SU(6) IN A BARYON BOOTSTRAP MODEL*

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There are several attractive features of the SU(3)-invariant, reciprocal bootstrap model, in which a degenerate $j=\frac{1}{2}$ baryon octet (B_8) and a degenerate $j^P=\frac{3}{2}^+$ baryon decuplet (B_{10}) are produced as resonance and bound-state poles by the B_8 and B_{10} exchange forces in scattering states of the type B_8 + pseudoscalar meson octet (P_8).[1,2] The predicted $\pi NN/\pi NN^*$ interaction ratio and $P_8B_8B_8$ interaction F/D ratio are in rough agreement with experiment. The model has one ugly feature, however. It is difficult to treat the B_8 and B_{10} multiplets on equal footing by including scattering states of the type P_8B_{10}. To the author's knowledge, this problem has not been solved, even in the limit in which the B_8-B_{10} mass difference is neglected and the static model is used. It is not even certain that a consistent solution exists.[3] The theoretical difficulty results from the large magnitude of the element of the P-meson–baryon, spin-crossing matrix referring to the force in the $j=\frac{3}{2}$ state caused by exchange of the spin $\frac{1}{2}$ baryon. One starts with a spin-$\frac{1}{2}$ baryon multiplet and some postulated internal symmetry, and is inevitably led to predicting a second ($j=\frac{3}{2}$) baryon multiplet, which complicates the model.

The purpose of this note is to construct a simpler baryon bootstrap model. We first complicate the standard model by adding to it still more particles, a vector-meson singlet and octet (V_1 and V_8). For simplicity, we neglect the B_8-B_{10} mass difference, and take all the mesons to have a common mass small enough that the static model may be used. The V mesons, like the P mesons, are emitted and absorbed in P waves. We look for an SU(3)-invariant bootstrap solution in which the B-exchange force produces the B multiplet in the coupled μB scattering states, where μ and B refer to the entire meson and baryon sets. We do not specify the exact energy dependence of the left-hand cut resulting from baryon exchange; our static assumption is used only in computing the spin dependence of the crossing matrix.

Various authors have pointed out that the isospin, hypercharge, and spin structure of these multiplets correspond to the representations $D^{35}(10001)$ and $D^{56}(30000)$ of SU(6).[4,5] The purpose of this paper is not to formulate SU(6) symmetry in some relativistically invariant way, but rather to use some properties of SU(6) to find a solution to the bootstrap problem. We make use of the fact that the mesons do not occur alone in the static model, so that the unit orbital angular momentum may be regarded as an intrinsic property of the mesons. The P-wave octet P_8 necessarily behaves as an effective axial-vector octet. We assume that the V-meson spins couple with the orbital angular momentum according to the following scheme:

$$P_8 \to A_8, \quad V_8 \to S_8, \quad V_1 \to A_1, \tag{1}$$

where S and A denote scalar and axial-vector effective particles. The effective particles A_8, S_8, and A_1 correspond to the SU(6) representation D^{35}. We take the μBB coupling to be given by SU(6) symmetry, so that standard group-theoretical techniques may be used to calculate Clebsch-Gordan coefficients and crossing-matrix elements. The wave functions for B_8 and B_{10} that result from this assignment may be written in the form

$$\begin{aligned}\psi(B_8) &= (2/5)^{1/2}[(5/9)^{1/2}(P_8B_8)_s + (4/9)^{1/2}(P_8B_8)_a] \\ &\quad + (2/15)^{1/2}(V_8B_8)_a + (4/9)^{1/2}(P_8B_{10}) \\ &\quad + (1/45)^{1/2}(V_1B_8), \\ \psi(B_{10}) &= (8/45)^{1/2}(P_8B_8) + (4/9)^{1/2}(P_8B_{10}) \\ &\quad + (4/15)^{1/2}(V_8B_{10}) + (1/9)^{1/2}(V_1B_{10}),\end{aligned} \tag{2}$$

where the subscripts s and a denote the symmetric and antisymmetric octet states. The states (P_iB_j), etc., are normalized to unity.

Gürsey, Pais, and Radicati have shown that SU(6) invariance leads to D/F ratios of zero and $\frac{3}{2}$ for the V and P octets, respectively. It is seen from Eq. (2) that we have not reversed the two octets in this sense. [$F/D=\frac{2}{3}$ corresponds to $\tan\theta=(4/5)^{1/2}$, where $\tan\theta$ is the relative amplitude of the antisymmetric and symmetric octet states.] The fact that the ratio of effective statistical weights of the

P and V mesons in our model is large, 24/11, is one of the main reasons that the predictions of the model, and the corresponding predictions of reference 4, are not far from those of the standard reciprocal bootstrap model.

The reduction of the direct product of the SU(6) μ and B representations is,[4]

$$D^{35}\otimes D^{56}=D^{56}(30000)\oplus D^{70}(11000)\oplus D^{1134}(21001)\oplus D^{700}(40001).$$

The bootstrap consistency condition requires that the force resulting from the exchange of D^{56} be more attractive in the D^{56} channel than in any other channel. We have calculated the elements of the SU(6) crossing matrix corresponding to the exchange of D^{56}; these elements are

$$C_{56}=11/15,\quad C_{70}=-2/5,$$
$$C_{1134}=-2/45,\quad C_{700}=2/15. \tag{3}$$

A positive crossing-matrix element corresponds to an attractive force, so the model is consistent; no second baryon multiplet is needed.[6]

The bootstrap condition does not require the existence of SU(6). Corresponding solutions exist for SU(2) and SU(3), if the mesons are identified with the adjoint representation, and the baryons with the completely symmetric representation formed from the cube of one of the fundamental representations.[7] (In a sense, the spin-$\frac{3}{2}$ decuplet is the most characteristic part of the physical 56-fold B multiplet.) In SU(2) and SU(3) [and presumably in SU(6)], inconsistency results if the postulated baryon multiplet corresponds to the fundamental representation.[8]

The manner in which the orbital angular momentum is coupled in Eq. (1) suggests an alternate physical interpretation in which the effective axial-vector singlet is formed from the 960-MeV X^0 particle[9] (assumed pseudoscalar), rather than from the vector SU(3) singlet (called here the ω). This is an unconventional assignment, since the physical μ multiplet would contain 33 spin states, rather than 35. The advantage of this assignment is that the ω can be a V meson coupled to the baryon-number current, as suggested originally by Sakurai.[10] Since the baryon number is constant within the 56-fold B multiplet, any particle coupled to the baryon-number current must interact effectively like an SU(6) singlet; the ω meson cannot be such a particle if it is part of the 35-fold representation.

The dependence on internal quantum numbers of the two $I_z=0$ members of the V octet is given by the statement that these particles are coupled to the hypercharge and I_z currents.[4] The $(\vec{\epsilon}\cdot\vec{p})$ spin dependence of these interactions assumed in Eq. (1) is unusual for models involving V mesons coupled to conserved currents. Such models are usually based (directly or indirectly) on an analogy with photon interactions; however, the masslessness of the photon forbids an $(\vec{\epsilon}\cdot\vec{p})$-type interaction. This unusual spin dependence may not be a weakness of the model. The masslessness of the particle coupled to the conserved current is one of the outstanding features that distinguishes the electromagnetic interaction from the strong and weak interactions; the implications of this feature are not completely understood. In our static model, states formed by combining the orbital angular momentum with the V spins in different ways behave like different particles; we have simply found a solution in which some of these "particles" are not coupled.

It has been shown in previous references that in the standard reciprocal bootstrap model, the experimentally observed P_8 mass splitting is such as to lead to the correct ordering of states within both the B_8 and B_{10} multiplets.[1,11] If this type of analysis is extended to the present model, the positive V-P mass difference leads to a positive B_{10}-B_8 mass difference. This follows because the total probability of P-meson states is greater in the wave function $\psi(B_8)$ than in $\psi(B_{10})$, as seen from Eq. (2).

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[1]R. E. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963).

[2]R. H. Capps, to be published.

[3]In one of the simplest known complete bootstrap models, the VVV model of R. E. Cutkosky [Phys. Rev. 131, 1888 (1963)], the bootstrap conditions, together with the assumption that the V mesons are not separable into noninteracting sets, requires that a solution corresponds to a single irreducible

representation (the adjoint representation) of a simple Lie group.

[4]F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964), and previous references by the same authors.

[5]B. Sakita, to be published.

[6]The representation of SU(2) and SU(3) multiplicities 6 and 35, discussed recently by R. H. Capps [Phys. Rev. Letters 13, 536 (1964)], is a member of the SU(6) representation D^{700}. As seen from Eq. (3), the force in this representation is attractive, but weak.

[7]The relevant crossing matrices for these SU(2) and SU(3) schemes are given in reference 2 and by D. E. Neville, Phys. Rev. 132, 844 (1963).

[8]This follows from the crossing matrices listed in reference 2.

[9]M. Goldberg et al., Phys. Rev. Letters 12, 546 (1964); G. R. Kalbfleisch et al., Phys. Rev. Letters 12, 527 (1964); P. Dauber, W. E. Slater, L. T. Smith, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 13, 449 (1964).

[10]J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).

[11]R. H. Capps, Phys. Rev. 134, B1396 (1964).

BARYON SUPERMULTIPLET

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In several recent papers[1-12] the remarkable correspondence between the 56-dimensional representation of SU(6) and the low-lying even-parity baryon states has been pointed out. Although the relations among these states have in the past[13-16] been described with some success in terms of a dynamical model which is based on dispersion relations and is an outgrowth of the Chew-Low model, the correspondence with SU(6) has been discussed in terms of unfamiliar dynamical concepts. We shall show here how the usual dynamical theory, with some minor extensions, can be interpreted as an SU(6) theory. Our model has the following features: (a) It is the natural approximation to a Lorentz-covariant theory pertaining to relatively heavy baryons. (b) The interactions of pseudoscalar mesons with the baryon octet and decuplet are taken to be the dominant interactions governing the structure of these states. (c) It incorporates a simple dynamical theory of the electromagnetic structure of the baryons.

We take the 35 SU(6) generators G_α to include the SU(3) generators as well as the total angular momentum in the baryon rest frame. In states containing a baryon and mesons we neglect the motion of the baryon; the total angular momentum is then decomposed into the spin of the central baryon and the angular momentum of the mesons with respect to this fixed baryon. Since pseudoscalar mesons are emitted by a baryon into P states, the pseudoscalar octet P provides a total of 24 mesonic states. To obtain the full complement of 35 states required by SU(6), we include provisionally an SU(3) singlet pseudoscalar meson X (which may be identified with the state at 960 MeV) and the states of the vector octet, V, which have $l=1$, $J=0$. We define coupling constants in such a way that the kinematical factors appear with the same coefficients in the vector and pseudoscalar states. These coupling constants are taken to be proportional to the generators $G_{ab\alpha}$.

The baryon-exchange force between a baryon and a meson is then proportional to

$$V_{a\alpha,b\beta}=G_{ac\beta}G_{cb\alpha}=G_{ac\alpha}G_{cb\beta}+F_{\alpha\beta\gamma}G_{ab\gamma}, \quad (1)$$

if we assume degeneracy, where the $F_{\alpha\beta\gamma}$ are the structure constants. In the representation N, this can be expressed in terms of the Casimir operators G^2 as

$$V(N)=G^2(N)\delta_{N,\underline{56}}+\tfrac{1}{2}[G^2(N)-G^2(\underline{56})-G^2(\underline{35})], \quad (2)$$

leading to[17] $V(\underline{56})=33/2$, $V(\underline{70})=-9$, $V(\underline{1134})=-1$, and $V(\underline{700})=+3$ (a positive sign denotes attraction). The strong attraction in $\underline{56}$ is consistent with a bootstrap picture. In contrast, the same calculation applied to the basic representation $\underline{6}$ gives $V(\underline{6})=-\frac{1}{6}$, $V(\underline{84})=-1$, and $V(\underline{120})=+1$, so no self-consistent model could be based on $\underline{6}$ alone.

To amplify this bootstrap discussion we may consider ladder-approximation equations in which the meson masses occur as parameters and the coupling constants and baryon masses are obtained by requiring self-consistency. The masses enter through integrals correspond-