamma(K_S\rightarrow 2\pi)\Gamma^*(K_L\rightarrow 2\pi)=\Gamma(K_S\rightarrow 3\pi)\Gamma^*(K_L\rightarrow 3\pi).$$

We expect $\Gamma(K_S \to 3\pi)$ and $\Gamma(K_L \to 3\pi)$ to be comparable in magnitude, but their ratio depends on the unknown relative phase of the 2π and 3π decay amplitudes. We may use this result to estimate the decay rate $\Gamma(K_L \to 2\pi)$, but we must remember that $\mathrm{Re}\Gamma(K \to 3\pi)$ is expected to be smaller than $\mathrm{Im}\Gamma(K \to 3\pi)$ because of the restricted phase space for $K \to 3\pi$. [An estimate of $\mathrm{Im}\Gamma(K \to 3\pi) = 4\,\mathrm{Re}\Gamma(K \to 3\pi)$ follows if a broad 2π resonance at ~300 MeV is invoked.] We obtain $\Gamma(K_L \to 2\pi) \approx 10^{-5}\Gamma(K_S \to 2\pi)$.

Summarizing, we have found that even if Φ is large, the $K_L \to 2\pi$ decay rate arising from each of the three effects discussed above is $\sim 10^{-5}\Gamma(K_S \to 2\pi)$. We have not established this result, but only made it plausible. Experimentally, our model requires the existence of considerable CP nonconservation in $\Lambda\,\beta$ decay and suggests observable CP nonconservation in all nonleptonic decays. Conversely, our model requires no CP nonconservation in K_{l3}.

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