LONG-RANGE MAGNETIC COUPLING IN METALS

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Experimental results have recently been obtained by Bruyere, Massenet, and Montmory' which seem to suggest that nonoscillating interactions ranging up to distances of several hundred angstroms exist between two ferromagnetic slabs through a thin film of nonmagnetic metal. Concerning these experimental results, Dreyfus, Maynard, and Quattropani² have pointed out that a nonoscillating spin-density fluctuation is induced in a nonmagnetic metal which decreases with the distance from the contact plane with a ferromagnetic metal.

Dreyfus, Maynard, and Quattropani consider two metals, F and N , which are in contact with each other at the plane $z = 0$ and assume that the average potential in metal N is higher by $\hbar^2 \alpha^2/2m$ than that in metal F. Describing the wave functions of the conduction electrons in the two metals by the following forms:

$$
\psi_F = [A \exp(ikz) + B \exp(-ikz)] \exp(ik \psi \psi), \quad (1)
$$

$$
\psi_N = [C \exp(ik'z) + D \exp(-ik'z)] \exp(ik \nvert \rho \nvert), \quad (2)
$$

they determine four parameters and k from the boundary conditions at the contact plane and the two end planes and the condition of normalization.

According to their calculations, for k smaller than α the wave function ψ_N falls off with increasing z as $\exp[-(\alpha^2-k^2)^{1/2}z]$. By collecting all the damping waves, they derived the following expression for the electron density in metal N :

$$
\rho(z) = \frac{1}{2\pi^2} \int_0^\alpha \exp(-2kz) \frac{k(\alpha^2 - k^2)^{1/2}}{\alpha^2} \times (k \frac{1}{\mathbf{F}}^2 - \alpha^2 + k^2) dk.
$$
 (3)

This density decreases asymptotically as $1/z^2$.

However, there remain contributions from the electrons with a value of k greater than α . Solving the same boundary conditions, we obtain the contributions from these electrons to the electron density in the nonmagnetic metal as

$$
\rho(z) = \frac{1}{2\pi^2} \int_0^{(k^2 - \alpha^2)^{1/2}} \frac{\sin^2 k(L - z)}{\alpha^2 \sin^2 kL + k^2} \times k(\alpha^2 + k^2)^{1/2} (k_F^2 - \alpha^2 - k^2) dk,
$$
 (4)

where L is the length of each metal in the z direction. For a large value of L , this integra can be rewritten as

$$
\rho(z) = \frac{1}{2\pi^2} \left\{ \frac{1}{\alpha^2} \int_0^{(k^2 - \alpha^2)^{1/2}} \cos(2kz) \left[k(\alpha^2 + k^2)^{1/2} \right] \right\}
$$

$$
- \frac{1}{2} (\alpha^2 + 2k^2) \left[k \frac{2}{r^2} - \alpha^2 - k^2 \right] dk
$$

$$
+ \frac{1}{2} \int_0^{(k^2 - \alpha^2)^{1/2}} (k \frac{2}{r^2} - \alpha^2 - k^2) dk \left\}.
$$
(5)

The z -dependent part of (5) can be expressed as

$$
\rho(z) = \frac{1}{2\pi^2} \left\{ \frac{1}{\alpha^2} \int_0^\infty \cos(2kz) k (\alpha^2 + k^2)^{1/2} (k \frac{1}{F}^2 - \alpha^2 - k^2) dk - \frac{1}{\alpha^2} \int_{(k \frac{1}{F}^2 - \alpha^2)^{1/2}}^{\infty} \cos(2kz) [k (\alpha^2 + k^2)^{1/2} - \frac{1}{2} (\alpha^2 + 2k^2)] (k \frac{1}{F}^2 - \alpha^2 - k^2) dk \right\}.
$$
 (6)

With the use of the contour integration in the complex plane, the first term of this expression can easily be shown to cancel completely with (3) , which arises from the contributions of the electrons with low energy. Thus, we can see that the boundary effect is given only by the second term of (6) which oscillates with z. For a small value of α , this term can be approximated by

$$
\rho(z) = \frac{\alpha^2}{8\pi^2} \int_{2k}^{\infty} \cos(qz) \frac{4k \frac{z}{F} - q^2}{4q^2} dq.
$$
 (7)

For small α , we can use the Ruderman-Kittel function for the density distribution of conduction electrons induced by one localized spin. Therefore, the electron density in the nonmagnetic metal can also be expressed by

$$
\rho(z) = \frac{1}{8\pi^2} \frac{2m}{\hbar^2} JSN^{-1} \sum \sum Q_{n} \sum \exp[iq(z - R_{n})] f(q),
$$

$$
q Z_{n} R_{\parallel n}
$$

$$
f(q) = 1 + \frac{4k_{F}^{2} - q^{2}}{4k_{F}^{q}} \ln \left| \frac{2k_{F} + q}{2k_{F} - q} \right|,
$$
 (8)

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where J is the exchange integral between conduction electrons and the localized spin S. The summation over lattice points on an xy plane can easily be taken, and (8) becomes

$$
\rho(z) = \frac{1}{8\pi^2} \frac{2m}{\hbar^2} J S \frac{1}{2\pi} \int_{-\infty}^{0} dZ \int_{-\infty}^{\infty} dq \exp[i q(z - Z)] f(q), \tag{9}
$$

where we neglect the contributions from the reciprocal lattice vectors which decay exponentially with z , as far as the reciprocal lattice vectors are larger than $2k_F$. The integral with respect to q in (9) can be evaluated by integrating $exp[iq(z-Z)]f'(q)$ along the path which avoids two branch points $q = \pm 2k_F$, where $f'(q)$ is the function derived by removing the symbol of absolute value from $f(q)$. The result is as follows:

$$
\rho(z) = \frac{1}{8\pi^2} \frac{2m}{\hbar^2} J S \int_{2k_{\text{F}}}^{\infty} \cos(qz) \frac{4k_{\text{F}}^2 - q^2}{4q^2} dq. \quad (10)
$$

If we put $\alpha^2 = 2mJS/\hbar^2$, (10) coincides with (7). For large z, (10) behaves as $\cos(2k_{\text{F}}z)/(2k_{\text{F}}z)^2$.

Thus, our conclusion is that in the neighborhood of the contact plane of two kinds of metals only oscillating density fluctuations are induced which originate from the Ruderman-Kittel interaction.

The authors are indebted to Professor Nagamiya who kindly suggested a simple technique for eliminating L from (4) .

¹J. C. Bruyere, O. Massenet, R. Montmory, and L. Néel, Compt. Rend. 258, 841, 1423 (1964); O. Massenet and R. Montmory, Compt. Rend. 258, 1752 (1964).

²B. Dreyfus, R. Maynard, and A. Quattropani, Phys. Rev. Letters 13, 342 (1964).

MAGNETIC VIRTUAL LOCAL MODE CONDENSATION AND FLUORESCENCE ANOMALIES

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This Letter proposes a mechanism to explain the very abrupt increase in the frequency, intensity, and lifetime of fluorescent emission, with decreasing temperature, in antiferromagnetic Mn salts. These fluorescent shifts, observed by Holloway et al.,¹ occur at a temper ature $T_H \sim \frac{1}{2}T_N$ (T_N is the Neel temperature), and are reminiscent of other fluorescent shifts which have been observed to occur at T_{N} .²

The experimentally observed fluorescent shifts are abrupt and suggest a cooperative transition. The absorption and emission processes in these Mn salts are a local phenomenon, however, as the excited state forms a self-trapped exciton. The concentration of excited sites is almost certainly too low for cooperative interactions to take place between the excited ions. It is proposed that a purely local effect, cooperative between local spin orientation and local lattice distortion, gives rise to these fluorescent shifts.

The proposed mechanism for altering the fluorescent emission is illustrated in Fig. 1. Curve a represents the energy of the ground state of the Mn^{2+} ion as a function of a gener-

LOCAL LATTICE SPACING

FIG. 1. ^A vibronic-state configuration diagram with curve a the ground state, curve b the excited state with spin aligned in the internal field, curve c the excited state with unaligned spin. Line 1 represents the emission of a photon from the aligned excited state, line 2 that from the unaligned state.