iltonian:

$$H=\langle 0|H|0\rangle+\sum_k\omega_k a_k{}^\dagger a_k+gH(\varphi K_1K_2)+\lambda_0{}^2H(\varphi^4)$$
$$+\alpha\lambda_0{}^2H(\varphi^3)+2m_kg\alpha\int K_1{}^0K_2{}^0dV. \quad (7)$$

Here $H(\varphi K_1K_2)$, $H(\varphi^3)$, and $H(\varphi^4)$ are normal ordered expressions of fourth and third order in the creation and destruction operators of the corresponding fields,

$$\omega_k{}^2=k^2+m_\varphi{}^2,\quad \Delta=3\lambda_0{}^2V^{-1}\sum_k\omega_k{}^{-1},$$
$$m_\varphi{}^2=\mu_0{}^2+\Delta+6\lambda_0\alpha^2,$$

and the equation for α is obtained from (6) as

$$\alpha(2\lambda_0{}^2\alpha^2+\mu_0{}^2+\Delta)=0.$$

Since Δ is divergent, $\mu_0{}^2$ has to be negative. If $\mu_0{}^2>-\Delta$, the only real root is $\alpha=0$; in this case no symmetry breakdown occurs. If, however, $\mu_0{}^2<-\Delta$, the minimum solutions are $\alpha=\pm0.5m_\varphi\lambda_0{}^{-1}$, which result in symmetry-breaking terms in the Hamiltonian (7). Supposing, e.g., $\lambda_0\sim1$, $m_\varphi\sim m_K$, one arrives at $g\sim10^{-17}$. While the electromagnetic field is coupled to a current with $Y=0$ with a strength 137^{-1}, and the weak current is characterized by $Y=0,\pm1$ and has a strength 10^{-11}, this φ field is coupled to a term with $Y=\pm2$ with a strength 10^{-34}.

We have attempted to show a possibility for the explanation of the $K_2{}^0$ puzzle by spontaneous breakdown of the CP symmetry. Our asymmetric vacuum was characterized by the minimum condition (6), using a CP-symmetric Hamiltonian. The principle of relativity and the principle of equivalence is maintained in this model. It may be that an observer can recognize the CP asymmetry of the universe with laboratory experiments, without being able to measure his absolute velocity.

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RESONANCE AND HIGH-ENERGY LIMIT OF SCATTERING*

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The purpose of this note is to point out that the usual analyticity assumption implies a strong correlation between a pronounced low-energy resonance (such as the 33 resonance in the pion-nucleon system) and the high-energy limit of the scattering amplitude. Assuming that the forward elastic-scattering amplitude is analytic in the usual sense, it is proved in this note that the forward amplitude cannot satisfy an unsubtracted dispersion relation, as long as a pronounced low-energy resonance appears in the direct and/or the crossed channels. If one assumes furthermore that the forward amplitude becomes pure imaginary sufficiently rapidly in the limit of infinite energy, it is then proved that the high-energy limit of the total cross section can only be a finite, nonzero limit, if there is a pronounced low-energy resonance. Here a pronounced resonance is such a resonance whose peak in the total cross section is sufficiently high and/or broad enough to make its sole contribution to the integral in (5) or (8) already exceed the right-hand sides of (5) and (8). The quantities on the right-hand sides of (5) and (8) are the S-wave scattering length and the residue of the bound-state pole (if there is any) and, therefore, are some measure of the strength of the coupling in the conventional sense. According to the proof in this note, if a resonance becomes pronounced in the above sense, the total cross section can no longer approach a zero limit but must tend to a finite, nonzero cross section in the limit of infinite energy. It is also pointed out in this note that the 33 resonance is pronounced in this sense, and the ρ resonance is also pronounced in the same sense as long as a_2, the S-wave pion-pion scattering length in the channel of total isospin 2, does not exceed a fairly high limit given in (6).

According to a recent work,[1] one can estimate the high-energy limit of the pion-nucleon total cross section in terms of the mass and the width of the 33 resonance. According to the proof in this note, this is not to be regarded as an accident but due to this strong correlation between a pronounced low-energy resonance and the high-energy limit of the scattering amplitude. It is also possible[2] to estimate in an analogous way the high-energy limit of the pion-pion total cross section in terms of the mass and the width of the ρ resonance, assuming a weak S-wave pion-pion interaction in the sense of (6).

The above correlation refers to the high-energy limit of the full amplitude. It is not immediately clear whether a similar correlation exists between a pronounced low-energy resonance and the high-energy limit of the partial-wave amplitude. It is, however, quite possible that the high-energy behavior of the partial-wave amplitude is no longer arbitrarily disposable when there appears a pronounced low-energy resonance. Therefore, some of the difficulties[3] often encountered in the low-energy dynamical approach to the resonance may in fact be due to this correlation.

In order to simplify the proof, $\pi^0\pi^+$ scattering is discussed here in detail. Application to the other cases is straightforward, as is explained below. The $\pi^0\pi^+$ forward amplitude $A(\omega)$, as a function of the laboratory pion energy, is analytic in ω except for two cuts extending from $\pm\mu$ (pion mass) to $\pm\infty$, and is even with respect to the sign change in ω. One normalizes $A(\omega)$ in such a way that $\mathrm{Im}A(\omega)=q\sigma(\omega)/4\pi$, for $\omega\geq\mu$, where q is the laboratory pion momentum and $\sigma(\omega)$ is the $\pi^0\pi^+$ total cross section. In this normalization, $A(\mu)=a_2$, the S-wave pion-pion scattering length in the channel of total isospin 2.

It is known that $A(\omega)$ has a phase representation[4] of the following form:

$$A(\omega)=P_n(\omega)\exp\left\{\frac{2\omega^2}{\pi}\int_\mu^\infty\frac{\delta(\omega')d\omega'}{\omega'(\omega'^2-\omega^2)}\right\}, \qquad (1)$$

where $P_n(\omega)$ is an even, real polynomial of ω, and $\delta(\omega)$ is the phase of $A(\omega)$ defined in such a way that $A(\omega)=\pm|A(\omega)|e^{i\delta(\omega)}$, for $\omega\geq\mu$, and also that $\delta(\mu)=0$. Since $\mathrm{Im}A(\omega)$ is positive-definite for all $\omega>\mu$, $\delta(\omega)$ is bounded by $\pi>\delta(\omega)>0$ when $A(\mu)>0$, or $0>\delta(\omega)>-\pi$ when $A(\mu)<0$, for all $\omega>\mu$. If $A(\omega)$ becomes pure imaginary at infinity, $\delta(\infty)=\pi/2$ when $A(\mu)>0$, or $\delta(\infty)=-\pi/2$ when $A(\mu)<0$.

It is also known[4] that the phase representation (1) has the following asymptotic form:

$$A(\omega)\xrightarrow[\omega\to+\infty]{}\propto\omega^n\omega^{-2\delta(\infty)/\pi}, \qquad (2)$$

provided the phase $\delta(\omega)$ approaches its limit $\delta(\infty)$ sufficiently rapidly. Even if this last condition fails, the asymptotic form (2) is modified at most by a factor which behaves essentially logarithmically in ω. Therefore, the fastest approach of $A(\omega)$ to zero as $\omega\to+\infty$ [or the gentlest behavior of $A(\omega)$ at infinity] materializes when $\delta(\infty)=\pi$ if $A(\mu)>0$, or $\delta(\infty)=0$ if $A(\mu)<0$. In both cases, the phase $\delta(\omega)$ is always smaller than $\delta(\infty)$, and, therefore, the logarithmic factor which may modify the asymptotic form (2) can only diverge logarithmically but cannot approach zero, say, as $1/\ln\omega$. Thus, in the case of the fastest approach, $A(\omega)$ behaves at infinity strictly as ω^{n-2} when $A(\mu)>0$, or ω^n when $A(\mu)<0$. If $A(\omega)$ becomes pure imaginary sufficiently rapidly as $\omega\to+\infty$, $A(\omega)$ can behave at infinity only as ω^{n-1} when $A(\mu)>0$, or ω^{n+1} when $A(\mu)<0$. In all the above cases, n can only be $0, 2, 4, \cdots$, by definition. It is therefore seen that, as long as $A(\omega)$ has at least a pair of zeros when $A(\mu)\geq0$, $A(\omega)$ can no longer satisfy an unsubtracted dispersion relation.[5] In the case when $A(\omega)$ becomes pure imaginary sufficiently rapidly as $\omega\to+\infty$, the high-energy limit of the total cross section can no longer be a zero limit, as long as $A(\omega)$ has at least a pair of zeros when $A(\mu)\geq0$.

The condition for $A(\omega)$ to have at least a pair of zeros can be derived from the dispersion relation for $A(\omega)$. For this purpose, it is sufficient to assume that $A(\omega)/(\omega^2-\mu^2)$ vanishes everywhere at infinity in the ω plane. One thus obtains

$$A(\omega)=A(\mu)+\frac{(\omega^2-\mu^2)}{2\pi^2}\int_0^\infty\frac{\sigma(\omega')dq'}{\omega'^2-\omega^2}, \qquad (3)$$

which is the usual once-subtracted dispersion relation. Because of the positive-definiteness of $\sigma(\omega)$, $A(\omega)$ increases monotonically as ω increases from zero to $+\mu$. Therefore, there is a pair of zeros on the gap $(+\mu \geqslant \omega \geqslant -\mu)$ provided

$$(\mu^2/2\pi^2)\int_0^\infty \sigma(\omega)dq/\omega^2 \geqslant A(\mu) \geqslant 0,$$

but otherwise there are no zeros on the gap. However, as $A(\mu)$ exceeds the above limit, a pair of zeros begins to appear on the imaginary axis. To see this, one rewrites the dispersion relation (3) for $\omega = ia$, a being real, as

$$A(ia) = A(\mu) - \frac{1}{2\pi^2}\int_0^\infty \left(\frac{a^2+\mu^2}{a^2+\omega^2}\right)\sigma(\omega)dq. \qquad (4)$$

Since $\sigma(\omega)$ is positive-definite, the integral in (4) increases monotonically as a increases from zero to infinity. Therefore, a pair of zeros must appear on the imaginary axis as long as $A(\mu)$ does not exceed the limit

$$(1/2\pi^2)\int_0^\infty \sigma(\omega)dq > A(\mu), \qquad (5)$$

where it is not necessarily meant that the integral is convergent.

Therefore, $A(\omega)$ cannot satisfy an unsubtracted dispersion relation, as long as the inequality (5) is satisfied. In the case when $A(\omega)$ becomes pure imaginary sufficiently rapidly as $\omega \to +\infty$, $A(\omega)$ behaves asymptotically only as $\omega^1, \omega^3, \cdots$, as long as the inequality (5) is satisfied. The behavior of $A(\omega)$ as ω^3 or higher violates the unitarity in the sense that the total elastic cross section would then exceed the total cross section. To see this,[6] one has only to estimate the total elastic cross section, $\sigma_{\text{el}}(\omega)$, as

$$\sigma_{\text{el}}(\omega) \propto \int^0 |A(\omega,t)|^2 \frac{dt}{\omega^2} \propto \frac{|A(\omega,0)|^2}{\omega^2} \propto \frac{|A(\omega)|^2}{\omega^2},$$

where $A(\omega,t)$ is the covariant scattering amplitude expressed in terms of the covariant momentum transfer squared, t. Therefore, as long as the inequality (5) is satisfied, the high-energy limit of the total cross section can only be a finite, nonzero limit.

If a low-energy resonance appears in the lth partial wave with the mass M_γ and the full width Γ_γ, one can estimate the contribution of this resonance to the integral in (5), assuming that the resonance cross section is given by $2\pi^2(2l+1)\Gamma_\gamma\delta(E-M_\gamma)/p^2$, where E is the c.m. total energy and p is the c.m. momentum. In the case of the ρ resonance, this contribution amounts to $0.96\mu^{-1}$ when $M_\rho = 760$ MeV and $\Gamma_\rho = 106$ MeV. Therefore, as long as the ρ resonance appears, the high-energy limit of the pion-pion total cross section can only be a finite, nonzero limit, provided a_2, the S-wave pion-pion scattering length in the channel of total isospin 2, does not exceed the limit

$$0.96\mu^{-1} > a_2. \qquad (6)$$

This is indeed a fairly high upper limit for a_2.

The foregoing analysis applies to $\pi^0\pi^0$ scattering without any change. If only the f resonance ($M_f = 1253$ MeV, $\Gamma_f = 100$ MeV, $l = 2$) is considered in the integral in (5), the inequality (5) becomes

$$0.28\mu^{-1} > (\tfrac{2}{3})a_2 + (\tfrac{1}{3})a_0, \qquad (7)$$

where a_0 is the S-wave pion-pion scattering length in the channel of total isospin 0. The upper limit in (7) can be substantially increased if the S-wave contribution to the integral in (5) is properly included. However, details of this shall not be discussed here.

In order to apply the preceeding analysis to $\pi^-\pi^+$ scattering, one combines the $\pi^-\pi^+$ amplitude with its crossed amplitude (the $\pi^+\pi^+$ amplitude) so that the amplitude becomes even[7] with respect to the sign change in ω. Similar symmetrization is necessary in the case of $\pi^\pm p$ scattering. Another complication arises in this case because of the nucleon poles. However, one can carry out essentially the same proof, because these poles more or less induce a pair of zeros and one has only to locate a pair of zeros on the imaginary axis. Thus, the same conclusion holds also in $\pi^\pm p$ scattering as long as the following inequality is satisfied:

$$\left(\frac{1}{2\pi^2}\right)\int_0^\infty \sigma(\omega)dq > A(\mu) + \frac{2\omega_0 g^2}{(\mu^2-\omega_0^2)}, \qquad (8)$$

where $\sigma(\omega)$ is the average of the $\pi^\pm p$ total cross sections; $A(\mu) = [1+(\mu/M)](a_1+2a_3)/3$, where M is the nucleon mass and a_1 and a_3 are the S-wave pion-nucleon scattering lengths in the channels of total isospins $\frac{1}{2}$ and $\frac{3}{2}$, respectively; $g^2 = 0.081$ is the renormalized pion-nucleon coupling constant; and $\omega_0 = \mu^2/2M$ is the location of the nucleon pole.

The contribution of the 33 resonance to the integral in (8) becomes $0.68\mu^{-1}$, when $M_{33} = 1237$ MeV and $\Gamma_{33} = 90$ MeV. This contribution is much larger than the right-hand side of (8), because the empirical figures[8] are $a_1 = (0.171 \pm 0.005)\mu^{-1}$, $a_3 = (-0.088 \pm 0.004)\mu^{-1}$, and $2\omega_0 g^2/(\mu^2-\omega_0^2) \simeq g^2/M = 0.012\mu^{-1}$. Therefore, as long as the 33 resonance exists, the high-energy limit of the pion-nucleon total cross section can only be a finite, nonzero limit.

*Work supported by the National Science Foundation.

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[3]For example, J. S. Ball and D. Y. Wong [Phys. Rev. Letters 7, 390 (1961)] point out that a far-away pole in the P-wave amplitude is indispensable in obtaining a resonance in the pion-pion problem. Recently, in their bootstrap approach to the pion-nucleon problem, K. Huang and F. E. Low [Phys. Rev. Letters 13, 596 (1964)] emphasize the necessity of introducing a subtraction into the partial-wave dispersion relation in order to obtain a bootstrap solution.

[4]M. Sugawara and A. Tubis, Phys. Rev. 130, 2127 (1963).

[5]The existence of a pair of zeros implies that $A(\omega)$ cannot vanish at infinity. In the case of the fastest approach of $A(\omega)$ to zero, however, $A(\omega)$ becomes pure real at infinity because $\delta(\infty)$ is then π or 0. Therefore, $A(\omega)$ cannot satisfy an unsubtracted dispersion relation. In the case when $A(\omega)$ approaches a finite pure imaginary limit, $A(\omega)$ can still satisfy an unsubtracted dispersion relation, as is discussed by M. Sugawara and A. Kanazawa, Phys. Rev. 123, 1895 (1961).

[6]The argument below is essentially the same as that given by M. Sugawara and A. Tubis, Phys. Rev. Letters 9, 355 (1962).

[7]If $A(\omega)$ is not even with respect to the sign change in ω, $A(\omega)$ is no longer real on the imaginary axis and, therefore, the preceeding argument concerning a pair of zeros on the imaginary axis no longer holds.

[8]J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963).

ERRATA

$Y = 2$ STATES IN SU(6) THEORY. Freeman J. Dyson and Nguyen-huu Xuong [Phys. Rev. Letters 13, 815 (1964)].

Experiment on π^--D scattering at 140 MeV[1] shows a large ratio of inelastic to elastic scattering, indicating that many angular momenta contribute. The peak at 170 MeV ($E^* = 2160$ MeV) is, therefore, not to be understood as belonging to a particular SU(6) representation. On the contrary, it seems to provide evidence that the splitting between SU(6) representation 490 and 1050 is small compared with the "SU(6)-violating" splitting between octet and decimet states of the single baryons. In this case, the hypothesis advanced in the second paragraph of our Letter is not valid. We thank Dr. R. Hofstadter for referring us to the experimental paper quoted above, and Dr. S. Treiman and Dr. W. Alles for criticism.

[1]E. G. Pewitt, T. H. Fields, G. B. Yodh, J. G. Fetkovich, and M. Derrick, Phys. Rev. 131, 1826 (1963).

STUDY OF EXCHANGE INTEGRAL OF CRYSTALLINE ^{3}He AT 0°K. L. H. Nosanow and William J. Mullin [Phys. Rev. Letters 14, 133 (1965)].

There is a typographical error in Eq. (12). It should read

$$V_{ij} = v_{ij} - (\hbar^2/2mf_{ij}^2)(f_{ij}\nabla_i^2 f_{ij} - \vec{\nabla}_i f_{ij} \cdot \vec{\nabla}_i f_{ij}).$$