B. Elsner, D. Harting, and W. Middelkoop, in addition to the author. The analysis was directed by B. Zacharov, and valuable contributions were made by E. Bleuler. At the University of Michigan, E. Coleman, B. Loo, and S. Powell have assisted in the analysis. R. Deck, S. Berman, S. Drell, A. Goldhaber, and M. Ross have all contributed concepts and clarifications on this subject in their valuable discussions with the author. Finally, the hospitality and collaboration of CERN in the execution of this experiment and subsequent data analysis is greatly appreciated.

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*Work supported in part by the U. S. Office of Naval Research and by the Ford Foundation.

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IMPLICATIONS OF SU(6) SYMMETRY FOR TOTAL CROSS SECTIONS*

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(Received 4 January 1965)

There has been much speculation recently about the possibility of applying to elementary-particle physics the Wigner[1] supermultiplet theory in nuclear physics. One considers the group SU(6),[2] which contains as a subgroup SU(2) $\times$ SU(3). The first factor refers to rotations in spin space, the second to the internal symmetry group. The difficult problems of reconciling these ideas with the principle of relativity are still open. As a consequence, the application of SU(6) notions has so far been confined to static questions, such as the identification of multiplets, mass formulas, electromagnetic properties, etc. Rules for operating with SU(6) become laid down in the course of these applications. In the above context the predictions of the theory have met with remarkable experimental success.

One would also like to apply (develop?) the theory to more general questions involving scattering reactions among particles. Here, in general, one must come to grips with the meaning of SU(6) in a relativistic context. However, even in advance of any full clarification of the theory in this direction, we suggest here that a limited class of new problems can be explored in something like the spirit of what has already been developed.

Namely, let us consider the forward elastic scattering of pseudoscalar mesons on baryons. In the laboratory frame, the initial and final baryons are at rest and the forward scattering amplitude can be written

$$f(0°) \sim \int d^4x\, e^{iq\cdot x} \langle 0,B|[j(x),j(0)]\theta(x_0)|0,B\rangle, \quad (1)$$

where $|0,B\rangle$ is the state of a baryon at rest. The operator $j(x)$ is the source function of the pseudoscalar meson field $\varphi(x)$. Precisely because the initial and final states describe baryons at rest in the same frame, it presumably has a meaning to regard the baryon states as belonging to a definite SU(6) multiplet—the 56-dimensional representation according to currently accepted assignments. But the question arises whether it makes sense to assign definite SU(6) transformation properties to the meson field $\varphi(x)$. The state $|0,M\rangle$ of a meson at rest is obtained from the vacuum state $|0\rangle$ according to the operation

$$|0,M\rangle \sim \int d^3x\, e^{ik\cdot x} \overleftrightarrow{\frac{\partial}{\partial x_0}} \varphi(x)|0\rangle \quad (x_0 \to \pm\infty), \quad (2)$$

where $\vec{k}=0$ and k_0 = meson mass. Since the meson states are supposed to have definite SU(6) transformation properties, so also must the above zero-momentum Fourier component of the meson-field operator. In a conventional situation one would be inclined to assign to the meson fields themselves, for all Fourier components, the same definite transformation properties as for the meson states. With SU(6) the situation is not so well defined. One can imagine, for example, that the meson field, expressed in terms of more fundamental (quark) fields, involves a total divergence—which would not contribute to the integral of Eq. (2).

Without claiming to understand the situation at a fundamental level, we nevertheless wish to explore here the implication of assigning to the full meson fields $\varphi(x)$ the definite SU(6) transformation properties of the meson states themselves—namely, those corresponding to the representation of dimension 35. Since only the $l=0$ (s-wave) part of the exponential $e^{iq\cdot x}$ contributes to the integral of Eq. (1), there is no further ambiguity in applying SU(6) notions to that expression. It bears a resemblance to corresponding expressions which arise in the SU(6) treatment of magnetic moments and electromagnetic mass shifts.

Since the direct product of the 35- and 56-dimensional representations can be decomposed according to $\underline{35}\otimes\underline{56}=\underline{56}\oplus\underline{70}\oplus\underline{700}\oplus\underline{1134}$, it is clear that all pseudoscalar-baryon amplitudes can be expressed in terms of four invariants. But there are six such reactions which are experimentally measurable in practice and which are unrelated by ordinary isotopic spin considerations (scattering off protons of K^+, K^-, K^0, $\bar{K}^0$, π^+, π^-). The corresponding six forward scattering amplitudes thus become connected, via SU(6), by two relations. These are found to be

$$\tfrac{1}{2}[f(K^+)-f(K^-)]=[f(K^0)-f(\bar{K}^0)]=[f(\pi^+)-f(\pi^-)], \quad (3)$$

where the target bayon in all cases is taken to be a proton. In particular, through the optical theorem, a corresponding set of relations is implied for total cross sections:

$$\tfrac{1}{2}[\sigma(K^+)-\sigma(K^-)]=[\sigma(K^0)-\sigma(\bar{K}^0)]=[\sigma(\pi^+)-\sigma(\pi^-)]. \quad (4)$$

The first part of Eq. (4) is discussed by Good and Xuong in the following Letter.[3] Concerning the relation

$$\Delta_K\equiv[\sigma(K^-)-\sigma(K^+)]=2\Delta_\pi\equiv 2[\sigma(\pi^-)-\sigma(\pi^+)],$$

there of course exist ample experimental data. Here, however, one is embarrassed by the undeniable mass difference between K and π mesons. This of course reflects a violation of SU(6) [and SU(3)], and makes uncertain how the comparison between Δ_K and Δ_π is to be effected. Provisionally, it seems to us reasonable to restrict attention to the high-energy region, where mass differences have less obvious reason to be important and where the cross sections are sufficiently smooth with energy so that it does not matter too much whether one compares Δ_K and Δ_π at similar energies, or similar momenta, etc. In the multi-BeV region the $K^\pm$ and $\pi^\pm$ total cross sections are slowly varying, Δ_K and Δ_π both decreasing slowly with increasing energy. These difference quantities are small, so that the errors are relatively rather large. But for momenta of a few BeV/c on up to 20 BeV/c it appears definite that Δ_K and Δ_π are nonvanishing and positive and that Δ_K is everywhere larger than Δ_π by a factor of order two or three.[4] For example, at $p=4.0$ BeV/c, $\Delta_K\approx 8.7$ mb, $\Delta_\pi\approx 2.7$ mb; at $p=5.5$ BeV/c, $\Delta_K\approx 6.4$ mb, $\Delta_\pi\approx 2.5$ mb; at $p=10$ BeV/c, $\Delta_K\approx 4.4$ mb, $\Delta_\pi\approx 1.7$ mb; at $p=15$ BeV/c, $\Delta_K\approx 3.0$ mb, $\Delta_\pi\approx 1.3$ mb; at $p=19.0$ BeV/c, $\Delta_K\approx 4.3$ mb, $\Delta_\pi\approx 1.5$ mb. The errors are typically of order one millibarn. The agreement with the prediction $\Delta_K=2\Delta_\pi$ is not obviously terrible, considering that the experimental uncertainty in the ratio Δ_K/Δ_π has a value of about 50%.

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*Work supported by the U. S. Air Force Office of Research, Air Research and Development Command.

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EXPERIMENTAL CHECK OF THE JOHNSON-TREIMAN RELATION ON THE K-N TOTAL CROSS SECTIONS*

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(Received 7 January 1965)

Using the SU(6) theory[1] and the fact that K and π mesons belong to an octet of the $\underline{35}$ multiplet, and the proton to an octet of the $\underline{56}$ multiplet, Johnson and Treiman[2] have shown that there are two relations between the Kp and πp total cross sections:

$$\tfrac{1}{2}[\sigma(K^+p)-\sigma(K^-p)]=[\sigma(K^0p)-\sigma(\bar{K}^0p)],\tag{1}$$

and

$$\tfrac{1}{2}[\sigma(K^+p)-\sigma(K^-p)]=[\sigma(\pi^+p)-\sigma(\pi^-p)].\tag{2}$$

Because of the mass difference between K and π mesons, the relation (2) is expected to hold only for the high-energy region. Johnson and Treiman[2] have pointed out that in the multi-BeV region, this relation agrees more or less with experiment. On the other hand, the relation (1) is expected to hold even at lower energy.[3] We would like to point out here that the relation (1) is supported by experiment in the region of momentum of the K meson from 0.6 to 3 BeV/c (in the laboratory).

Using charge symmetry one gets

$$\sigma(K^0p)=\sigma(K^+n)\text{ and }\sigma(\bar{K}^0p)=\sigma(K^-n).\tag{3}$$

Therefore, relation (1) with charge symmetry gives the relation

$$R\equiv\frac{\sigma(K^++n)-\sigma(K^-+n)}{\sigma(K^++p)-\sigma(K^-+p)}=\frac{1}{2}.\tag{4}$$

Because charge symmetry is known to hold for strong interactions, a check of relation (4) is equivalent to a check of relation (1).

In Fig. 1 we have plotted all the data known to us on the total cross section of K^-p, K^-n, K^+p, and K^+n interactions[4,5] below 4 BeV/c.

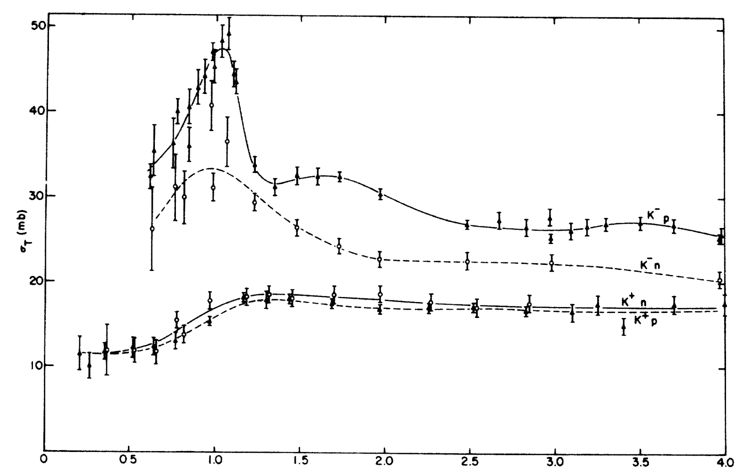

FIG. 1. Total cross sections of K-N interaction versus the laboratory momentum of the K meson. All data come from references 4 and 5.