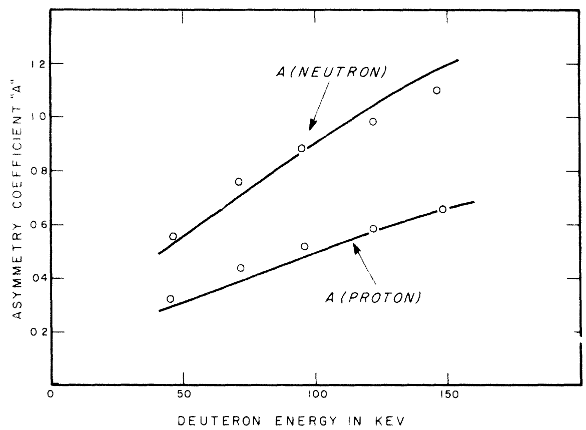

FIG. 3. The experimental asymmetry coefficients for the reaction products compared with theory assuming unequal He^3 and H^3 mass radii.

rameters which are dependent on the three-nucleon wave function and which include the effects of higher orbital angular momentum and quintic spin states are in progress at this laboratory.

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ELECTRON SCATTERING FROM THE MAGNETIC DIPOLE AND OCTOPOLE MOMENTS OF BERYLLIUM-9 AND BORON-11†

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The magnetic form factors of Be^9 and B^{11}, both spin-$\frac{3}{2}$ P-shell nuclei, have been measured in the q^2 range up to 6 F^{-2} (where $\hbar q$ is the four-momentum transfer) by electron scattering at an angle of 180°. The results can be interpreted in terms of form factors for the magnetic dipole and octopole moments of these nuclei. These experiments give for the first time a method of studying the properties of the octopole moment in light nuclei.

The cross sections for elastic electron scattering from nuclei may be represented by a multipole expansion, in which time reversal and parity conservation permit only even Coulomb and odd magnetic multipoles.[1] The general cross section may be written[1,2]

$$d\sigma/d\Omega = (d\sigma/d\Omega)_C + (d\sigma/d\Omega)_M$$

where, in first Born approximation,

$$\left(\frac{d\sigma}{d\Omega}\right)_C = \frac{\sigma_{NS}}{(Ze)^2}\left\{\sum_{\text{even }\lambda}\frac{4\pi q^{2\lambda}}{[(2\lambda+1)!!]^2}B(C\lambda,q)\right\} \qquad (1)$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{e^2}{8E_0^{\,2}}\frac{1}{\eta}\frac{(1+\sin^2\frac{1}{2}\theta)}{\sin^4\frac{1}{2}\theta} \times\left\{\sum_{\text{odd }\lambda}\frac{4\pi q^{2\lambda}(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}B(M\lambda,q)\right\}, \qquad (2)$$

where $B(C\lambda,q)$ and $B(M\lambda,q)$ are defined[2] in terms of the electric and magnetic moments of the nucleus. In Eqs. (1) and (2),

$$\sigma_{NS} = \left(\frac{Ze^2}{2E_0}\right)^2\frac{1}{\eta}\frac{\cos^2\frac{1}{2}\theta + (1/\gamma^2)\sin^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}$$

and

$$\eta = 1 + \frac{2E_0}{Mc^2}\sin^2\tfrac{1}{2}\theta,$$

where M is the mass of the nucleus, E_0 is the total energy of the incident electron, and $\gamma = E_0/m_ec^2$, where m_e is the mass of the electron. λ is restricted to $\lambda \leq 2J$, where J is the nuclear spin.

Formulas (1) and (2) indicate that in order to measure the magnetic terms it is advantageous to use a scattering angle near 180°.[3] Writing $\xi = \pi - \theta$, where $\xi \lesssim 0.1$, say, (1) becomes

$$\left(\frac{d\sigma}{d\Omega}\right)_C \simeq \left(\frac{Ze^2}{2E_0}\right)^2 \frac{1}{\eta_{\theta=\pi}}\left(\tfrac{1}{4}\xi^2 + \frac{1}{\gamma^2}\right)F_C^{\,2}(q_{\theta=\pi}^{\,2}), \quad (3)$$

where $F_C(q^2)$ is the "charge" form factor which is normalized to unity at $q^2 = 0$.

Expression (2) may now be written

$$\left(\frac{d\sigma}{d\Omega}\right)_M \simeq \frac{2}{3}\frac{e^2}{\eta_{\theta=\pi}^{\,2}}\left(\frac{e}{2M_Nc}\right)^2\left(\frac{J+1}{J}\right)\mu_0^{\,2}F_M^{\,2}(q_{\theta=\pi}^{\,2}), \quad (4)$$

where M_N is the nucleon mass and μ_0 is the static magnetic dipole moment of the nucleus in nuclear magnetons. $F_M(q^2)$, the magnetic form factor, is given by

$$\mu_0^{\,2}F_M^{\,2}(q^2) = \mu_0^{\,2}F_{M1}^{\,2}(q^2) + \frac{2}{1575}\left(\frac{2J+3}{2J-1}\right)\left(\frac{2J+4}{2J-2}\right) \times \Omega_0^{\,2}q^4F_{M3}^{\,2}(q^2) + \cdots, \quad (5)$$

where $F_{M1}(q^2)$ and $F_{M3}(q^2)$ are also normalized to unity at $q^2 = 0$, and Ω_0 is the magnetic octopole moment in nuclear magneton (Fermi)^2.

The different angular dependencies of (3) and (4) enable one to separate the Coulomb from the magnetic scattering. For $J = \frac{3}{2}$, (5) becomes

$$F_M^{\,2}(q^2) = F_{M1}^{\,2}(q^2) + \frac{2}{75}\frac{\Omega_0^{\,2}}{\mu_0^{\,2}}q^4F_{M3}^{\,2}(q^2). \quad (6)$$

In practice, further terms must be introduced[4] into (3) to account for the finite experimental solid angle, multiple scattering in the target, and finite angular spread of the incident beam. (3) then becomes

$$\left(\frac{d\sigma}{d\Omega}\right)_C \simeq \left(\frac{Ze}{2E_0}\right)^2 \frac{F_C^{\,2}(q_{\theta=\pi}^{\,2})}{\eta_{\theta=\pi}}\{\alpha_0^{\,2} + \tfrac{1}{3}[(\Delta\alpha)^2 + (\Delta\beta)^2] + 2\langle\theta^2\rangle_{\text{MS}} + \langle\theta^2\rangle_B + 4/\gamma^2\}, \quad (7)$$

where ξ has been written in terms of its projected components α and β ($\xi^2 \simeq \alpha^2 + \beta^2$), and (3) has been integrated within the limits of a solid angle defined by $\alpha_0 \pm \Delta\alpha$ and $\pm\Delta\beta$.

$\langle\theta^2\rangle_{\text{MS}}$ represents the effect[4] of folding the multiple scattering distribution of the electrons in the target into the cross section (3). Near 180°, the scattered electrons must leave the target from the same face by which they entered. Consequently, electron paths in the target vary between zero and approximately twice the target thickness, the corresponding distribution of energy loss giving rise to a very broad "elastic peak." For electrons which penetrate the target to a particular depth t_p, $\langle\theta^2\rangle_{\text{MS}}$ is approximately equal to the mean-square multiple scattering angle for the path t_p, evaluated up to an angular cutoff where single scattering is dominant, plus a term to account for large-angle scattering above the cutoff. Similarly, $\langle\theta^2\rangle_B$ is the mean-square angular spread of the beam at the target.

180° scattering is achieved with an apparatus similar to that described by Peterson and Barber.[5] In the present experiment[4] electrons from the Stanford Mark-III linear accelerator enter a circular uniform magnetic field, and after being deflected through an angle of about 35°, pass out of the field region and through the target.

Those electrons scattered at ~180° pass through the field a second time, bending away from the incident beam, and are analyzed by a vertical 180°, $n = \frac{1}{2}$ magnetic spectrometer (with a mean orbit radius of 72 inches), with a 10-channel scintillator ladder as detector. In order to vary the scattering angle around 180°, the method of moving the deflection magnet, described previously by Rand and Hofstadter,[6] is used.

The beam current is measured with an aluminum-foil secondary-emission monitor located before the deflection magnet. This can be calibrated by passing the beam into a Faraday cup. Absolute calibration of the apparatus (detector efficiency, solid angle, etc.) is achieved by scattering electrons at 180° from protons in polyethylene and using the proton form factors determined by Janssens et al.[7]

The usable target thicknesses were limited by the necessity of restricting the width of the elastic peaks so that the first excited nuclear levels were resolved. The present results have been obtained using a Be^9 target of 0.229 g/cm^2 and B^{11} targets of 0.275 and 0.218 g/cm^2.

For electron energies up to 150 MeV ($q_{\theta=\pi}^2 \simeq 2.2$ F^{-2}), elastic peaks were measured at various values of α_0. At each angle the counts in each momentum channel were corrected to a standard solid angle (the effective solid angle is, in general, a function of both momentum and scattering angle with this type of apparatus), and a least-squares fit of (4) and (7) was made to the data to extract the counts due to magnetic scattering at 180°. Finally, a radiation correction was made to obtain the magnetic form factors. The analysis was checked by measuring backward electron scattering from a spin-zero nucleus, C^{12}. Magnetic scattering at 180° consistent with zero was obtained.[6] At electron energies above ~150 MeV, the charge-scattering contribution was so small that it was sufficient to take data at 180° only and use the accepted values[8] of $F_C^2(q^2)$ in (6).

Results for Be^9 and B^{11} are shown in Figs. 1 and 2, respectively, where the magnetic form factor is plotted as a function of q^2. The theoretical curves given by expression (6) have not been fitted to the data, but represent the magnetic form factors calculated using the extreme single particle, harmonic well, shell model. This model gives[9] for the last $P_{3/2}$ neutron in Be^9

$$F_{M1}(q^2) = (1-0.800x)e^{-x}$$

and

$$F_{M3}(q^2) = e^{-x},$$

where

$$x = \tfrac{1}{4}q^2 a_0^2$$

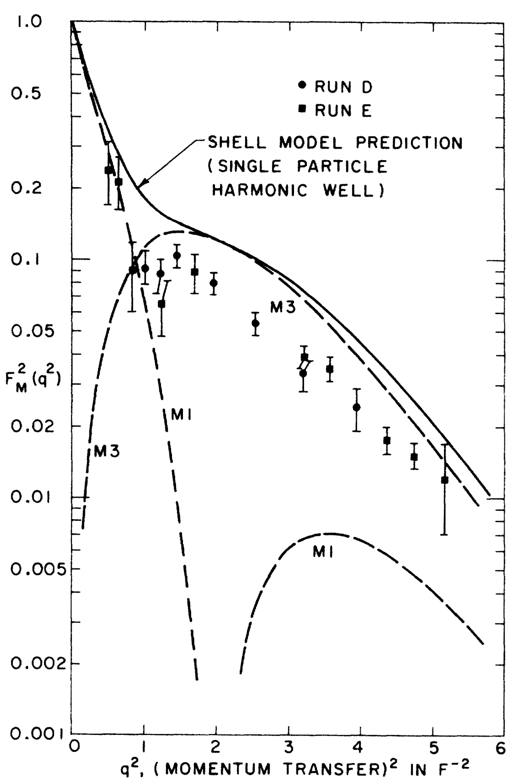

FIG. 1. The magnetic form factor of Be^9. The experimental data are calculated from expression (4), while the theoretical curves are calculated using the extreme single-particle shell model. The separate contributions of the dipole and octopole terms are shown (dashed lines) together with the total form factor (solid line).

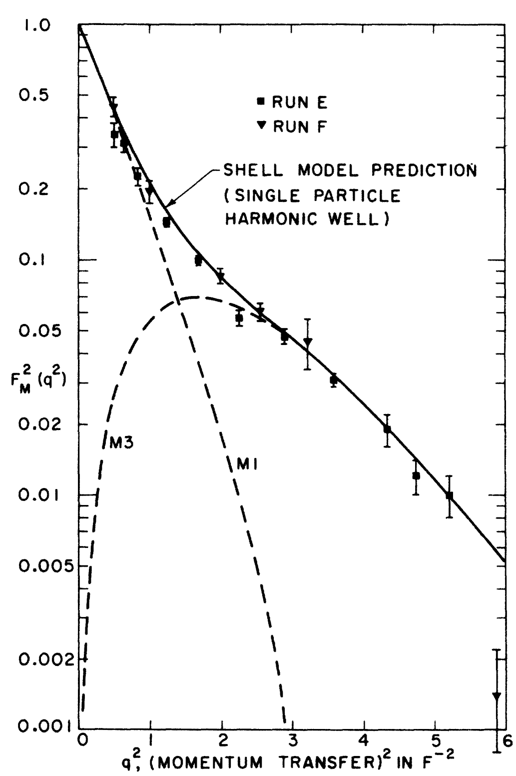

FIG. 2. The magnetic form factor of B^{11}. See caption to Fig. 1.

and a_0 is the radius parameter of the harmonic well. The octopole moment is given by[10]

$$\Omega_0 = -\tfrac{3}{2}\times 1.91 a_0^2.$$

For the odd-$P_{3/2}$ proton in B^{11} one obtains

$$F_{M1}(q^2) = (1-0.460x)e^{-x},$$

$$F_{M3}(q^2) = e^{-x},$$

and

$$\Omega_0 = \tfrac{3}{2}\times 2.79 a_0^2.$$

$F_M^2(q^2)$ was calculated from (6) using the Schmidt values of the magnetic dipole moments ($-1.91\ \mu_N$ for Be^9 and $3.79\ \mu_N$ for B^{11}). Values of a_0 were taken from the measurements of the charge form factors for these nuclei by Meyer-Berkhout, Ford, and Green,[8] viz. Be^9, $a_0 = 1.60$ F; B^{11}, $a_0 = 1.55$ F.

Experimental points were calculated from (4) using the established experimental values of the magnetic dipole moments to ensure that $F_M^2(q^2=0)=1$. Thus agreement between the experimental points and the theoretical curves would mean that the single-particle model could be made to fit the data by multiplying the dipole and octopole moments by the same factor, $\mu_0(\text{exptl.})/\mu_0(\text{Schmidt})$. This assumption would agree with measurements of octopole moments in much heavier nuclei.[10]

The results show that this assumption is consistent with the B^{11} data above $q^2 = 2\ F^{-2}$, where the octopole term dominates the form factor. It is remarkable that the general shape of the B^{11} form factor is so well described by this crude model. In Be^9, however, the octopole moment is apparently smaller than this assumption would indicate. More refined theories are now being calculated.[9] Other P-shell nuclei, Li^6, Li^7, C^{13}, and N^{14}, are also being investigated and will form the subject of further reports.

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