lead. Preliminary experiments of deforming the crystal, i.e., increasing the dislocation density, show an enhancement of the amplitude dependence for moderate deformations and a reduction for large deformations. In the latter case, it is assumed that the dislocation density Λ becomes so great that for many dislocations the node distance L_n is smaller than L_c, the impurity pinning distance. As a result, the number of unpinning processes is drastically reduced and the amplitude dependence becomes very weak.

So far no investigation has been made about the nature of pinning-point imperfections. The pinning force has to be weak in order to explain the breakaway at such low temperatures where the assistance by thermal phonons is almost negligible.

A detailed experimental and theoretical account of this effect will be published soon.

The authors wish to express their gratitude to Dr. Paul Shlichta for orienting several of the samples. The authors also wish to thank Mr. Floyd Lacy for his help, especially in the sample preparation. One of us (B.R.T.) would like to thank Professor W. F. Libby for support out of National Aeronautics and Space Administration Grant No. NASA NSG-237 during part of this work.

---

*This research is supported in part by Grant No. NSF GP 2391 from the National Science Foundation.

[1]R. E. Love and R. W. Shaw, Rev. Mod. Phys. 136, 260 (1964).

[2]R. W. Morse, Progress in Cryogenics, edited by K. Mendelssohn (Academic Press, Inc., New York, 1959), Vol. I, pp. 221-259, for example.

[3]A. Granato and K. Lücke, J. Appl. Phys. 27, 583 (1956).

[4]A. Granato and K. Lücke, J. Appl. Phys. 27, 789 (1956).

[5]The authors are indebted to Professor T. Holstein for the calculation of the electron-dislocation relaxation time.

[6]W. P. Mason, Bell System Tech. J. 34, 903 (1955).

POSSIBILITY OF LONG-RANGE SPIN POLARIZATION IN A DEGENERATE ELECTRON GAS*

A. Bardasis, D. S. Falk, R. A. Ferrell, M. S. Fullenbaum, and R. E. Prange
University of Maryland, College Park, Maryland

and

D. L. Mills
University of California, Berkeley, California

(Received 22 January 1965)

In a recent issue of this journal, Dreyfus, Maynard, and Quattropani[1] have arrived at the conclusion that under certain circumstances it is possible for an induced long-range spin-density distribution to exist in a nonferromagnetic metal. This novel result is surprising when regarded from the standpoint of the theory of Ruderman and Kittel[2] and Yosida.[3] In this theory the spin density generated at large distances from a localized spin perturbation is of an oscillatory nature. Dreyfus, Maynard, and Quattropani suggest that the difference between their result and that of Ruderman and Kittel results from the nonperturbative nature of the problem which they treated. But we find this explanation not completely convincing, as the behavior of the free-electron gas in the asymptotic region at considerable distance from the disturbance would nevertheless be expected to be characteristic of an only slightly perturbed degenerate gas, regardless of the strength of the distant perturbation.

For this reason we have re-examined the calculation of Dreyfus, Maynard, and Quattropani,[1] with the goal of reconciling it with the Ruderman-Kittel theory. In particular, we have evaluated an additional contribution to the total spin density which, although discussed by Dreyfus, Maynard, and Quattropani, was evidently not computed by them. This is the spin density contributed by the "nonevanescing waves." The motivation of our calculation was to explore the possibility that the nonevanescing waves might contribute a long-range polarization which just cancels that coming from the "evanescing waves"—thereby leaving only the expected Ruderman-Kittel spatial dependence. The outcome of our investigation is that this is precisely what happens. The purpose of this note is to give a brief sketch of the calculation and to exhibit explicitly the cancellation which takes place between the long-range portions of the spin density.

We use identically the same model and notation as Dreyfus, Maynard, and Quattropani. A semi-infinite ferromagnet is represented by a spin-exchange field which lowers the energies of the "up"-spin electrons while leaving unaffected the energies of the "down"-spin ones, which will be henceforth disregarded. Contact exists with a normal metal (free-electron gas) along the boundary $z=0$. The problem is to find how the spin density is modified by the free passage of the polarized conduction electrons across the boundary into the normal region (positive z). As the boundary is taken to be perfectly smooth, the electron momentum parallel to the boundary remains a good quantum number, and the problem can be studied in one dimension. The height of the step potential at $z=0$ is taken to be such that electrons of wave number α in the ferromagnetic region can just surmount the barrier and enter the normal region. Electrons of wave number slightly less than α have only evanescing wave functions in the classically forbidden normal region, and will contribute a term to the spin density for $z>0$ of

$$n_e(z)=\frac{2}{\pi}\int_0^\alpha d\kappa'\frac{\kappa' k}{\alpha^2}e^{-2\kappa' z}\approx(2\pi\alpha z^2)^{-1}. \qquad (1)$$

This one-dimensional result is easily converted to three dimensions by multiplying by the cross-sectional area of the Fermi sphere. This yields the three-dimensional distribution found by Dreyfus, Maynard, and Quattropani [their Eq. (7)],

$$\rho_e(z)\approx\frac{1}{8\pi^2}\frac{k_F^2-\alpha^2}{\alpha}\frac{1}{z^2}. \qquad (2)$$

Unprimed quantities refer to the ferromagnetic region, and primed quantities to the normal region. Thus the numerator in Eq. (2) can equally well be written as $k_F'^2$, the square of the Fermi wave number in the normal region.

It is now necessary to add to the contribution of the evanescing waves the contribution to the spin density from the freely propagating non-evanescing waves. This can be computed by putting impenetrable walls in both the ferromagnetic and nonferromagnetic regions, parallel to the boundary and at great distance from it. Then the energy eigenfunctions are all real and form a dense but discrete spectrum. These standing waves contribute to the electron density a part which fluctuates rapidly as a function of energy. These fluctuations arise from interference with the waves reflected from the distant boundary. Averaging over the fluctuations (this occurs automatically when the total density is summed) gives the following expression, which can alternatively be derived more directly from continuum waves in the absence of external walls:

$$\begin{aligned} n_n(z)&=\frac{1}{\pi}\int_0^{k_F'}dk'\left[1-\frac{(k-k')^2}{\alpha^2}\cos 2k'z\right]\\ &\cong\frac{k_F'}{\pi}-\frac{1}{\pi}\int_0^{k_F'}dk'\frac{\alpha^2-2kk'+2k'^2}{\alpha^2}\cos 2k'z\\ &\approx\frac{k_F'}{\pi}-\frac{1}{2\pi\alpha z^2}+O(z^{-4})\\ &\qquad+\text{oscillating terms}. \end{aligned} \qquad (3)$$

The oscillating terms are exhibited below in Eq. (5). The constant term is the equilibrium density associated with the nonferromagnetic electron gas, and is matched by an equal density of "down"-spin electrons. The second term, proportional to the inverse square of the distance from the boundary, just cancels the contribution of the evanescent waves [Eq. (1)]. Thus there remains no long-range effect of the adjacent ferromagnetic gas.

The cancellation of the long-range terms can be exhibited more exactly, without the approximations involved in Eqs. (1) and (3). This is accomplished by rotating the contour of integration in Eq. (1) by 90°:

$$\begin{aligned} n_e(z)&=\frac{1}{\pi}\operatorname{Im}\int_0^\infty d\kappa'\frac{(k+i\kappa')^2}{\alpha^2}e^{-2\kappa' z}\\ &=\frac{1}{\pi}\operatorname{Im} i\int_0^\infty dk'\frac{(k-k')^2}{\alpha^2}e^{-2ik'z}\\ &=\frac{1}{\pi}\int_0^\infty dk'\frac{(k-k')^2}{\alpha^2}\cos 2k'z. \end{aligned} \qquad (4)$$

Because of the functional connection $k=(\alpha^2-k'^2)^{1/2}$, it is permitted to extend the limit of integration to infinity. Addition of this form of the evanescing-wave contribution to Eq. (3)

gives for the total "up"-spin density

$$n(z)=n_n(z)+n_e(z)$$
$$=\frac{k_F'}{\pi}+\frac{1}{\pi}\int_{k_F'}^{\infty}dk'\frac{(k-k')^2}{\alpha^2}\cos 2k'z$$
$$=\frac{k_F'}{\pi}+\frac{\alpha^2}{\pi}\int_{k_F'}^{\infty}\frac{\cos 2k'z}{(k+k')^2}dk'$$
$$\approx\frac{k_F'}{\pi}-\frac{\alpha^2}{2\pi(k_F+k_F')^2}\frac{\sin 2k_F'z}{z}+\cdots. \quad (5)$$

The fact that the integral now only involves wave numbers in excess of the Fermi value makes it clear that the spatial dependence is entirely of the Ruderman-Kittel type, and is a consequence of the sharpness of the Fermi surface. By appropriate integration over the above one-dimensional formula, the density for the corresponding three-dimensional problem is found to be

$$\rho(z)=\frac{k_F'^3}{6\pi^2}+\frac{\alpha^2 k_F'}{8\pi^2(k_F+k_F')^2}\frac{\cos 2k_F'z}{z^2}+\cdots. \quad (6)$$

For completeness we also list the changes in density of "up"-spin electrons which occur in the ferromagnetic region for one dimension:

$$\Delta n(z)=\frac{\alpha^2}{\pi}\int_{k_F}^{\infty}\frac{\cos 2kz}{(k+k')^2}dk$$
$$\approx\frac{\alpha^2}{2\pi(k_F+k_F')^2}\frac{\sin 2k_F z}{z}, \quad (7)$$

and in three dimensions

$$\Delta\rho(z)\approx-\frac{\alpha^2 k_F}{8\pi^2(k_F+k_F')^2}\frac{\cos 2k_F z}{z^2}. \quad (8)$$

Summarizing, we see that the spatial dependence of the excess spin density resulting in a nonferromagnetic electron gas from contact with a ferromagnetic one (and vice versa) is entirely of the Ruderman-Kittel oscillatory type. There is no net slowly varying component. As a matter of fact, in the limit $\alpha \ll k_F \approx k_F'$ when the exchange potential is weak enough to be treated as a perturbation, then Ruderman-Kittel theory yields precisely the above equations. In this limit only one Fermi wave number enters the analysis. Step potentials too large to treat as a small perturbation in this way have the effect of introducing distinct wave numbers k_F and k_F' into the Ruderman-Kittel formulas without otherwise intorducing qualitative changes in the spatial dependence. That the Ruderman-Kittel–type results hold completely generally for a degenerate electron gas, independently of any specific spatial variation of the potential, can be seen by using closure.[4] We write the sum over occupied states in terms of a sum over the unoccupied states of energy eigenvalue E greater than the Fermi value E_F:

$$\sum_{E<E_F} u_E^*(\vec{x}')u_E(\vec{x})$$
$$=\delta^{(3)}(\vec{x}'-\vec{x})-\sum_{E>E_F} u_E^*(\vec{x}')u_E(\vec{x}). \quad (9)$$

$u_E(\vec{x})$ is a member of a complete set of energy eigenfunctions, and $\delta^{(3)}$ is the Dirac delta function. Now take the difference between the problem at hand and a reference case in which the potential has everywhere the value it has at point $\vec{x}$. Then allowing $\vec{x}'\to\vec{x}$, we obtain

$$\Delta n(\vec{x})=-\sum_{E>E_F}\Delta|u_E(\vec{x})|^2. \quad (10)$$

This equation establishes that the density change induced by spatial variations in the potential is independent of the low-energy properties of the energy eigenfunctions.

*Research supported at the University of Maryland in part by the U. S. Air Force Office of Scientific Research and the Advanced Research Projects Agency, and at the University of California in part by the National Science Foundation.

[1]B. Dreyfus, R. Maynard, and A. Quattropani, Phys. Rev. Letters 13, 342a (1964).

[2]M. A. Ruderman and C. Kittel, Phys. Rev. 96, 99 (1954).

[3]K. Yosida, Phys. Rev. 106, 893 (1957).

[4]It is well known that for any given problem the continuum wave functions are intimately connected with the bound-state wave functions, in several important ways. It appears that the crucial connection in this instance is that of completeness.