MESON COUPLINGS IN THE INTRINSICALLY BROKEN U(6) SYMMETRY SCHEME

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The groups U(6)[1-3] and $W_6 = U(6)\otimes U(6)$[2,3] have been recently proposed as symmetry groups of hadron interactions. The group U(6) in particular leads to remarkable predictions for meson-baryon interactions[4] and electromagnetic properties of baryons.[5] We wish to discuss here the implications of the groups U(6) and W_6 for trilinear meson interactions. Specifically, we shall show that good values for parameters like the $\omega\rho\pi$, $\varphi\rho\pi$, $K^*K^*\pi$, and $K^*\rho K$ coupling constants are obtained from U(6) symmetry. Indeed, based on the U(6) values of these parameters, we can calculate the decay rates for $\omega\rightarrow\pi^++\pi^-+\pi^0$ and $K^*\rightarrow K+\pi+\pi$ and obtain results in agreement with experiment. We also obtain the selection rule $\varphi\not\rightarrow\rho+\pi$. On the theoretical side we shall give a systematic method for the construction of interactions that are both U(6) and Lorentz invariant. In this context a method for handling the intrinsic symmetry breaking due to kinetic-energy effects will be given.

We start by considering the following U(6)-invariant four-quark interaction[2]:

$$\mathcal{L}_Q = g_Q[\bar\psi\gamma_\mu\lambda_i\psi\bar\psi\gamma^\mu\lambda_i\psi - \bar\psi\gamma_\mu\gamma_5\lambda_i\psi\bar\psi\gamma^\mu\gamma_5\lambda_i\psi]. \quad (1)$$

Introduce a set of 72 fields V_i^μ, A_i^μ ($\mu=0, 1,\cdots,3$; $i=0,1,\cdots,8$) that transform under U(6) like the currents $\bar\psi\gamma^\mu\lambda_i\psi$ and $\bar\psi\gamma^\mu\gamma_5\lambda_i\psi$, respectively.[6] The meson-quark interaction

$$\mathcal{L}_{MQ} = g_{MQ}[\bar\psi\gamma_\mu\lambda_i\psi V_i^\mu - \bar\psi\gamma_\mu\gamma_5\lambda_i\psi A_i^\mu] \quad (2)$$

is then U(6) invariant. Define

$$V_i^\mu = \tilde V_i^\mu + \frac{i}{m_+}\partial^\mu S_i, \quad \partial_\mu \tilde V_i^\mu = 0;$$

$$A_i^\mu = \tilde A_i^\mu + \frac{i}{m_-}\partial^\mu P_i, \quad \partial_\mu \tilde A_i^\mu = 0. \quad (3)$$

The fields $\{\vec{A}_i, V_i^0\}$ form an $L=0$, parity $P=+1$ 36-plet of U(6) of mass m_+, whereas the fields $\{\vec{V}_i, A_i^0\}$ belong to an $L=0$, $P=-1$ 36-plet of mass m_-. Of course in general $m_+\neq m_-$. It is at this point that an important feature of relativistic U(6) invariance makes itself apparent. The U(6) transformations

$$\psi\rightarrow U(\alpha_i^\mu)\psi = \exp(i\alpha_i^\mu\Sigma_\mu\lambda_i)\psi$$

$$(i=0,\cdots,8,\ \mu=0,\cdots,3),$$

$$\Sigma_\mu = 1 \qquad \text{for } \mu=0,$$

$$= \tfrac{1}{2}\epsilon_{\alpha\beta\mu}\sigma_{\alpha\beta} \text{ for } \mu=1,2,3 \ (\alpha,\beta=1,2,3), \quad (4)$$

when combined with the homogeneous Lorentz group lead to the group GL(6).[2] This extension is responsible for the parity doubling in Eq. (3). Classifying the representations of GL(6) by means of the "unitary trick" we see from the invariance of (2) that $\{V_i^\mu, A_i^\mu\}$ belong to the representation $(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})\oplus(\underline{1},\underline{1})\oplus(\underline{1},\underline{1})$.[2] It is useful to introduce the meson matrix[7]

$$M = \gamma_\mu\lambda_i V_i^\mu - \gamma_\mu\gamma_5\lambda_i A_i^\mu, \quad (5)$$

in terms of which $\mathcal{L}_{MQ}$ can be rewritten as

$$\mathcal{L}_{MQ} = g_{MQ}\bar\psi M\psi. \quad (2a)$$

Alternatively, we could have assigned the mesons to the representation $(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$, in which case we would have obtained the meson matrix

$$M' = \tfrac{1}{2}\sigma_{\mu\nu}\lambda_i T_i^{\mu\nu} + \frac{i}{2}\sigma_{\mu\nu}\gamma_5\lambda_i U_i^{\mu\nu} + \lambda_i S_i + i\gamma_5\lambda_i P_i, \quad (5')$$

with

$$T_i^{\mu\nu} = (\partial_\mu\tilde V_i^\nu - \partial_\nu \tilde V_i^\mu)/m_-,$$

$$U_i^{\mu\nu} = (\partial_\mu\tilde A_i^\nu - \partial_\nu\tilde A_i^\mu)/m_+, \quad (6)$$

and $\{T_i^{\mu\nu}, P^i\}$ and $\{U_i^{\mu\nu}, S^i\}$ belonging to $L=0$, $P=-1$ and $L=0$, $P=+1$ U(6) 35-plets, respectively. Before we consider meson couplings, let us explicitly spell out the transformation properties of the meson matrices M and M' under U(6) transformations $U(\alpha_i^\mu)$ [Eq. (4)], space inversion P, and charge conjugation C.

Under $U(\alpha_i{}^\mu)$

$$\mathfrak{M} \to U(\alpha_i{}^\mu)\mathfrak{M}U^{-1}(\alpha_i{}^\mu), \quad \mathfrak{M}=M,M'; \tag{7}$$

under P

$$\mathfrak{M} \to \tfrac{3}{2}\gamma_0\lambda_0\mathfrak{M}\gamma_0\lambda_0; \tag{8}$$

and under C

$$\mathfrak{M} \to \tfrac{3}{2}\mathfrak{C}\lambda_0\mathfrak{M}^{\mathrm{T}}\mathfrak{C}^{-1}\lambda_0; \tag{9}$$

where T stands for transposition in both Dirac and SU(3) spaces and $\mathfrak{C}$ is the well-known charge-conjugation matrix. The factors $\frac{3}{2}$ in (8) and (9) come from the definition of $\lambda_0=(\frac{2}{3})^{1/2}1$. The meson-quark interaction will induce effective interactions between mesons, which of course would be U(6) invariant in the limit in which the intrinsic breaking due to the kinetic energy of the virtually propagating quarks were ignored. The problem now is to explicitly construct these interactions.

Under the assignment $(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$ the trilinear meson vertex satisfying the requirements that (1) it is Lorentz invariant, (2) it is P and C invariant, and (3) it is U(6) invariant [and because of (1) then formally GL(6) invariant], is

$$G_3{}'=\mathrm{Tr}M'M'M'.$$

It forbids vector-pseudoscalar-pseudoscalar meson ($\tilde{V}PP$) couplings,[8] but allows $\tilde{V}\tilde{V}P$ and $\tilde{V}\tilde{V}\tilde{V}$ couplings. With symmetry breaking this would imply $\tilde{V}PP$ couplings small compared to $\tilde{V}\tilde{V}P$ and $\tilde{V}\tilde{V}\tilde{V}$ couplings. This is of course in contradiction with experiment $[\Gamma(\rho\to\pi+\pi)\cong 100\ \mathrm{MeV}]$, and we discard the assignment $(\underline{6}^*,\underline{6})\oplus(\underline{6},\underline{6}^*)$. With the assignment $(\underline{35},\underline{1})\oplus(\underline{1},\underline{35})$, all trilinear meson couplings are forbidden in the limit of "exact U(6)" invariance (insofar as such a limit is meaningful[2]). In this limit only vertices with an even number of external meson lines are allowed. E.g., the four-meson vertex satisfying (1)-(3) is

$$G_4=g_1\,\mathrm{Tr}MMMM+g_2\,\mathrm{Tr}MM\,\mathrm{Tr}MM +g_3\,\mathrm{Tr}MM\gamma_5\,\mathrm{Tr}MM\gamma_5. \tag{10}$$

The trilinear meson couplings appear upon U(6) symmetry breaking and, as we shall see, to lowest order in symmetry breaking the $\tilde{V}PP$ couplings will not be suppressed. The fact that the absolute intensity of trilinear meson couplings is rather large is not in contradiction with their symmetry-breaking character. U(6) symmetry breaking is intrinsic,[2] after all.

We now have to find the most general trilinear meson vertex to lowest order in the symmetry breaking due to the kinetic energy. This is most easily done by defining a kinetic spurion[2] (kineton)

$$K=(\tfrac{3}{2})^{1/2}\gamma_\mu\lambda_0\partial^\mu \tag{11}$$

that transforms under U(6) like the kinetic energy. The most general vertex G_3 trilinear in M and linear in K that obeys (1) and (2) is then[9]

$$\begin{aligned}G_3&=(\tfrac{1}{2})^{9/2}g\,\mathrm{Tr}MM\overleftrightarrow{K}M\\&=\frac{\sqrt{3}}{32}g\,\mathrm{Tr}[MM\gamma_\mu\lambda_0(\partial^\mu M)\\&\qquad -M(\partial^\mu M)\gamma_\mu\lambda_0 M]\end{aligned} \tag{12}$$

[where the normalization factor $(\frac{1}{2})^{9/2}$ is inserted for convenience]. In terms of the fields (3) G_3 can be written as

$$\begin{aligned}G_3=g\underset{\mathrm{SU}(3)}{\mathrm{Tr}}\,&\{V^\alpha[(\partial_\alpha V_\beta)V^\beta-V_\beta(\partial_\alpha V^\beta)]\\&-(1/m)\epsilon_{\alpha\beta\mu\nu}\partial^\alpha P[(\partial^\mu V^\nu)V^\beta+V^\beta\partial^\mu V^\nu]\\&+[(\partial^\alpha P)P-P(\partial^\alpha P)]V_\alpha\\&-(1/m^2)\partial^\alpha P\partial^\beta P(\partial_\alpha V_\beta-\partial_\beta V_\alpha)]\}+\cdots,\end{aligned} \tag{13}$$

where we have omitted the couplings of the axial-vector and scalar particles in view of the inconclusive experimental evidence concerning their existence. From the ρ-meson width $\Gamma(\rho\to\pi+\pi)=100$ MeV, we find

$$g^2/4\pi\cong 4/9. \tag{14}$$

From (13) we now find

$$g_{\varphi\rho\pi}=0, \tag{15}$$

$$\frac{g_{\omega\rho\pi}{}^2}{4\pi}\cong\frac{32}{9}, \tag{16}$$

and

$$\frac{g_{K^{*+}K^{*-}\pi^0}{}^2}{4\pi}=\frac{g_{K^{*+}\rho^0K^-}{}^2}{4\pi}\cong\frac{8}{9}. \tag{17}$$

In the above estimates we have used the $\omega\varphi$

mixing angle θ given by $\cos\theta = (\frac{2}{3})^{1/2}$. This would come about if before the onset of the SU(3) breaking the nine vector mesons were degenerate.

Equation (15) leads to a selection rule forbidding $\varphi \to \rho + \pi$. This selection rule has been previously derived from U(6) invariance (conservation of spin of the strange quark) by Lipkin.[10] He did not include symmetry breaking in his derivation, and as we know this latter is intrinsic. It is therefore gratifying that the selection rule $\varphi \not\to \rho + \pi$ can be proved also when symmetry breaking is included [Eq. (15)]. Experimentally this selection rule is very well obeyed.

The coupling constants given by (16) and (17) cannot be directly measured. It is, however, commonly believed[11] that three-particle decays of vector mesons such as $\omega \to \pi^+ + \pi^- + \pi^0$ or $K^* \to K + \pi + \pi$ are dominated by VP intermediate states. This then permits an indirect determination of these coupling constants. Using the results of reference 11, we find

$$\Gamma(\omega \to 3\pi) = 0.425 \frac{g_{\omega\rho\pi}^2}{4\pi} \frac{1}{m^2} 10^6 \text{ MeV}.$$

Here m is, as above, the common mass of the 35 odd-parity mesons in the limit of "exact U(6)." Reasonable values of m are expected to range somewhere between 500 and 700 MeV.[12] Using the experimental result[13] $\Gamma(\omega) \sim (1/0.85) \times \Gamma(\omega \to 3\pi)$, we find with the value (16) for $g_{\omega\rho\pi}^2/4\pi$ in the above-mentioned range of m values[14]

$$4 \text{ MeV} \lesssim \Gamma(\omega) \lesssim 7 \text{ MeV}, \qquad (18)$$

to be compared with the experimental value[13]

$$\Gamma(\omega) = 9.4 \pm 1.7 \text{ MeV}. \qquad (19)$$

Our result (18) exceeds by a whole order of magnitude the original result of reference 11.[14] It is of interest to analyze the source of this discrepancy. Reference 11 ignored ω-φ mixing in the evaluation of the process $\pi^0 \to \rho + \omega \to 2\gamma$, and this resulted in an underestimate, by a factor $\frac{1}{3}$, of $g_{\omega\rho\pi}^2/4\pi$. Furthermore, new measurements[13] of the $\pi^0 \to 2\gamma$ width yield 6 eV rather than 3 eV, the value used in reference 11. With these two corrections the method of reference 11 yields (for $500 < m < 700$ MeV)

$$1.2 \lesssim \frac{g_{\omega\rho\pi}^2}{4\pi} \lesssim 2.1, \qquad (20)$$

which is in reasonable agreement with the U(6) value (16). A similar mechanism for the decay $K^* \to K + \pi + \pi$ using the coupling constants (17) and $m \sim 600$ MeV yields[15]

$$\Gamma(K^* \to K + \pi + \pi, \text{ all charge modes}) \simeq 5 \text{ keV}, \qquad (21)$$

to be compared with the present experimental upper bound[13]

$$\Gamma(K^* \to K + \pi + \pi) \lesssim 100 \text{ keV}. \qquad (22)$$

All other relations among experimentally measurable coupling constants that follow from (13) can be derived from SU(3) alone using the ω-φ mixing angle predicted by U(6) and have been previously discussed in the literature.[16] There is one more important feature of the vertex (13) which we wish to discuss now. As can be seen, e.g., from the last term in (13) the VPP coupling is not of the form one would expect in a "gauge theory"[8] of vector mesons. Even if one attributed a deeper meaning to the fact that the coupling of vector mesons to their unitary-spin current and to the unitary-spin current of pseudoscalar mesons proceeds with equal strength, it is clear that the "universal coupling constant" of vector mesons cannot be obtained directly from the $\rho \to \pi + \pi$ width. One first has to subtract the contribution of the last term in (13). Thus, e.g., the $\rho\rho\rho$ coupling constant predicted by (13) and (14) is

$$\frac{g_{\rho\rho\rho}^2}{4\pi} = \left(\frac{g_\rho^2}{4\pi}\right)_{\text{univ}} \approx \frac{8}{9},$$

as opposed to the usually quoted value $(g_\rho^2/4\pi)_{\text{univ}} \cong 2$.[17] We wish to emphasize that the above discussion gives the necessary techniques for the general construction of relativistic U(6)-invariant interactions and of the kinetic-energy corrections to these. Further applications of this method will be discussed elsewhere.

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13, 173 (1964); B. Sakita, Phys. Rev. 136, B1756 (1964). See also A. Pais, Phys. Rev. Letters 13, 175 (1964).

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[6]M. Gell-Mann, Phys. Rev. Letters 14, 77 (1965). Couplings of the $L=2,4,\cdots$ Regge recurrences of the meson 35-plet can be discussed along the lines of the present paper. Of course these couplings will be of the derivative type.

[7]The matrix M for low-energy mesons has been first written down by M. A. B. Bég and A. Pais, Phys. Rev. (to be published). Our derivation is not restricted to small meson momenta. For the trilinear and quadrilinear meson couplings (as opposed to meson-baryon couplings) no static limit can be envisaged, and a momentum-independent derivation is crucial to our argument.

[8]Note that this situation is essentially different from the case of a "gauge theory" [C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954); J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960)], where vector mesons couple to the conserved total unitary-spin current. This is not surprising since gauge theories are based on the kinetic-energy term of the Lagrangian whereas it is exactly this term that breaks U(6). The fact that the trilinear meson couplings identically vanish in the limit of U(6) symmetry is a particular case of a general result due to Sakurai (to be published) stating that this must be the case in any theory in which the nine vector mesons are degenerate. One of the authors (B.W.L.) is grateful to Professor J. J. Sakurai for reminding him of this result.

[9]This can most easily be seen by replacing one of the matrices M in (10) by K and observing that Tr $MK\gamma_5 \equiv$ Tr $MK \equiv 0$. The meaning of the vertex (12) can be better understood at a pictorial level. Consider an arbitrary Feynman graph with three external meson lines. In the limit of "exact U(6)" in which symmetry-breaking kinetic-energy effects are ignored (γp terms in quark propagators and $k_\mu k_\nu$ terms in meson propagators are dropped), this vertex would be ~Tr MMM $\equiv 0$. Considering the kinetic energy as a spurion can be visualized by kineton emission from the three-meson vertex. It is readily checked that any graph with three external meson lines and one external kineton line leads to a contribution of the form (12) to the vertex.

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[11]M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters 8, 261 (1962).

[12]The tentative value $m=615$ MeV has been suggested by M. A. B. Bég and V. Singh, Phys. Rev. Letters 13, 681 (1964).

[13]A. H. Rosenfeld et al., Rev. Mod. Phys. 36, 977 (1964).

[14]The result of reference 11 is $\Gamma(\omega)=0.4$ MeV. For the effects of ω-φ mixing, see also R. Dashen and D. Sharp, Phys. Rev. 133, B1518 (1964); and R. Socolow, to be published.

[15]In the estimate of Eq. (21), the contribution from the diagram $K^* \rightarrow \rho + K$ is also included. The relativistic correction factor $W(m_{K^*})$ of reference 11 is taken to be $\simeq 2$.

[16]See, e.g. J. J. Sakurai, Phys. Rev. 132, 434 (1963).

[17]J. J. Sakurai, Rendiconti della Scuola Internazionale di Fisica "Enrico Fermi" XXVI Corso, p. 42. In particular, the agreement with experiment of the relation between $g_{\rho\pi\pi}$ and $g_{\pi N}$ obtained in reference 4 assuming both universality in the usual sense $[(g_\rho^2/4\pi)_{\rm univ} \cong 2]$ and U(6) symmetry is presumably fortuitous.

RELATIVISTIC, CROSSING-SYMMETRIC, SU(6)-INVARIANT S-MATRIX THEORY

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In recent papers[1,2] we have described several aspects of the relativistic completion of SU(6). The procedure is meant to apply only to effective vertices and matrix elements ("S-matrix theory"). For local Lagrangian field theory there is a specific breakdown[1,2] of SU(6). In this Letter we indicate how the systematic application of our rules leads to an S-matrix description which is fully covariant, crossing symmetric, and SU(6) invariant. As before,[1,2] we take as our starting definition for SU(6) a group property of zero three-momentum one-particle states.

It is crucial to note that the relativistic completion (boosting[3]) of an SU(6) representation is not unique, except for the trivial case[4] of the 6, which of course is to be treated like the representation $[\frac{1}{2},0]$ of $\mathcal{L}^\uparrow$, the orthochronous Lorentz group including space reflections. Thus we call it $\underline{6}[\frac{1}{2},0]$. This is given by $u^{ia}(p)t^A$ where the notation is as follows: $i=1,2$ denotes spin states, $A=1,2,3$ refers to unitary spin,