dicted to be independent of the Treiman-Yang angle φ_π, and to vary as $\cos^2\theta_\pi$ as a function of the $\pi\pi$ scattering angle. The data show a much more complicated dependence on φ_π and $\cos\theta_\pi$.

[10]The value $\Gamma_\epsilon = 140$ MeV corresponds to the full width of the resonance. Since the only decay modes of the ϵ^0 which are expected to be significant for present purposes are $\epsilon^0 \rightarrow \pi^+ + \pi^-$ and $\epsilon^0 \rightarrow 2\pi^0$, with a 2:1 branching ratio, the effective $\epsilon^0\pi^+\pi^-$ coupling constant used in the calculation corresponds to the partial width for the $\pi^+\pi^-$ decay mode, $\Gamma_{\epsilon+-} = 93$ MeV.

[11]The lack of exact agreement between the present theory and experiment is hardly surprising considering the uncertainties in the low partial waves in the $\pi^- + p \rightarrow \rho^0 + n$ and $\pi^- + p \rightarrow \epsilon^0 + n$ amplitudes. We have also neglected the small real $T = 0, J = 2^+$ amplitude expected from the tail of the f^0 resonance. Although this will have very little effect on the cross section it can change the $\pi^+\pi^-$ decay distribution significantly, particularly the forward-to-backward ratio. On the other hand, it cannot, by itself, explain the data. It is interesting also to note that the absorptive effects reduce the forward-to-backward asymmetry which would be predicted by the unmodified single-pion-exchange model. The odd term in $\cos\theta_\pi$ in the angular distribution results from the interference between the ϵ^0 and ρ^0 contributions to the state of the di-pion system with $J_z = 0$ (the z axis is taken along the direction of motion of the incoming pion as seen in the rest system of the di-pion). In the unmodified theory, the fact that the incoming pion is spinless implies that the ρ^0 is always produced in this state. This need not be the case in the modified theory because of the initial- and final-state interactions. For example, at 3 BeV/c for $-t < 10m_\pi^2$, the modified theory predicts that the $\rho^\pm$ and ρ^0 spins are 60% in the state with $J_z = 0$, and 40% in the state with $J_z = \pm 1$. As we have seen, this prediction yields a good fit to the ρ^--decay angular distribution, and we expect it to be essentially correct for the ρ^0 as well. The amplitude of the odd term in $\cos\theta_\pi$ is thus smaller by roughly $0.78 = (0.6)^{1/2}$ than is the case for the unmodified model, and the forward-to-backward asymmetry is reduced accordingly. On this basis, it seems unlikely that one can account for the asymmetry in the modified theory using a constant $J = 0$ phase shift of ~60°, as was proposed in the first paper of reference 4.

[12]R. W. Birge, R. P. Ely, Jr., T. Schumann, Z. G. T. Guiragossián, and M. N. Whitehead, Proceedings of the International Conference on High Energy Physics, Dubna, 1964 (to be published). The authors are greatly indebted to Dr. Guiragossián and Dr. Birge for supplying their data prior to publication, and for several comments thereon.

[13]A missing-mass spectrum for the reaction $\pi^+ + d \rightarrow p + p + \mathrm{MM}$ has been reported by N. Gelfand, G. Lütjens, M. Nussbaum, J. Steinberger, H. O. Cohn, W. M. Buss, and G. T. Condo, Phys. Rev. Letters 12, 568 (1964). The ϵ^0 would appear in this experiment as a bump with a width of ~140 MeV containing ~25 events. The situation is confused by the strong contributions to the missing-mass spectrum from the neutral decay modes of the ω^0 and η^0 mesons. Although these contributions could, in principle, be removed by measuring the ω^0 and η^0 components of the reaction $\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0$, and using the known branching ratios for the neutral and charged decay modes, this subtraction was apparently not attempted. The results are thus inconclusive for present purposes. We note only that the neutral background in the neighborhood of the ρ^0 or ω^0 mass appears to be rather high, and that the apparent ω^0 peak is shifted toward the high-mass region.

INTERNAL SYMMETRY AND LORENTZ INVARIANCE

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Recently a number of papers[1] have appeared in which the possibility of combining internal symmetry and Lorentz invariance is discussed. Most of these papers have discussed the possibility within the framework of some specific model, or under some specific assumptions. It may be of interest, therefore, to consider the question from a general point of view and to try to determine the most general conditions under which a combination is possible. The purpose of the present note is to consider this problem. The point of departure for our discussion is the general result[2] that any Lie algebra E can be expressed as the semidirect sum of a semisimple subalgebra G (the Levi factor) and an invariant solvable subalgebra S (the radical), i.e.,

$$E = G \oplus S. \tag{1}$$

Using this result, we establish the following theorem:

Theorem.—Let E be any Lie algebra with Levi factor G and radical S. Let L be the Lie algebra of the inhomogeneous Lorentz group, consisting of the homogeneous part M and translation part P. If L is a subalgebra of E, then either

$$\text{(a) } M \subset G : P \subset S, \tag{2}$$

or

$$\text{(b) } L \cap S = 0, \tag{3}$$

where $\subset$ denotes "is a subalgebra of" and $\cap$ denotes intersection.

Proof.—By the Marcev–Harish-Chandra theorem,[2] and the semisimplicity of M, we can define G so that, in any case,

$$M \subset G. \tag{4}$$

On the other hand, from the Lorentz relation

$$[M, P] = P, \tag{5}$$

and the invariance of S, we have

$$[M, P \cap S] = P \cap S. \tag{6}$$

Thus, with respect to M, $P \cap S$ is an invariant subspace of the space P. But P is irreducible with respect to M. Hence

$$P \cap S = P \text{ or } P \cap S = 0. \tag{7}$$

This establishes the theorem.

The relevance of this theorem is that it allows us to classify the ways in which L can be imbedded as a subalgebra of a larger Lie algebra E. To make this classification, it is convenient to subdivide case (a) above into the three cases (i) $S = P$, (ii) S Abelian, but larger than and containing P, (iii) S solvable, but not Abelian, and containing P. If we add to this the other possibility, (b) above, or (iv) $S \cap P = 0$, we see that we have, in all, four classes. Practically all of the proposals made so far for combining L and internal symmetry belong to classes (i) and (ii). All four classes will be discussed in more detail in a forthcoming paper. Here we shall merely summarize some of the more outstanding results.

In case (i), the relation

$$[G, S] = S \tag{8}$$

implies that S is a representation space for G. But since the space $S = P$ is four dimensional, and G contains M, it can be shown that G can then be written in the form

$$G = G_0 \oplus G_r, \tag{9}$$

where $\oplus$ here denotes direct sum; $\tilde{G}_0$, the complex extension of G_0, is isomorphic to A_3, B_2, or $A_1 \oplus A_1$ (in the Cartan notation); the transformations

$$[G_0, P] = P \tag{10}$$

contain the Lorentz transformations (5); and G_r (r = remainder) is a semisimple algebra satisfying

$$[G_r, P] = 0. \tag{11}$$

On account of the low order of G_0, which already contains the Lorentz transformations, any nontrivial internal-symmetry algebra must be contained in G_r, which commutes with G_0 and P. In this sense, case (i) is trivial. This is one of the reasons for the negative results obtained earlier by other authors.

Case (ii) is not trivial in the same sense, since in this case the dimensionality of S is not restricted, but it has the disadvantage of introducing a translation group of more than four dimensions. This is not easy to interpret physically. Case (iii) appears to be rather unphysical, as solvable algebras are not usually considered by physicists. This is because, for solvable algebras, Hermitian conjugation cannot be defined for all elements, at least for finite-dimensional representations. Case (iv) is possible, although it involves imbedding an algebra isomorphic to L in a simple algebra. However, the simple algebra is necessarily extremely noncompact, which may lead to difficulties in defining multiplets.

With regard to the question of mass splitting, which is one of the reasons for attempting to imbed L in a larger Lie algebra E, we can make the following statement: In cases (i) and (ii) of our classification, there can be no mass splitting. More specifically, we establish the following theorem:

Theorem.—Let R be the vector space of any irreducible representation of E, and R_m any subspace of R which is an eigenspace of the mass operator $p_\mu p^\mu$ belonging to the discrete eigenvalue m^2. Then in cases (i) and (ii) above, the space R_m is invariant with respect to E. Hence, $R = R_m$, and there is no mass splitting.

Proof.—Let $|R_m\rangle$ be any vector in R_m, E any element of E, and consider the vector

$$|\psi\rangle = E\,|R_m\rangle. \tag{12}$$

Since

$$(p_\mu p^\mu - m^2)\,|R_m\rangle = 0, \tag{13}$$

we have

$$(p_\mu p^\mu - m^2)^2 |\psi\rangle$$

$$= [p_\mu p^\mu, [p_\sigma p^\sigma, E]] |R_m\rangle$$

$$= [p_\mu p^\mu, p_\sigma [p^\sigma, E] + [p_\sigma, E] p^\sigma] |R_m\rangle. \quad (14)$$

Using the invariance of S and the fact that for cases (i) and (ii) S contains P, we have

$$(p_\mu p^\mu - m^2)^2 |\psi\rangle = [p_\mu p^\mu, p_\sigma S^\sigma + S_\sigma p^\sigma] |R_m\rangle, \quad (15)$$

where S_σ is some element of S. But every element in the commutator on the right-hand side of (15) is contained in S, which in cases (i) and (ii) is Abelian. Hence, this commutator is zero. Thus

$$(p_\mu p^\mu - m^2)^2 |\psi\rangle = 0. \quad (16)$$

Multiplying this equation to the left by $\langle\psi|$, we have, using the Hermiticity of p_μ,

$$\langle\varphi|\varphi\rangle = 0, \quad (17)$$

where

$$|\varphi\rangle = (p_\mu p^\mu - m^2) |\psi\rangle. \quad (18)$$

Hence, from the positive definiteness of the matrix, we have

$$(p_\mu p^\mu - m^2) |\psi\rangle = 0, \quad (19)$$

whence

$$|\psi\rangle = E |R_m\rangle \subset R_m. \quad (20)$$

Since R is irreducible, this implies that $R = R_m$; hence, $p_\mu p^\mu$ is equal to m^2 throughout R, and there is no mass splitting.

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[2]N. Jacobson, Lie Algebras (Interscience Publishers, Inc., New York, 1962), p. 24-25.

[3]The definitions of a semisimple Lie algebra as a Lie algebra which does not contain an Abelian invariant subalgebra or a Lie algebra which does not contain a solvable invariant subalgebra are equivalent.[2]

DECAY $K_L{}^0 \to 2\pi$ AND SPONTANEOUS BREAKDOWN OF THE CP SYMMETRY

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To explain the decay $K_L{}^0 \to 2\pi$ observed recently,[1] a very attractive possibility has been suggested by Bell and Perring, and by Bernstein, Cabibbo, and Lee[2]: The apparent CP violation may be a consequence of the CP asymmetry of the state of our world. Starting with the CP-symmetric interaction

$$H_{\text{int}} = ig\varphi_\mu(\bar{K}^0\partial_\mu K^0 - K^0\partial_\mu \bar{K}^0), \quad (1)$$

and supposing that in the local vacuum state

$$\langle 0|\varphi_0|0\rangle \neq 0, \quad \langle 0|\varphi_j|0\rangle = 0 \text{ for } j = 1, 2, 3, \quad (2)$$