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SU(6)⊗O(3) STRUCTURE OF STRONGLY INTERACTING PARTICLES

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Recently there has been considerable interest in the possibility of an SU(6) structure of the strongly interacting particles. Significant success has been achieved in the application of the SU(6) structure to multiplet assignments,[1] mass spectra,[2] meson-baryon coupling structure,[1] and electromagnetic[3,4] and weak interaction[5] properties of baryons. The group SU(6), as defined in the present context, contains the intrinsic spin group $SU(2)_S$ and the internal symmetry group SU(3). The theory is thus naturally a nonrelativistic one. The basic group of a nonrelativistic theory (as contrasted with a Galilean or a Lorentz group) is the Newtonian group of space and time translations and rotations in the three-dimensional space. It is thus natural to combine the SU(6) structure with this Newtonian group to construct the relevant combined space-time and internal symmetry group. As the first step in this direction, we postulate the invariance of the strong interactions under SU(6)⊗O(3), where O(3) is a group of rotations in the three-dimensional space, independent of the spin group $SU(2)_S$ contained in SU(6). We then examine the various consequences of this postulate. We find the following results:

(i) A unique SU(6)⊗O(3)-invariant (parity-conserving) Yukawa coupling, bilinear in the baryon supermultiplet components, can be constructed.

(ii) There exist a 0^- nonet, a 1^- nonet, and a 2^- nonet of mesons. The octet part of the 0^- nonet is identified with π, K, $\bar{K}$, and η. The 1^- nonet is identified with ρ, K^*, $\bar{K}^*$, φ, and ω.

(iii) In addition to the above submultiplets, there exists a second 1^- octet.

(iv) The vector-nonet decay into two pseudoscalar mesons is no longer forbidden in the limit of exact symmetry.

(v) Within the framework of our mass formula the 0^- singlet and the 2^- singlet are degenerate in mass among themselves and with the (unphysical) degenerate mass of φ and ω. This predicted mass differs by 3 percent from the observed mass of the X_0 (960 MeV) meson. No appreciable η-X_0 mixing is expected.

Multiplet assignments and coupling structure.—We shall denote the irreducible representations of SU(6)⊗O(3) by (a_6, b), where a_6 is the dimensionality of the representation of SU(6) and $b = 2l+1$ is the dimensionality of the representation of O(3). As the assignment of the baryons and baryon resonances according to SU(6) is reasonable, we assign these to (56, 1). For the mesons, we require an assignment such that we can construct SU(6)⊗O(3)-invariant (parity-conserving) Yukawa interactions. The simplest such assignment is (35, 3). This supermultiplet decomposes under the SU(3)⊗SU$(2)_J$ classification [in which the spin group SU$(2)_S$ and the O(3) group structures are identified] into a vector (1^-) octet and the three nonets (octet + singlet), respectively pseudoscalar (0^-), vector (1^-), and pseudotensor (2^-). In the symmetric limit all these particles are degenerate in mass. Now the meson matrix can be written as

$$\Phi_{\beta,m}^{\alpha} \equiv \Phi_{sB,m}^{rA} = (\sigma^k)_s^{\ r} R_{B,m}^{A,k} + \delta_s^{\ r}\{S_{B,m}^{A} - \tfrac{1}{3}\delta_B^{A} S_{C,m}^{C}\}, \tag{1}$$

where r, s are two-valued spinor indices; A, B, C are three-valued SU(3) indices; and k, m are three-valued vector indices; summation over repeated indices is understood. We can indicate the SU(3)⊗SU$(2)_J$ reduction by writing

$$R_{B,m}^{A,k} = \tfrac{1}{3}\delta_m^{\ k} R_{B,n}^{A,n} + \tfrac{1}{2}(R_{B,m}^{A,k} - R_{B,k}^{A,m}) + \tfrac{1}{2}(R_{B,m}^{A,k} + R_{B,k}^{A,m} - \tfrac{2}{3}\delta_m^{\ k} R_{B,n}^{A,n}), \tag{2}$$

which displays the nonet of 0^-, 1^-, and 2^- unnormalized meson wave functions. The parity of the particle states is entirely due to the property of the three-dimensional space.[6] The baryon-meson interaction could now be immediately written down:

$$\frac{G}{i}\bar{B}_{\alpha\gamma\delta}\nabla_m B^{\beta\gamma\delta}\Phi_{\beta,m}^{\alpha}, \tag{3}$$

which is seen to be SU(6)⊗O(3) invariant. Using the expressions (1) and (2), we see that 0^-, 1^-, and 2^- mesons are, respectively, coupled to baryons via $\vec{\sigma}\cdot\nabla$, $\vec{\sigma}\times\nabla$, and $(\vec{\sigma}\nabla)$. Thus the vector nonet is coupled to baryons through a magnetic-type coupling, whereas the vector octet is coupled through an electric-type coupling.[7] The coupling structure (3) leads to two immediate results:

(a) In the coupling of the nonets, the D/F ratio for the octet coupling to baryons is unique and is given by[1] $\frac{3}{2}$. In contrast, the second vector octet is coupled via pure F-type coupling.

(b) Assuming π-N coupling $g_{ps}^2/4\pi \sim 15$, one obtains the N^* width to be ~75 MeV.[1]

In analogy with (3) we may write down the trilinear meson coupling

$$\frac{\lambda}{i}\Phi_{\gamma,l}^{\beta}\nabla_m\Phi_{\alpha,l}^{\gamma}\Phi_{\beta,m}^{\alpha}, \tag{4}$$

which is also SU(6)⊗O(3) invariant and allows the decays $\rho \to \pi + \pi$, $K^* \to K + \pi$, and $\omega \to K + \bar{K}$.

Mass formula.—The highly symmetric SU(6) ⊗O(3) discussed so far is broken, as evidenced by the lifting of the mass degeneracy between the vector and the pseudoscalar particles prior to the stage at which SU(3) invariance is broken. Before deriving a mass formula from general considerations, we obtain a suggestive mass formula for mesons with the following reasoning: The symmetry could be broken by invoking the spin-orbit coupling term $\vec{L}\cdot\vec{S}$ Also, the symmetry breaking could split a nonet into an octet and a singlet through the appearance of $C_2^{(3)}$, the quadratic Casimir operator of SU(3), but we must guarantee that the φ and the ω of the vector nonet remain degenerate. The simplest way to accomplish this gives rise to the following mass formula involving a single symmetry-breaking parameter[8]:

$$m(a_6, a_3, a_2, b, J) = m_0(a_6, a_2, b) + m_1(1 + L\cdot S)C_2^{(3)}, \quad (5)$$

where a_3 and a_2 are, respectively, the dimensionalities of SU(3) and $SU(2)_S$ contained in SU(6). The symmetry-breaking chain is SU(6) $\otimes O(3) \supset SU(3)\otimes SU(2)_J$. Further breaking of SU(3) to U(2) by a neutral octet tensor gives a Gell-Mann–Okubo splitting of the SU(3) multiplets and the standard ω-φ mixing. The mass formula (5) gives rise to a large separation of the respective octet and singlet members in the 0^- and the 2^- nonets. In fact the degenerate mass of the 1^- nonet, 0^- singlet, and 2^- singlet is ~930 MeV; the mass of the 0^- octet is ~550 MeV and that of the 2^- octet is ~1400 MeV.[10] Hence there is no analog of the φ-ω mixing for either the 0^- or the 2^- nonet. We note that the mass of the second 1^- octet is not related by the mass formula to the nonets, as m_0 is dependent on the instrinsic spin which is unity for the nonets and zero for the octet. Under the SU(3) breaking, the known relation

$$m_{K^*}^2 - m_\rho^2 = m_K^2 - m_\pi^2$$

is valid in this model also; it can be generalized to the prediction that the difference in $(\text{mass})^2$ values of the isodoublet and isotriplet of all the meson octets is the same.

The Casimir operators $C_2^{(3)}$ and $(\vec{L}\cdot\vec{S})C_2^{(3)}$ in the mass formula (5) are contained in (35, 1) $\otimes[(35,1)\oplus(35,3)]$. Under the assumption that the symmetry-breaking interaction transforms like $(35,1)\otimes[(35,1)\oplus(35,3)]$, we can get a general mass formula which will be discussed in detail elsewhere. We note here that for the baryons which have been assigned to (56, 1) there is no contribution from $(35,1)\otimes(35,3)$, and the general mass formula reduces to the one already obtained.[2]

So far we have discussed only (56, 1) and (35, 3) for baryons and mesons. In addition to the various other representations with different values of a_6 to which they could be assigned,[1] we could assign them to representations with any arbitrary value of $b = 2l + 1$.

Electromagnetic properties.—The structure of the nonrelativistic electromagnetic interaction consists of the electric charge and current interactions which transform, respectively, like (35, 1) and (35, 3) belonging to the spin singlet term in its $SU(2)_S\otimes SU(3)\otimes O(3)$ reduction; and the magnetic interaction which transforms like (35, 3) but belongs to the spin triplet term under this reduction. In the limit of exact $SU(6)\otimes O(3)$, we observe that the lowest order matrix elements of the electromagnetic interactions between the states of (56, 1) baryon multiplet, which would define their magnetic moment and electric and magnetic form factors (and transition moments and form factors), vanish, with the exception of the charge form factor, which is universal and proportional to electric charge. We have to consider then the interference between the term which breaks the $SU(6)\otimes O(3)$ symmetry and the primitive electromagnetic interactions. We have already invoked the mechanism of symmetry breaking by the spin-orbit interaction in connection with the mass formula. A more primitive symmetry breaking is provided by the recoil of the (spinning) baryon. The generator K_j of the transformation to a frame moving with an increase of velocity in the jth direction (for both the Galilei and the Lorentz sysetms) transforms as (1, 3) for free spinless particles, and as $(1,3)\oplus(35,3)$ for free spinning particles[9] (in Lorentz systems). In any case, by virtue of the commutation relation

$$(1/i)[K_j, H] = P_j$$

being valid for arbitrary interacting Lorentz or Galilei systems, the generator K_j or the Hamiltonian H (or both!) has parts which transform as the three-component (vector) representation of O(3). Assuming that this transforms as (1, 3), we deduce in a very transparent fashion that the leading contributions to the linear electromagnetic properties transform as the (35, 1) representation. This yields the results already obtained[3] for the ratio of the neutron and proton magnetic moments, $\mu_n/\mu_p = -\frac{2}{3}$, and also gives rise to similar relations for the baryon magnetic moments and transition moments. We also get the result that all the magnetic form factors are multiples of a common magnetic form factor, and that the electric (current) form factors are simple multiples of a common electric (current) form factor, the electric charge being the multiplicative factor. The latter result implies that the electric (current) form factor of a neutron vanishes. It is to be noted that the nonrelativistic magnetic interaction gets contributions from both the relativistic electric and magnetic form factors. In the present model we note that the magnetic contribution comes mainly from the coupling of baryons to the vector nonet, which is magnetically coupled, in contrast to the vector octet, which is electrically coupled.

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SU(6) AND NONLEPTONIC HYPERONIC DECAYS

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Recently Gürsey and Radicati[1] have suggested the possibility that the strong interactions might be approximately spin-unitary-spin independent. This corresponds to the invariance of the strong-interaction Hamiltonian under an SU(6) group which includes spin independence and unitary-spin independence. The static properties of particles like mass, charge, and magnetic moment seem to have rather simple transformation properties under this group.[2-5] Thus it looks plausible to assume simple transformation properties for the Hamiltonian responsible for nonleptonic weak decays of hyperons also under this group.

Within the framework of SU(3) it is usually assumed that this Hamiltonian transforms like a member of the adjoint representation. It has been shown by Coleman, Glashow, and Lee[6] that it is possible to have seven types of interaction terms for s-wave (p-v – parity-nonconserving) decay amplitudes, and seven more for the p-wave (p-c – parity-conserving) decay amplitudes consistent with CP invariance. Thus one is not able to obtain any sum rule among the p-v or the p-c amplitudes of the four observed decay amplitudes $[\Lambda \to N+\pi,\ \Sigma \to N+\pi\ (I=\frac{1}{2},\frac{3}{2}),$ and $\Xi \to \Lambda+\pi]$.

As a natural generalization we assume that the relevant Hamiltonian transforms like a member of the adjoint representation of SU(6). Now we have only four possible interaction terms for the p-v amplitudes and four more for the p-c amplitudes. But out of these eight terms, only four contribute to the decays of interest