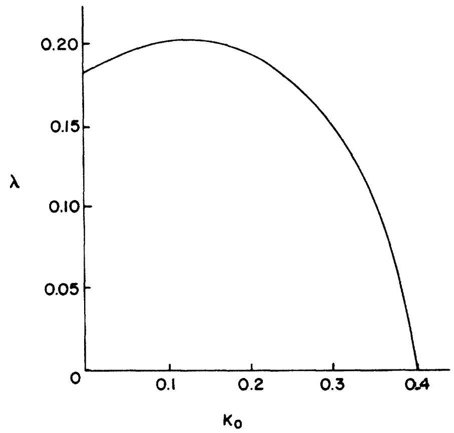

FIG. 2. Curve depicting the value of k_0 at which a phase transition occurs for various values of λ. The associated densities are given by Eqs. (4) and (5).

and there is also an intermediate-density formula given by Nozières and Pines.[10] In a report to be published elsewhere, it will be shown that the presently described phase transition is contained within all of these formulas at approximately the same density. This can be easily confirmed by noting that since the density fluctuations $\partial n/\partial\mu$ must be non-negative, it follows that the curve for the total energy as a function of density must always have positive curvature. And each of the above formulas has negative curvature for some range of densities, thus indicating at least the inapplicability of the formula for this region of densities. Further, the phase transition described here is also present if one includes lowest order temperature effects.

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[5]See, for example, Kerson Huang, Statistical Mechanics (John Wiley & Sons, Inc., New York, 1963), p. 319.

[6]This condition is somewhat stronger than required but is sufficient for our purposes.

[7]Recall that, according to Eqs. (5) and (6), all negative values of μ may by associated with the solution of zero density.

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NEUTRON STARS*

John N. Bahcall and Richard A. Wolff†
California Institute of Technology, Pasadena, California
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Several authors[1-5] have suggested that the recently discovered[5-7] extraterrestrial sources of x rays may be hot neutron stars. The plausibility of this suggestion, and in fact the likelihood that astronomers will ever be able to observe neutron stars by their x-ray emission, depend critically upon the cooling times of the hot stars. The main purposes of this note are (i) to present the results, and suggest the implications, of some approximate calculations for the neutrino cooling rates of neutron stars, and (ii) to point out that some current ideas regarding the constituents of neutron stars should be revised. Our description of the states of a neutron star and the reactions by which it cools differ from the work of previous authors[2,3,8] in that we include in an approximate way the effects of the strong interactions among all the hadrons (strongly interacting particles) present. The principal new results obtained (for densities not much greater than nuclear densities) are[9] (i) the existence of effective masses for all the hadrons, (ii) differential shifts in the threshold densities at which various kinds of particles are produced, and (iii) much faster cooling rates than previous workers have estimated. At densities greater than 10 times nuclear densities, unsolved matters of principle are of primary importance.[10] We have therefore attempted to phrase our initial questions in terms of quantities that can be defined independent of any specific model for the interaction among the particles that constitute a neutron star. Our practical results are, of course, calculated on the basis of a specific model that has a limited domain of validity which we attempt to estimate.

The ground state of a neutron star can be determined[8] by minimizing the total energy subject to the constraints of conservation of charge and baryon number. Other authors[2,3,8] have adopted a noninteracting gas model for all the particles in the star in order to carry out this minimization; their approach is valid only in the low-density limit. We have estimated the effects of the strong interactions using a picture based on an independent pair model (IPrM), which is similar to the self-consistent independent pair model used by Gomes, Walecka, and Weisskopf[11] for discussing the ground state of nuclear matter. We suggest that a necessary criterion for the validity of any independent particle model is that the average separation, d, between hadrons satisfy the following inequality:

$$d > 0.5 \times 10^{-13}\ \text{cm}. \tag{1}$$

If inequality (1) is not satisfied (i.e., the stellar density is $\gtrsim$ eight times nuclear densities), then pairs of hadrons spend most of their time within each other's hard cores, and the concept of distinct strongly interacting particles is not meaningful.[12]

The new results we have obtained by including the strong interactions via IPrM are (i) the neutron has an effective mass of 0.9 and the proton has an effective mass of 0.6 for densities not too different[13] from nuclear density; and (ii) the threshold densities for producing various hadrons are differentially shifted from the threshold values predicted by the noninteracting gas model.

The effective masses for neutrons and protons enter in an important way the numerical calculations for neutrino cooling rates.

The reason for the shifts in the threshold densities for hadrons is most clearly understood by considering a specific example. Sigmas are produced ($e^- + n + n \to \Sigma^- + \nu_e + n'$) at densities such that

$$E_F(e) + E_F(n) \geq (m_\Sigma - m_n) + [B_0(\Sigma^-) - B_F(n)], \tag{2}$$

where the terms on the left-hand side of Eq. (2) are the electron and neutron Fermi energies, respectively, $B_0(\Sigma^-)$ is the binding energy of a zero energy Σ^-, and $B_F(n)$ is the average potential energy of a neutron. Equation (2) reduces to the usual result[3,8,14] if the term $[B_0(\Sigma^-) - B_F(n)]$ is ignored.

Pions are produced ($e^- + n \to \pi^- + \nu_e + n'$) at densities such that

$$E_F(e) \geq m_\pi + B_0(\pi^-). \tag{3}$$

In the noninteracting gas model, Σ^-'s are produced at much lower densities than π^-'s.[3,8,14] One can show with Eqs. (2) and (3) and the equilibrium relation between protons, electrons, and neutrons that pions are actually produced before sigmas if, for $\rho \leq 3\rho_{\text{nucl}}$,

$$B_0(\pi^-) \leq 0.5\{[m_\Sigma - m_p - 2m_\pi] + [B_0(\Sigma^-) - B_0(p) - E_F(p)]\}, \tag{4a}$$

or, expressing all energies in MeV,

$$B_0(\pi^-) \lesssim -11 - 2.5(\rho/\rho_{\text{nucl}})^{4/3} + 0.5[B_0(\Sigma^-) - B_0(p)]. \tag{4b}$$

If inequality (4) is satisfied, then pions are produced before sigmas, and the numbers of particles of various kinds present in neutron stars are very different from the numbers previously obtained[3,8,14] by neglecting the strong interactions. This result would have great practical importance since, as we shall show later, the presence of a significant number of pions in a neutron star changes the predicted cooling rates of a hot neutron star by a large factor ($\sim 10^{+8}$).

Equations (2)-(4) are valid for any model that assumes the existence of individual particles in a neutron star. Unfortunately, one must invoke a detailed theory of strong interactions in order to calculate quantities such as $B_0(\pi^-)$ or $B_0(\Sigma^-)$. We hope that some high-energy theorists will apply their methods to the calculation of these binding energies which are vital to an understanding of neutron stars.[15]

In order to compute cooling times, one must consider the various excited states of a neutron star. One can imagine that these excited states are populated (according to the usual Boltzmann factor) by placing the system in contact with a thermal bath at a finite temperature T. The star then has a definite baryon number and total electric charge but does not have a definite energy. The rate of energy loss (cooling) by neutrino emission is given by an expression of the form

$$L_\nu = \text{const} \times \sum_\nu \sum_{\beta<\alpha} \sum_\alpha |\langle S_\beta ; \nu | H_w | S_\alpha \rangle|^2 E_\nu \times \delta(E_\alpha - E_\beta - E_\nu) \exp(-E_\alpha/kT), \tag{5}$$

where S_α and S_β are states of the entire star, H_w is the weak-interaction Hamiltonian, E_ν is the energy of the neutrino ν that is radiated, and the summation over β is limited to states for which $E_\beta < E_\alpha$.

In practice, cooling times must be computed by assuming a model; we adopt IPrM. We also approximate the thermal average [Eq. (5)] over the states of the star by assigning a Fermi-Dirac or Bose-Einstein distribution to each kind of particle in the star. The most important cooling reactions are

$$n + n \to n' + p + e^- + \bar{\nu}_e, \tag{6a}$$

$$n + \pi^- \to n' + \mu^- + \bar{\nu}_\mu, \tag{6b}$$

and their inverses, $p + e^- + n' \to n + n + \nu_e$ and $n' + \mu^- \to n + \pi^- + \nu_\mu$. Unlike photons, neutrinos produced in the interior of a neutron star escape from the surface with a negligible probability[17] of having been absorbed or scattered. Reaction rates for processes (6) and their inverses (which have equal rates within our approximation) have been estimated[9] by distorted-wave Born approximations using empirically determined scattering potentials and weak-interaction matrix elements. The exclusion principle for fermions was included in the phase space with a Fermi-Dirac distribution function. We find for the rate of energy loss, from (6a) and its inverse,

$$L_\nu \cong 10^{+38} \left(\frac{M}{M_\odot}\right) \left(\frac{T_c}{10^{+9}\,{}^\circ\mathrm{K}}\right)^8 \times \left(\frac{\rho_{\text{nucl}}}{\rho}\right) \text{erg sec}^{-1}, \tag{7a}$$

and from (6b) and its inverse [assuming $m_\pi + B_0(\pi^-) \geq m_\mu$],

$$L_\nu \cong 10^{+46} \left(\frac{n_\pi}{n_n}\right) \left(\frac{M}{M_\odot}\right) \left(\frac{T_c}{10^{+9}\,{}^\circ\mathrm{K}}\right)^6 \times \left(\frac{\rho_{\text{nucl}}}{\rho}\right)^2 \text{erg sec}^{-1}, \tag{7b}$$

where M, $M_\odot$, T_c, ρ, ρ_{nucl}, n_π, and n_n are, respectively, the mass of the neutron star, the mass of the sun, the central temperature of the star, the central density of the star, $3.7 \times 10^{+14}$ g cm^{-3}, the number density of pions, and the number density of neutrons. If $m_\pi + B_0(\pi^-) < m_\mu$ (=106 MeV), then Reaction (6b) is energetically forbidden. In this case, Reaction (6b) should be replaced by $n + \pi^- \to n' + e^- + \bar{\nu}_e$, and the energy loss of (7b) should be multiplied by $(m_e/m_\mu)^2 = 2 \times 10^{-5}$.

Our rate, (7a), for cooling by Reactions (6a) is two orders of magnitude faster, in the important temperature-density range, than the rate estimated by Chiu and Salpeter.[2,18] One can show, by using the models of neutron stars given in references 1 and 3, that the neutrino luminosity from Reactions (6a) exceeds the photon luminosity for effective (i.e., surface) temperatures $\gtrsim$ two million degrees. The energy loss, Eq. (7b), from neutrino emission by pions is much greater than from all previously known cooling processes. The basic reason that the energy loss from Reactions (6b) is so much faster than from, for example, Reactions (6a) is that pions are bosons. Each fermion that participates in a cooling reaction introduces a factor in the cooling rate of $kT/E_\mathrm{F} \sim 10^{-3}$ to 10^{-4}, where E_F is the Fermi energy of the particle; this factor occurs because only the small fraction ($\sim kT/E_\mathrm{F}$) of the fermions that are on the tail of the Fermi-Dirac distribution can make transitions allowed by the exclusion principle. No such restriction exists for bosons. Note that Reaction (6a) involves two more fermions than Reaction (6b).

The cooling rate (7b) is so great that it seems very unlikely that neutron stars will be observable with present techniques[5-7] if this rate applies [i.e., inequality (4) is satisfied and $B_0(\pi^-) \geq -34$ MeV]. For example, one can show with the models of reference 3 that the x-ray source in Scorpius, which Friedman[7] indicates might have a surface temperature of the order of 2 or 3 million degrees, would decrease in photon luminosity by a factor of 10 in a period of less than or of the order of a week if Eq. (7b) applies. Our fast cooling rates are also inconsistent with the hypothesis that a hot neutron star exists in the Crab nebula, which is a remnant of a supernova explosion that occurred in 1054.

Models of the type (IPrM) used in our calculations, which rely heavily on the concept of individual particles, will fail at densities $\gtrsim 10$ times nuclear densities [cf. Eq. (1)]. In particular, the distinction between fermions and bosons probably disappears at such high densities that hadrons are continually within a hard-core distance of each other.[12] Thus the tables that some authors have given which describe the state of a neutron star in terms of the number of, e.g., Ξ^-'s present at $\rho > 25\rho_{\rm nucl}$ are unjustified. At $\rho_{\rm stellar} > 10\rho_{\rm nucl}$, it presumably makes sense to specify a definite baryon number and charge, but one can at best hope to calculate, for example, an expectation value for the strangeness per unit volume. The problem of how to describe the state of matter at very high densities correctly and in a manner suitable for calculation is fascinating but unsolved. In any event, the hadronic constituents of matter (if this phrase continues to have an approximate meaning) at very high densities will be vastly different from their free-particle analogs.

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[11]L. C. Gomes, J. D. Walecka, and V. F. Weisskopf, Ann. Phys. (N.Y.) 3, 241 (1958).

[12]Imagine, for example, a collection of alpha particles at a density for which $d > R_\alpha$, where R_α is the "radius" of an alpha particle. If the density of alpha particles is now increased so that $d \lesssim R_\alpha$, the alpha particles will come apart into their constituents, primarily neutrons and protons.

[13]There is some variation of the neutron and proton effective masses with density. This variation can be estimated (reference 9) from the work of K. A. Brueckner [Phys. Rev. 97, 1353 (1955)] and L. C. Gomes and J. D. Walecka (to be published). The effective masses of the neutron and proton are different in a neutron star because the stellar matter contains many more neutrons than protons.

[14]E. E. Salpeter, Ann. Phys. (N.Y.) 11, 393 (1960).

[15]One such calculation is currently underway (W. G. Wagner, private communication). The result of H. Miyazawa [J. Phys. Soc. Japan 11, 393 (1964)] cannot be used since he omits a subtraction term and a Born term which are important for neutron stars. One also cannot use directly the pion-nucleus scattering data since the neutron excess is much smaller for nuclei than for neutron stars.

[16]If inequalities (4) are satisfied for $\rho \lesssim 3\rho_{\rm nucl}$, then the large number of pions present form a degenerate Bose gas; their presence affects (reference 9) the theoretical equation of state (lowers the pressure) and the sequence in which various particles are produced.

[17]The neutrino opacity of a neutron star can be determined from formulas given by J. N. Bahcall, Phys. Rev. 136, B1164 (1964), and J. N. Bahcall and S. C. Frautschi, Phys. Rev. 136, B1547 (1964). The largest contribution to the opacity [for neutrinos from Reaction (6a)] arises from neutrino-electron scattering. When applying Formula (57) of the paper by Bahcall, note that $\mu_e > 10$ for a neutron star.

[18]A. Finzi [Phys. Rev. 137, B472 (1965)] has calculated the rate of Reaction (6a) for a specific density, $\rho = 1.5\rho_{\rm nucl}$. He treats the neutrons and protons as free particles (neglecting nuclear-matter effects) and estimates, in second-order perturbation theory, the energy loss due to pairs of successive virtual transitions. We have used, on the other hand, first-order perturbation theory with empirically determined matrix elements and potentials and have included, in an approximate way, nuclear-matter effects. However, our result for $\rho = 1.5\rho_{\rm nucl}$ is in surprisingly good agreement with his value.