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MICROWAVE PHONON-ASSISTED TUNNELING IN SUPERCONDUCTORS

Edward Lax and Frank L. Vernon, Jr.
Aerospace Corporation, El Segundo, California
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This Letter describes experimental evidence for the influence of coherent microwave phonons on the current-voltage characteristics of superconductive tunneling junctions.[1] The types of junctions tested were Al-Al_2O_3-Pb, Al-Al_2O_3-Sn, Pb-Al_2O_3-Sn, and Pb-Al_2O_3-Pb. In principle, these experiments are analogous to the photon-assisted tunneling measurements of Dayem and Martin,[2] in which the quantum effects of direct electromagnetic excitation of tunneling junctions were clearly observed. In the situation discussed here, well-defined quantum effects were not observable because of the lower microwave frequency involved (8.6×10^9 cps) and the resultant thermal smearing ($h\nu/kT\sim0.25$). The results, however, can be explained from theoretical considerations which yield, in first order, the same type of analytical expression as obtained for the electromagnetic case.

The phonons were generated in a quartz crystal 3 cm in length and 3 mm in diameter. Longitudinal waves were generated in X-cut quartz and shear waves in AC-cut quartz. The ends were flat and parallel to within $\frac{1}{10}$ of an acoustic wavelength (approximately 6000 Å for 10-Gc/sec longitudinal waves). On one end of the quartz rod the thin-film tunneling junction was deposited. The thickness of each metal film was about one-half an acoustic wavelength in that metal. The other end of the quartz rod was positioned in the high electric field of a superconducting re-entrant microwave cavity. The transduction efficiency between microwave and acoustic energy was usually greater than −25 dB. The acoustic waves generated by the electric field in the cavity traveled down the rod and excited the tunneling junction, while the dc I-V characteristic of the junction was plotted simultaneously. Since the excitation energy was small, the resulting I-V curve was almost indistinguishable from the I-V curve without acoustic excitation. Therefore the microwave signal was square-wave modulated at a 1000-cps rate, and the excess tunneling current ΔI_A due to the microwave phonons was measured with a narrow-band lock-in amplifier and plotted by an X-Y recorder.

Originally all the acoustic modes of the quartz within the bandwidth of the microwave cavity were driven randomly by a low-stability oscillator. In more recent experiments a high-stability oscillator was used to resonate the quartz rod (which has a Q of about 10^6) in a single mode, in order to enhance ΔI_A.

All the experiments were performed below the λ point of the liquid helium to prevent frequency instability due to helium gas bubbles in the re-entrant cavity.

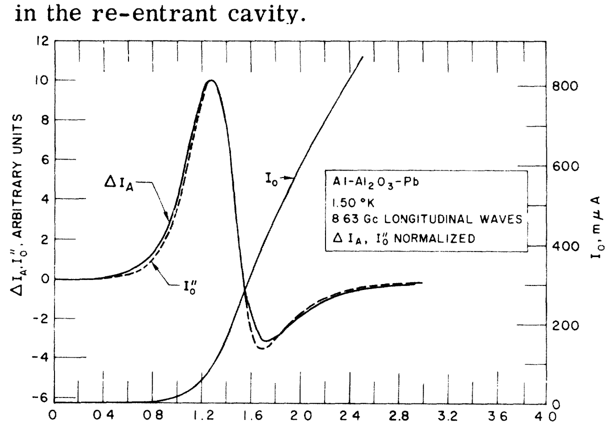

FIG. 1. Experimental tunneling current I_0, second derivative I_0'' of tunneling current, and phonon-assisted tunneling current ΔI_A versus bias voltage, in an Al-Al_2O_3-Pb junction.

Figure 1 shows a typical measurement of ΔI_A versus V for an Al-Al_2O_3-Pb tunneling junction under longitudinal-wave excitation at $T = 1.5°K$. Also shown are the I-V and d^2I/dV^2-V characteristics, both measured without phonon excitation (I_0 and I_0'', respectively). The I_0'' measurement was made by a well-known ac second-harmonic technique. The I_0'' and ΔI_A curves (normalized for shape comparison) show a close correlation. ΔI_A measurements taken at other temperatures in the range of 1.15-2°K show this the same correlation with I_0''. For these measurements dependence of the magnitude of ΔI_A on phonon intensity was found to be essentially linear at the power levels available. The quantitative estimate of the magnitude of the effect is difficult because of the uncertainty of the acoustic properties of the films and the efficiency with which acoustic energy is transmitted to the films. However, it is estimated that the ΔI_A characteristic produced by an acoustic power level of 0.1 mW has approximately the same magnitude as the second-harmonic (I_0'') curve resulting from a 50-μV ac signal impressed across the junction.

Measurements of ΔI_A versus V for Al-Al_2O_3-Sn, Sn-Al_2O_3-Pb, and Pb-Al_2O_3-Pb junctions using both longitudinal and transverse phonons were also made. For all of these cases and for both types of acoustic excitation the proportionality of ΔI_A to I_0'' was observed. In addition, in some experiments excitation power levels were large enough that nonlinearities were noticeable in the dependence of ΔI_A on the acoustic power P_A. Figure 2 shows the variation for a Pb-Al_2O_3-Pb junction of ΔI_A measured at voltages corresponding to the two peaks. For low power levels ΔI_A is proportional to P_A, consistent with all previous measurements. Further experiments of this type are currently being performed.

The experimental results for longitudinal acoustic-wave excitation can be understood by analyzing a normal metal film for the case of uniform strain. The effects of finite values of acoustic wavelength and the case of transverse waves requiring more lengthy consideration will be discussed in a later publication. The Hamiltonian to be considered is

$$H = \sum_k \epsilon_k c_k^\dagger c_k + H'(t), \tag{1}$$

where the ϵ_k represent single-particle electron

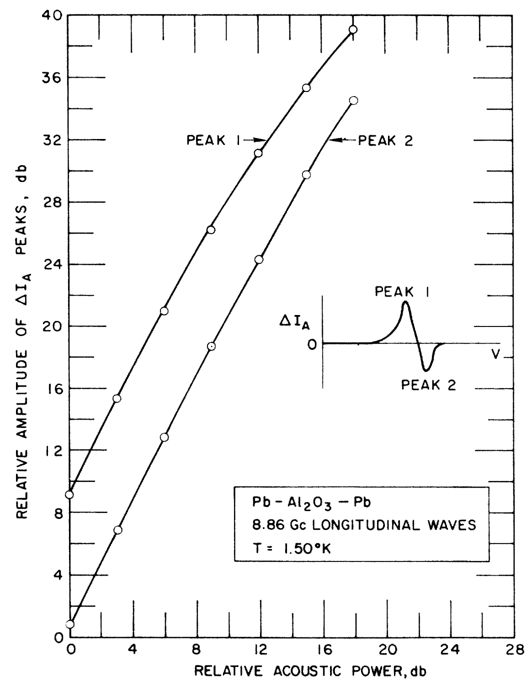

FIG. 2. Variation of the peaks of the phonon-assisted tunneling current ΔI_A as a function of acoustic power in a Pb-Al_2O_3-Pb junction showing nonlinearity.

energy levels of the unperturbed system and $H'(t)$ the interaction-energy term resulting from the applied lattice vibration. If it is assumed that the propagation direction of the acoustic wave is along the x direction perpendicular to the surfaces of the film, then displacement of a lattice point at position x can be expressed by

$$\delta x = A \cos\omega t \cos qx, \tag{2}$$

where q is the magnitude of the phonon wave vector. $H'(t)$ can be expressed in second quantized form as

$$H'(t) = \sum_k \langle k+q | H'(t) | k \rangle c_{k+q}^\dagger c_k + \text{c.c.}$$

$$= \tfrac{1}{2} \sum_k CqA \cos\omega t\, c_{k+q}^\dagger c_k + \text{c.c.}, \tag{3}$$

where C is regarded as a constant of the same order as the Fermi energy.[3] For free electrons $C = \frac{2}{3}\epsilon_F$ as evaluated from the ultrasonic attenuation theories of Kittel and Pippard.[4,5] As the case of uniform strain is approached q tends to zero, but the product qA remains finite and is the strain s. Then

$$H'(t) = \sum_k Cs \cos\omega t\, c_k^\dagger c_k,$$

and the total Hamiltonian becomes

$$H=\sum_k[\epsilon_k+Cs\cos\omega t]c_k^\dagger c_k. \tag{4}$$

The form of this expression is the same as obtained by Tien and Gordon in their analysis of the electromagnetic case.[6] The modification of the single-particle states indicated by Eq. (4) leads to an effective density of states

$$\rho(\epsilon)_{\text{eff}}=\sum_{-\infty}^{\infty}J_n^{\,2}(\alpha)\rho(\epsilon+n\hbar\omega), \tag{5}$$

where $J_n(\alpha)$ is the nth-order Bessel function and $\alpha=Cs/\hbar\omega$. For the case of the superconductor the analysis carries over except that $\rho(\epsilon)$ becomes the superconducting density of states. From Eq. (5) it follows that the tunneling current with acoustic excitation, $I_A(V)$, can be expressed in terms of $I_0(V)$ as

$$I_A(V)=\sum_{-\infty}^{\infty}J_n^{\,2}(\alpha_1-\alpha_2)I_0(V+n\hbar\omega/e), \tag{6}$$

where α_1 and α_2 represent the values of $Cs/\hbar\omega$ appropriate to each metal. The lowest order terms in this expression, which are appropriate for small values of $\alpha_1-\alpha_2$ and $\hbar\omega/e$, are

$$\Delta I_A=I_A(V)-I_0(V)\approx\tfrac{1}{4}[\hbar\omega(\alpha_1-\alpha_2)/e]^2I_0''. \tag{7}$$

As observed experimentally this expression shows the fundamental shape of ΔI_A to be that of I_0'' and its magnitude to be proportional to $(\alpha_1-\alpha_2)^2$, that is, to the acoustic intensity. In addition, the equivalent voltage across the junction is given by

$$eV_{\text{eq}}=\hbar\omega(\alpha_1-\alpha_2)/\sqrt{2}=(C_1s_1-C_2s_2). \tag{8}$$

In our case the strains produced in the quartz were about 2×10^{-7} for the acoustic power level of 0.1 mW. From the free-electron theory $C_1=6.4$ eV and $C_2=7.6$ eV for Pb and Al, respectively. In addition, if the strains ($s_1\approx6\times10^{-7}$, $s_2\approx2\times10^{-7}$) are assumed to be oppositely directed in the two films, then $V_{\text{eq}}\approx6$ μV. This calculated value is about an order of magnitude lower than the observed value of excitation, probably because of the approximate nature of the theory.

Because of the low conversion efficiency from electromagnetic to acoustic energy and the resulting small values of ΔI_A, it was necessary to design experiments which could eliminate the possibility that the measured results were caused by electromagnetic leakage. This possibility was eliminated by very careful shielding of the sample. In addition, to prove that ΔI_A was caused by excitation from the acoustic modes of the quartz, a very stable microwave oscillator was tuned through the acoustic resonances. These modes were separated by approximately 100-kc/sec intervals for longitudinal waves. Clearly defined maxima were observed in ΔI_A at 100-kc/sec intervals. Thus, the energy causing ΔI_A could only be coming through the quartz rod. In addition, to eliminate the possibility that electric fields associated with the acoustic wave in the quartz were inducing a microwave voltage on the junction, a superconducting shield 1.5 microns thick was deposited over the end and sides of the rod. The tunneling junction was then placed on top of the shield. Measurements on an Al-Al_2O_3-Pb junction fabricated in this manner yielded the same results for ΔI_A as have been discussed.

Finally, in addition there are many minor mechanisms (i.e., variation in junction capacitance, resistance, temperature, or energy gap) which could cause results similar to those measured. However, all could be eliminated as either having much too small an influence to explain the magnitude of ΔI_A or having a different shape from the measured variation of ΔI_A versus V. Therefore, it is felt that the measured effect has conclusively been identified as microwave phonon-assisted tunneling.

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