DECAY OF THE ρ^0 MESON, AND THE POSSIBLE EXISTENCE OF A $T=0$ SCALAR DI-PION*

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It has been known for some time that the decay angular distribution of the "ρ^0 meson" observed in the reaction $\pi^- + p \to \pi^+ + \pi^- + n$ is inconsistent with the assumption that the $\pi^+\pi^-$ system is in a pure $T=1, J^P=1^-$ state at the resonant energy.[1] This is not the case for the charged ρ mesons observed through the reactions $\pi^\pm + p \to \pi^\pm + \pi^0 + p$. The ρ-decay angular distributions in the latter reactions may in fact be accounted for in detail using the single-pion-enchange model for ρ production, modified, however, to take properly into account the effects of competing processes on the reaction amplitude.[2] The absence of any notable peculiarities in the decays of the charged ρ mesons appears to eliminate final-state interactions between the decay pions and the nucleon as the cause of the ρ^0 anomaly. It has consequently been suggested by several authors[3,4] that the anomalies in the "ρ^0"-decay distribution and, in particular, the large forward-to-backward asymmetry in the decay, result from the interference of the resonant $T=1, J=1^-$ amplitude in the $\pi^+\pi^-$ system with a $T=0$ amplitude with $J=0^+$ or 2^+. We have explored this suggestion in detail using the modified single-pion-exchange model to describe the production process. The results obtained assuming the $\pi^+\pi^-$ system to result from the decay of a superposition of the ρ^0 resonance and a resonant $T=0, J=0^+$ state ϵ^0 with $m_\epsilon \sim m_\rho$ and $\Gamma_\epsilon \sim \Gamma_\rho$ are sufficiently striking that we wish to report them at this time, and to suggest that a search for the $2\pi^0$ decay mode of the ϵ^0 be included in future experiments.

The possible explanations of the forward-to-backward asymmetry in the $\pi^+\pi^-$ angular distribution in terms of interference between the $J=1^-$ state and a neighboring state with $J=0^+, 2^+, \cdots$, are severely restricted by the observation that this ratio is large at the mass of the ρ, is approximately symmetric around that mass, and decreases only slowly in the wings of the ρ peak.[1,5,6] Since the $J=1^-$ amplitude is purely imaginary at the ρ mass, whatever background is present must have a large imaginary component at that mass. On the other hand, the absence of a marked asymmetry in the forward-to-backward ratio about the ρ mass precludes the presence of a large real component in the background unless that component, like the real component of the 1^- amplitude, changes sign in the ρ region. Finally, the rapid decrease of the imaginary part of the 1^- amplitude away from the ρ mass probably requires that some real background be present to account for the rather large forward-to-backward ratios obtained in the wings of the "ρ^0" mass spectrum. These considerations suggest strongly that the main background term is in fact resonant at a dipion mass near that of the ρ, and that the width of the resonance is comparable to Γ_ρ. However, a large or resonant $\pi\pi$ scattering amplitude in a $T=0$ state with $J \geq 2$ would lead through the modified single-pion-exchange model to a cross section for $\pi^+\pi^-$ production much too large to be consistent with the near equality of the experimental $\pi^+\pi^-:\pi^\pm\pi^0$ ratios for the reactions $\pi^\pm + p \to \pi + \pi + N$, and the theoretical ratio 2:1 expected for pure ρ production. We have consequently assumed the background to arise from a resonant $T=0, J=0^+$ state ϵ^0 of the $\pi^+\pi^-$ system with $m_\epsilon \sim m_\rho$ and $\Gamma_\epsilon \sim \Gamma_\rho$.

The differential reaction cross section and the $\pi^+\pi^-$ angular distribution to be expected in the reaction $\pi^- + p \to \pi^+ + \pi^- + n$ have been calculated at several energies using the single-pion-exchange model for p^0 and ϵ^0 production as modified for the effects of initial- and final-state interactions. Details of the model have been discussed elsewhere.[2,7] The results of the calculation at 3 BeV/c are compared with experimental data[8] at that energy in Figs. 1 and 2 and Table I. To indicate the reliability of the model, we have included the results obtained at the same energy for the ρ^- reaction $\pi^- + p \to \pi^- + \pi^0 + p$; results of similar quality have been obtained previously for the ρ^+ reaction $\pi^+ + p \to \pi^+ + \pi^0 + p$ at 2.08 and 4 BeV/c.[2,7] The prediction for the ρ^- production cross section in Fig. 1(a) is absolute: The pion-nucleon and $\rho\pi\pi$ coupling constants are known, the latter in terms of the ρ decay width, and the absorption factors which describe the initial- and final-state interactions are known within reasonable limits.[2,7] Despite some uncertainties in the low partial-wave amplitudes, the

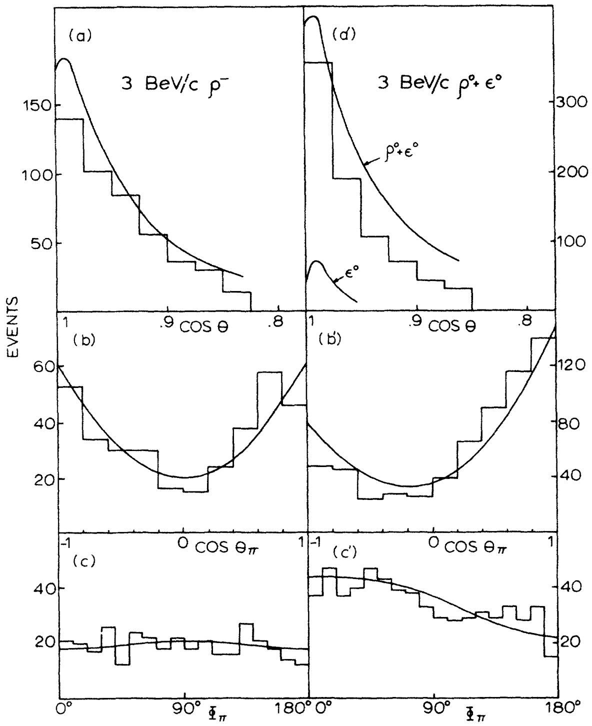

FIG. 1. Comparison of (a), (a′) the predicted di-pion production angular distribution; (b), (b′) the predicted decay distribution in $\cos\theta_\pi$; and (c), (c′) the predicted Treiman-Yang distribution, with the corresponding experimental results for the reactions $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ and $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 3 BeV/c. The data are from reference 8, and are restricted to di-pion masses in the range 700 MeV $\leq m_{\pi\pi} \leq$ 850 MeV and, for the decay distributions, to momentum transfer less than $10 m_\pi^2$. The theoretical predictions for (a) and (a′) are absolute.

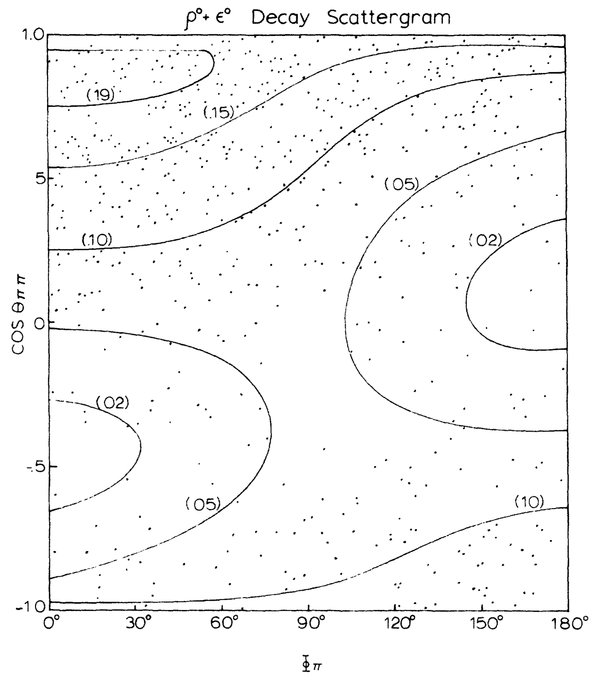

FIG. 2. Joint di-pion decay distribution in $\cos\theta_\pi$ and φ_π for the reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 3 BeV/c. The experimental points are from reference 8. The theoretical contour plot corresponds to the ρ^0 and ϵ^0 parameters given in the text.

model is rather successful in predicting the shape of the angular distribution, and the absolute magnitude of the cross section. The slight residual discrepancy in the magnitude would be absent had we used a smaller value for the ρ^- decay width, $\Gamma_\rho \approx 105$ MeV instead of $\Gamma_\rho \approx 120$ MeV. As has been emphasized by Gottfried and Jackson,[2] a more sensitive test of the theory is provided by the ρ^--decay angular distribution.[9] The predictions for the distributions in the $\pi\pi$ scattering angle θ_π and the Treiman-Yang angle φ_π shown in Figs. 1(b) and 1(c) are in excellent agreement with the data. The prediction for the joint distribution in $\cos\theta_\pi$ and φ_π is also essentially correct. The complete distribution is not shown, but the theoretical and experimental results are compared by quadrants in Table I. The rather striking success of the modified single-pion-exchange model for the $\rho^\pm$ reactions suggests strongly that the results to be presented for

Table I. Comparison by quadrants of the theoretical and experimental results for the normalized $\pi\pi$ angular distributions in the reactions $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ and $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 3 BeV/c. The di-pion masses were restricted to the range 700 MeV $\leq m_{\pi\pi} \leq$ 850 MeV, and the momentum transfer to the nucleon, to the range $-t \leq 10 m_\pi^2$. The data are from reference 8.

	$\pi^-\pi^0$		$\pi^+\pi^-$	
Quadrant	Theory	Expt.	Theory	Expt.
$0 < \cos\theta_\pi < 1,\ 0 < \varphi_\pi < \pi/2$	109	113	249	285
$-1 < \cos\theta_\pi < 0,\ 0 < \varphi_\pi < \pi/2$	64	68	93	76
$0 < \cos\theta_\pi < 1,\ \pi/2 < \varphi_\pi < \pi$	64	68	137	164
$-1 < \cos\theta_\pi < 0,\ \pi/2 < \varphi_\pi < \pi$	109	95	139	93

the ρ^0, ϵ^0 reaction should also be reliable.

The predictions for the production and decay angular distributions for the $\pi^+\pi^-$ system in the reaction $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ are compared with the experimental data at 3 BeV/c in Figs. 1 and 2 and in Table I. The theoretical results correspond to the parameters $m_\rho = 760$ MeV, $\Gamma_\rho = 120$ MeV, $m_\epsilon = 770$ MeV, and $\Gamma_\epsilon = 140$ MeV,[10] the parameters for the ϵ^0 lying in the region suggested by our previous arguments. The fit is rather insensitive to small variations in the mass of the ϵ^0, provided it is close to that of the ρ^0. However, the quality of the fits to the $\pi^+\pi^-$ angular distributions is significantly reduced if Γ_ϵ is changed by more than about 25%. We have not attempted thus far to obtain an optimum set of parameters within these rather wide limits. The amount of $\pi^+\pi^-$ production which proceeds through the ϵ^0 is small, about 10% of that which proceeds through the ρ^0 for events in the forward peak, $1 \geq \cos\theta \geq 0.9$. It is thus not surprising that the experimental branching ratio for $\pi^+\pi^-:\pi^-\pi^0$ production is close to the value 2:1 expected for pure ρ production. The theoretical prediction for the differential production cross section for the $\pi^+\pi^-$ system shown in Fig. 1(a) is good in both shape and magnitude, but does not differ markedly from that obtained for the $\pi^+\pi^0$ reaction. However, the decay angular distribution of the di-pion in the $\pi^+\pi^-$ reaction is changed rather drastically by the presence of the small admixture of the ϵ^0. The agreement with the data is very good. The forward-to-backward asymmetry in $\cos\theta_\pi$, Fig. 1(b′), has the correct sign, and is fitted reasonably well in magnitude.[11] The fit to the Treiman-Yang distribution, which now contains a term in $\cos\varphi_\pi$ as well as $\cos 2\varphi_\pi$, is excellent [Fig. 1(c′)]. The joint distribution in $\cos\theta_\pi$ and φ_π is shown in Fig. 2 as a contour plot superposed on the experimental scattergram. The results are certainly correct qualitatively. The predicted and actual numbers of events are again listed by quadrants in Table I. We have obtained equally good results for the reactions $\pi^- + p \rightarrow \pi^- + \pi^0 + p$ and $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 3.9 BeV/c. In particular, the modified single-particle-exchange model, including the ϵ^0 state with the foregoing parameters, yields an excellent fit to the di-pion–decay angular distribution in the recent high-statistics experiment of Birge et al.[12]

The foregoing results are sufficiently impressive overall that we feel the suggested existence of a $T=0, J=0^+$ di-pion with $m_\epsilon \sim m_\rho$ and $\Gamma_\epsilon \sim \Gamma_\rho$ should be taken seriously. This particle could be detected unambiguously by the observation of the neutral decay mode $\epsilon^0 \rightarrow \pi^0 + \pi^0$ in such reactions as $\pi^- + p \rightarrow \pi^- + p + \pi^0 + \pi^0$, $\pi^+ + p \rightarrow \pi^+ + p + \pi^0 + \pi^0$, and $\pi^+ + d \rightarrow p + p + \pi^0 + \pi^0$.[13] Since these reactions can all proceed through single-pion exchange, the ϵ^0:ρ^0 production ratios should be similar to that noted above, about 0.15, with $\frac{1}{3}$ of the ϵ^0's decaying by the $\pi^0\pi^0$ mode.

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[1]The experimental situation has been summarized by W. Selove, Proceedings of the Conference on Particles and High Energy Physics, Boulder, Colorado, 1964 (to be published).

[2]K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964). L. Durand, III, and Y. T. Chiu, Proceedings of the Conference on Particles and High Energy Physics, Boulder, Colorado, 1964 (to be published); and Phys. Rev. (to be published). I. Derado, V. P. Kenney, and W. D. Shephard, Phys. Rev. Letters 13, 505 (1964). Aachen-Birmingham-Bonn-Hamburg-London (I.C.)-Munich Collaboration, to be published.

[3]V. Hagopian and W. Selove, Phys. Rev. Letters 10, 533 (1963); Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 29, 515 (1963).

[4]M. M. Islam and R. Piñon, Phys. Rev. Letters 12, 310 (1964); S. H. Patil, Phys. Rev. Letters 13, 261 (1964).

[5]Saclay-Orsay-Bari-Bologna Collaboration, to be published.

[6]In contrast, the forward-to-backward asymmetry in the $\pi^\pm\pi^0$ angular distribution passes through zero close to the apparent $\rho^\pm$ mass, and is small throughout the ρ mass region. The behavior is consistent with that expected from the interference of a dominant $J = 1^-$ $\pi\pi$ scattering amplitude with a small real background in the $T=2, J=(0^+ \text{or } 2^+)$ state.

[7]L. Durand, III, and Y. T. Chiu, Phys. Rev. (to be published).

[8]The data at "3 BeV/c" are from a compilation of the 2.75-BeV/c data of the Saclay-Orsay-Bari-Bologna Collaboration, reference 5, and the 3-BeV/c data of Hagopian and Selove, reference 3. The authors would like to thank Professor Walter Selove for supplying these data, and for comments thereon.

[9]The single-pion-exchange model used in the calculations in reference 4 is unable to account for the ρ decay distribution. This model includes arbitrary form factors at the vertices and on the propagator of the exchanged pion, but ignores the absorptive modifications of the low to medium partial-wave amplitudes. Although it is possible to fit the production angular distribution using the form factors, these do not change the ρ decay distribution. $I(\cos\theta_\pi, \varphi_n)$ is therefore pre-

dicted to be independent of the Treiman-Yang angle φ_π, and to vary as $\cos^2\theta_\pi$ as a function of the $\pi\pi$ scattering angle. The data show a much more complicated dependence on φ_π and $\cos\theta_\pi$.

[10]The value $\Gamma_\epsilon = 140$ MeV corresponds to the full width of the resonance. Since the only decay modes of the ϵ^0 which are expected to be significant for present purposes are $\epsilon^0 \to \pi^+ + \pi^-$ and $\epsilon^0 \to 2\pi^0$, with a 2:1 branching ratio, the effective $\epsilon^0\pi^+\pi^-$ coupling constant used in the calculation corresponds to the partial width for the $\pi^+\pi^-$ decay mode, $\Gamma_{\epsilon+-} = 93$ MeV.

[11]The lack of exact agreement between the present theory and experiment is hardly surprising considering the uncertainties in the low partial waves in the $\pi^- + p \to \rho^0 + n$ and $\pi^- + p \to \epsilon^0 + n$ amplitudes. We have also neglected the small real $T=0, J=2^+$ amplitude expected from the tail of the f^0 resonance. Although this will have very little effect on the cross section it can change the $\pi^+\pi^-$ decay distribution significantly, particularly the forward-to-backward ratio. On the other hand, it cannot, by itself, explain the data. It is interesting also to note that the absorptive effects reduce the forward-to-backward asymmetry which would be predicted by the unmodified single-pion-exchange model. The odd term in $\cos\theta_\pi$ in the angular distribution results from the interference between the ϵ^0 and ρ^0 contributions to the state of the di-pion system with $J_z = 0$ (the z axis is taken along the direction of motion of the incoming pion as seen in the rest system of the di-pion). In the unmodified theory, the fact that the incoming pion is spinless implies that the ρ^0 is always produced in this state. This need not be the case in the modified theory because of the initial- and final-state interactions. For example, at 3 BeV/c for $-t < 10m_\pi^2$, the modified theory predicts that the $\rho^\pm$ and ρ^0 spins are 60% in the state with $J_z = 0$, and 40% in the state with $J_z = \pm 1$. As we have seen, this prediction yields a good fit to the ρ^--decay angular distribution, and we expect it to be essentially correct for the ρ^0 as well. The amplitude of the odd term in $\cos\theta_\pi$ is thus smaller by roughly $0.78 = (0.6)^{1/2}$ than is the case for the unmodified model, and the forward-to-backward asymmetry is reduced accordingly. On this basis, it seems unlikely that one can account for the asymmetry in the modified theory using a constant $J = 0$ phase shift of $\sim 60°$, as was proposed in the first paper of reference 4.

[12]R. W. Birge, R. P. Ely, Jr., T. Schumann, Z. G. T. Guiragossián, and M. N. Whitehead, Proceedings of the International Conference on High Energy Physics, Dubna, 1964 (to be published). The authors are greatly indebted to Dr. Guiragossián and Dr. Birge for supplying their data prior to publication, and for several comments thereon.

[13]A missing-mass spectrum for the reaction $\pi^+ + d \to p + p + \mathrm{MM}$ has been reported by N. Gelfand, G. Lütjens, M. Nussbaum, J. Steinberger, H. O. Cohn, W. M. Buss, and G. T. Condo, Phys. Rev. Letters 12, 568 (1964). The ϵ^0 would appear in this experiment as a bump with a width of ~140 MeV containing ~25 events. The situation is confused by the strong contributions to the missing-mass spectrum from the neutral decay modes of the ω^0 and η^0 mesons. Although these contributions could, in principle, be removed by measuring the ω^0 and η^0 components of the reaction $\pi^+ + d \to p + p + \pi^+ + \pi^- + \pi^0$, and using the known branching ratios for the neutral and charged decay modes, this subtraction was apparently not attempted. The results are thus inconclusive for present purposes. We note only that the neutral background in the neighborhood of the ρ^0 or ω^0 mass appears to be rather high, and that the apparent ω^0 peak is shifted toward the high-mass region.

INTERNAL SYMMETRY AND LORENTZ INVARIANCE

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Recently a number of papers[1] have appeared in which the possibility of combining internal symmetry and Lorentz invariance is discussed. Most of these papers have discussed the possibility within the framework of some specific model, or under some specific assumptions. It may be of interest, therefore, to consider the question from a general point of view and to try to determine the most general conditions under which a combination is possible. The purpose of the present note is to consider this problem. The point of departure for our discussion is the general result[2] that any Lie algebra E can be expressed as the semidirect sum of a semisimple subalgebra G (the Levi factor) and an invariant solvable subalgebra S (the radical), i.e.,

$$E = G \oplus S. \tag{1}$$

Using this result, we establish the following theorem:

Theorem.–Let E be any Lie algebra with Levi factor G and radical S. Let L be the Lie algebra of the inhomogeneous Lorentz group, consisting of the homogeneous part M and translation part P. If L is a subalgebra of E, then either

$$\text{(a) } M \subset G : P \subset S, \tag{2}$$