MEASUREMENT OF THE RECOIL PROTON POLARIZATION IN ELASTIC π^+p SCATTERING AT $T_\pi = 410$ AND 492 MeV

P. Bareyre, C. Bricman,* M. J. Longo,† G. Valladas, and G. Villet
Laboratoire de Physique Corpusculaire à Haute Energie, Centre d'Etudes Nucléaires de Saclay, Saclay, France

and

G. Bizard, J. Duchon, J. M. Fontaine, J. P. Patry, J. Seguinot, and J. Yonnet
Laboratoire de Physique Corpusculaire de l'Université de Caen, Caen, France
(Received 18 December 1964)

The angular distribution of the recoil-proton polarization in elastic pion-nucleon scattering has been measured at the synchrotron "Saturne," at the pion energies of 410 and 492 MeV. These energies have been chosen to extend the range where the pion-nucleon interaction is fairly well known. We present here the measurements corresponding to the scattering of positive pions.

Without a polarized proton target, a double scattering is needed to measure the polarization. We used a carbon-plate spark chamber, as the analyzing power of this material is rather well known.[1,2]

Experimental setup (Fig. 1).—The spark chamber[3] was made of 28 carbon plates, 1 and 0.5 cm thick. Six thin aluminum foils were used to improve the measurements of the recoil angle of the proton. The useful area was 80 $\times 60$ cm^2. The pion beam of momentum resolution 2% and angular divergence 0.5° was incident on a liquid-hydrogen target 35 cm thick. The spark chamber was triggered by the coincidence $C_1C_2\bar{C}_3C_4C_5\check{C}_1$ or $C_1C_2\bar{C}_3C_4C_5\check{C}_2$. The water Čerenkov counters $\check{C}_1$ and $\check{C}_2$ selected the scattered pion. The counters C_4, C_5 guaranteed an approximate coplanarity.

To cover an angular range as wide as possible, the spark chamber was placed in two angular positions, with its axis at 30° and 43°, respectively. All measurements have been repeated symmetrically with the chamber on the left and on the right of the beam.

For the run with positive pions we triggered about 1.5 million times. The chamber was photographed once per cycle (and later twice per cycle). Each photograph corresponds to an average of four triggers.

Data analysis.—The following data were recorded: the coordinate of the entry point, the plate in which the scattering occurred, and the plate in which the proton stopped. In addition, certain scanning criteria were required. The angles were measured with a digitized protractor.

A reconstruction program checked that the first scattering had taken place in the hydrogen

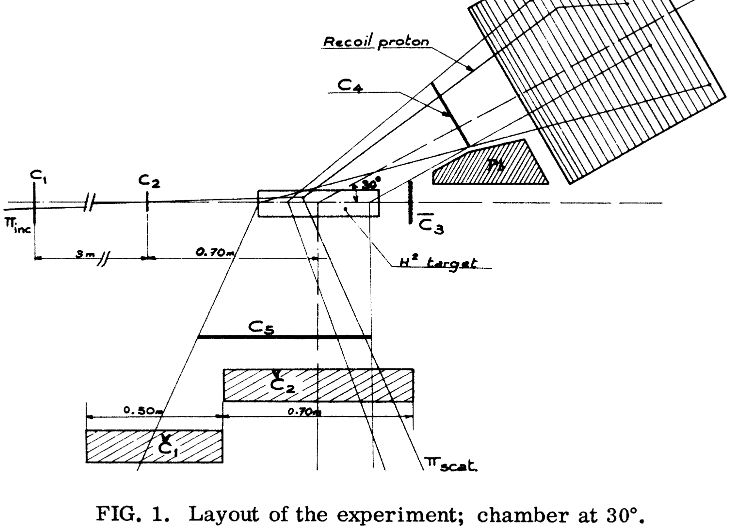

FIG. 1. Layout of the experiment; chamber at 30°.

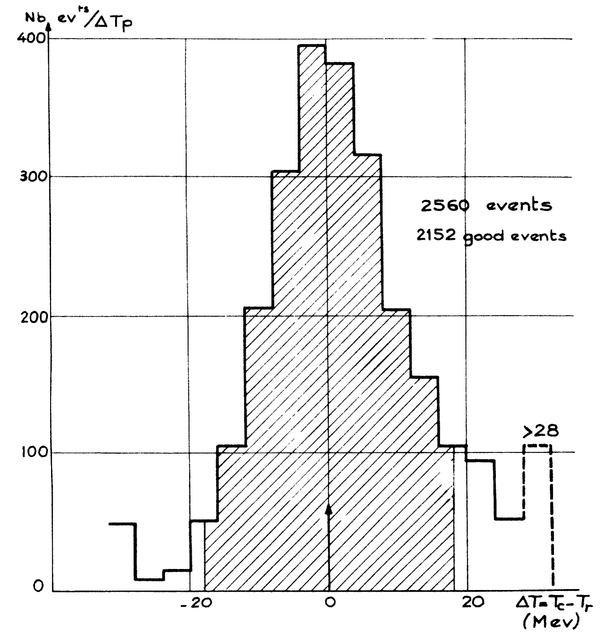

FIG. 2. Distribution of the calculated energy loss in the p-C scattering (second scattering). The events in the cross-hatched region are considered as being elastic scatters.

target and that the selected events formed an unbiased sample. (In particular, if the proton has scattered to the other side, the event would still fulfill the scanning criteria.)

Each event is characterized by the following quantities: the scattering angle θ_1 of the p in the π-p collision; the scattering angle θ_2 in the p-C collision; the angle between the two scattering planes φ; the kinetic energy T_2 of the second scattering; the kinetic energy T_C calculated assuming an elastic π^+p scattering; and the kinetic energy T_r evaluated from the range in matter and assuming an elastic p-C scattering. The events for which the difference $|T_C - T_r| < 18$ MeV are considered as elastic.

Figure 2 shows a typical histogram of the energy distribution (half-width at half-maximum = 12 MeV). This width is essentially due to the thickness of the plates, to the momentum dispersion of the beam, and to the uncertainty in the scattering angles.

The polarization of the protons as a function of the cosine of the pion scattering angle in the center of mass has been computed by the maximum likelihood method. The most probable value of the polarization P_1 is the one which maximizes the product

$$\prod_{i=\Lambda}^{n} [\Lambda + P_\Lambda \cdot P_2(\theta_{2i}, T_{2i}) \cos\theta_i].$$

The analyzing power of the carbon, $P_2(\theta, T)$, is taken from a table similar to that of Peterson,[4] which we have established using the known experimental values in the range $71 < T_2 < 313$ MeV. Our table takes into account the effect of contamination by inelastic p-C scattering. Since the analyzing power of carbon is imperfectly known, we have carried out a calibration

Table I. Experimental results. For $T_\pi = 410$ and 492 MeV, and for each interval of $\cos\theta_\pi^*$, we give the average energy T_p of the protons when entering the chamber; the polarization as measured with the chamber at the right and at the left of the incident beam; and the resulting average polarization [Figs. 3(c) and 4(b)]. The error corresponds to the rms of the likelihood curve.

	410 MeV				492 MeV			
		Polarization				Polarization		
$\cos\theta_\pi^*$	T_p (MeV)	Left position	Right position	Average	T_p (MeV)	Left position	Right position	Average
−0.8	248	−0.19 ± 0.12	−0.21 ± 0.21	−0.20 ± 0.10				
−0.7	224	−0.36 ± 0.09	−0.41 ± 0.12	−0.375 ± 0.075	274	−0.61 ± 0.09	−0.52 ± 0.09	−0.545 ± 0.07
−0.6	210	−0.27 ± 0.09	−0.32 ± 0.11	−0.29 ± 0.07	258	−0.64 ± 0.09	−0.71 ± 0.07	−0.68 ± 0.06
−0.5	196	−0.48 ± 0.09	−0.32 ± 0.11	−0.415 ± 0.07	240	−0.73 ± 0.11	−0.79 ± 0.07	−0.765 ± 0.065
−0.4	182	−0.31 ± 0.10	−0.41 ± 0.12	−0.35 ± 0.075	224	−0.92 ± 0.12	−0.81 ± 0.09	−0.85 ± 0.08
−0.3	168	+0.17 ± 0.10	−0.02 ± 0.12	+0.085 ± 0.08	205	−0.29 ± 0.11	−0.29 ± 0.09	−0.29 ± 0.07
−0.2	153	−0.02 ± 0.10	+0.07 ± 0.10	+0.025 ± 0.07	188	−0.19 ± 0.11	−0.13 ± 0.07	−0.155 ± 0.06
−0.1	140	+0.17 ± 0.10	−0.05 ± 0.10	+0.065 ± 0.07	172	−0.31 ± 0.09	−0.12 ± 0.07	−0.20 ± 0.06
0	124	+0.12 ± 0.12	+0.02 ± 0.11	+0.06 ± 0.08	155	−0.06 ± 0.09	−0.07 ± 0.07	−0.065 ± 0.06
+0.1	112	+0.12 ± 0.15	−0.02 ± 0.15	+0.05 ± 0.10	138	−0.15 ± 0.10	−0.02 ± 0.07	−0.055 ± 0.06
+0.2	98	−0.38 ± 0.20	+0.24 ± 0.23	−0.075 ± 0.15	123	−0.05 ± 0.13	−0.08 ± 0.08	−0.075 ± 0.065
+0.3	84	+0.05 ± 0.36	−0.12 ± 0.38	−0.04 ± 0.27	105		−0.07 ± 0.13	−0.07 ± 0.13
+0.4					90		−0.07 ± 0.17	−0.07 ± 0.17

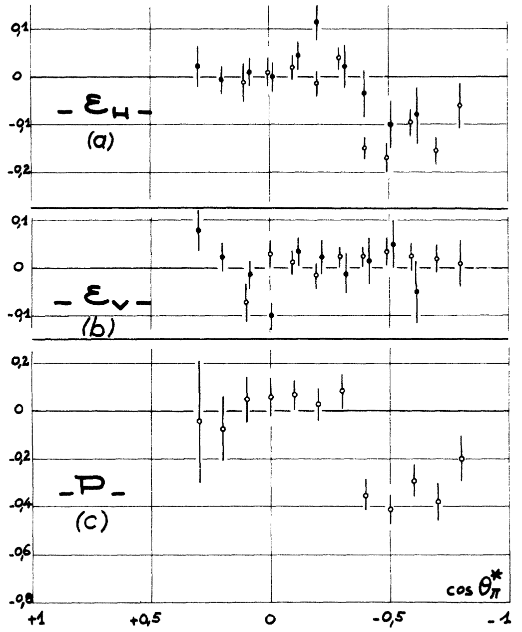

FIG. 3. Angular distribution of the asymmetries and of the polarization at $T_\pi = 410$ MeV. Solid points represent the data obtained with the chamber at 30°, and open circles those with the chamber at 43°. The points represent the average over the results with the chamber at the left and at the right of the beam.

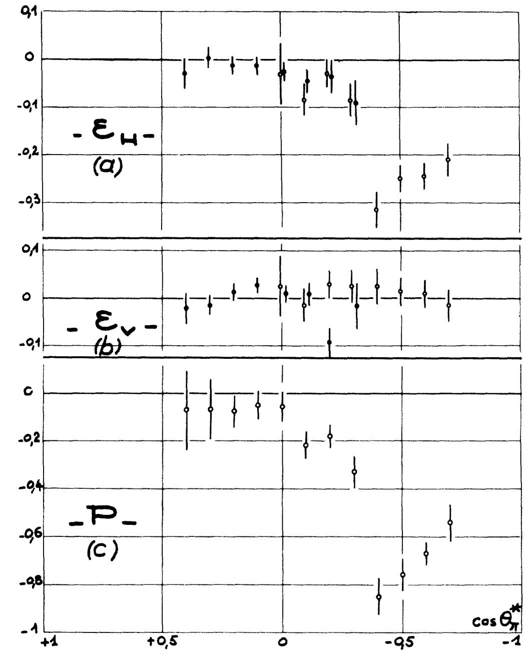

FIG. 4. Angular distributions of the asymmetries and the polarization at $T_\pi = 492$ MeV. Symbols are the same as in Fig. 3.

experiment with protons polarized by a first scattering from a carbon target. The preliminary results show that the corrections to the table will not change the value of the polarization in π-p scattering by more than 0.05.

Results.—At the pion energy of 410 MeV, 36 700 proton tracks have been selected by the scanning from a total of 640 000 tracks; 18 000 of these events satisfied the criteria for geometry and for elasticity and were used for computing the polarization. At 492 MeV the corresponding figures are, respectively, 49 000, 910 000, and 23 000. In Table I we give our main results.

Figures 3(a) and 3(b) show, at $T_\pi = 410$ MeV, the horizontal and vertical asymmetries obtained for each angular position of the chamber. These asymmetries are defined by $\epsilon_{\text{hor}} = (N_L - N_R)/(N_L + N_R)$ and $\epsilon_{\text{ver}} = (N_U - N_D)/(N_U + N_D)$. N_L and N_R are the numbers of tracks scattered to the left or to the right of the plane which is normal to the plane of the first scattering and which contains the direction of the proton. N_U and N_D are the number of tracks scattered up or down with respect to the plane of the first scattering. Figure 3(c) shows the angular distribution of the polarization given by the maximum-likelihood method, combining the results from all four positions of the chamber: two angles at each side of the incident beam. In Fig. 4 we give the same distribution for $T_\pi = 492$ MeV.

The results show good agreement between the measurements with the chamber at the left and at the right. There is also good agreement in the angular interval which was covered by both positions of the chamber.

The vertical asymmetry is consistent with zero. We can conclude, therefore, that our measurement of the polarization is not affected by instrumental biases.

We wish to express our appreciation to Professor A. Berthelot and Professor M. Scherer for having supported this research. Our thanks go also to all of the people who assisted us in carrying out this experiment, particularly the group "Techniques Nucléaires," the scanners, the crew of "Saturne," and the "Service de Calcul Electronique."

---

*Chercheur agrée IISN, Bruxelles, Belgium.

†Present address: University of Michigan, Ann Arbor, Michigan.

[1]E. F. Beall, B. Cork, P. G. Murphy, W. A. Wenzel, C. M. P. Johnson, and L. J. Koester, Jr., Phys.

Rev. 126, 1554 (1962).
[2]R. D. Eandi, T. J. Devlin, R. W. Kenney, P. G. McManigal, and B. J. Moyer, Phys. Rev. 136, B536 (1964).
[3]P. Autones, P. Bareyre, J. M. Gaillard, G. Robert, and R. Seige, Nucl. Instr. Methods 24, 418 (1963).
[4]V. Z. Peterson, University of California Radiation Laboratory Report No. UCRL-10622 (unpublished), and references quoted there.

PHASE-SHIFT ANALYSIS IN π^+p SCATTERING AT $T_{\pi\,\mathrm{lab}} = 410$ AND 492 MeV

P. Bareyre, C. Bricman,* J. Seguinot,† and G. Villet
Laboratoire de Physique Corpusculaire à Haute Energie, Centre d'Etudes Nucléaires de Saclay, Saclay, France
(Received 18 December 1964)

Many phase-shift analyses have been performed in the energy range $T_\pi \leq 200$ MeV. The cross-section results are well described by S and P waves only. In addition to the Fermi-type solution, other solutions have appeared because of the intrinsic ambiguities. They have been theoretically rejected by using dispersion relations. For slightly higher energies, D waves are no longer negligible and give rise to new ambiguities which cannot be resolved by the scattering angular distributions alone. Supplementary experimental data such as recoil proton polarization are necessary. Some polarization measurements have been done,[1,2] and a more complete analysis was realized at 310 MeV.[2] Two phase-shift sets giving a good fit to the experimental results have been obtained for the $\frac{3}{2}$ isotopic spin component. For energies higher than 300 MeV, the inelastic scattering increases the number of parameters by a factor 2.

For our analysis of positive pion-proton scattering at 410 and 492 MeV, we have the following data: σ_{tot},[3] $(d\sigma/d\Omega)_{\mathrm{el}}$,[4] and recoil proton polarization,[5] which seem sufficient to carry out the analysis.

Search for solutions. –With an IBM-7094 II computer we have used a search program[6] which seeks to minimize the quantity

$$\chi^2 = \sum_i \left[\frac{P_i^{\mathrm{calc}} - P_i^{\mathrm{exp}}}{\Delta P_i^{\mathrm{exp}}}\right]^2 + \left[\frac{\sigma_{\mathrm{tot}}^{\mathrm{calc}} - \sigma_{\mathrm{tot}}^{\mathrm{exp}}}{\Delta\sigma_{\mathrm{tot}}^{\mathrm{exp}}}\right]^2 + \sum_j \left[\frac{\left(\frac{d\sigma}{d\Omega}\right)_j^{\mathrm{calc}} - \left(\frac{d\sigma}{d\Omega}\right)_j^{\mathrm{exp}}}{\Delta\left(\frac{d\sigma}{d\Omega}\right)_j^{\mathrm{exp}}}\right]^2,$$

where, for example, P_i^{calc} is the value of the polarization at the angle θ_i calculated from a phase-shift set with the usual formulas for π^+-p scattering[7]; P_i^{exp} and $\Delta P_i^{\mathrm{exp}}$ are the experimental value and error on the polarization for the same angle. A gradient method is used to minimize χ^2. This method gives the phase-shift set corresponding to a relative minimum of the hypersurface χ^2, starting from an initial set. The minimization must be repeated many times, using a random starting set, to be certain to find all the minima. For a good solution one expects that the final value of χ^2 will not be too different from the number of degrees of freedom N, i.e., the number of experimental data minus the number of parameters.

$T_\pi = 410$ MeV. –We use 19 values of $d\sigma/d\Omega$, including the value at zero degrees obtained from dispersion relations; 12 values of the recoil proton polarization; and one value for σ_{tot}.

(1) *SPD* analysis: Approximately 150 starting sets have been used which yield two solutions that have an acceptable value for χ^2/N. These solutions are given in Table I. Eight other solutions were found, which have $\chi^2/N > 2.3$.

The fact that we have two solutions reflects the uncertainty of the measurements (see Fig. 1). It should be noted that, if the polarization measurements are not utilized, an ambiguity of the Fermi-Yang type exists in the D wave.

(2) *SPDF* analysis: Using 60 starting sets no new solution appeared. Beginning with either the solution *SPD* Fermi I or *SPD* Fermi II, only one *SPDF* solution was found. The F waves do not seem to be necessary; the agreement with the experimental results is almost the same as for *SPD* solutions; however, the errors of the phase shifts are increased.

$T = 492$ MeV.– For this energy we use the same number of data as for 410 MeV.

(1) *SPD* analysis: Table II shows the results