[7]P. Roman and J. J. Aghassi (to be published) have also introduced a U(2) group but with a different rationale from ours. More recently, K. Bardakci, J. Cornwall, P. Freund, and B. Lee [Phys. Rev. Letters 13, 698 (1964)] have found a noncompact group $M(12)$ whose maximal compact subgroup is W(6); our U(12) group is, of course, compact.

[8]This may possibly be regarded as an illustration of a more general theorem proved by L. S. O'Raifeartaigh, private communication.

[9]R. E. Marshak and S. Okubo, Nuovo Cimento 19, 1226 (1961) (see Appendix); cf. also R. E. Marshak, N. Mukunda, and S. Okubo, Phys. Rev. 137, B698 (1965). If we limit ourselves to the free Hamiltonian for a triplet of quark fields, then the underlying symmetry is larger than R(6). This can be seen most readily by expanding $\chi_\mu(x)$ in terms of the ordinary creation and annihilation operators:

$$H_0=K_0+H_0(m)=\sum_{\mu=1}^{3}\sum_{n=1}^{2}\int d^3p\,E_p[a_{\mu r}^*(p)a_{\mu r}(p)+b_{\mu r}^*(p)b_{\mu r}(p)],$$

and defining the 12-component annihilation operator

$$A(p)=\begin{pmatrix}a_{\mu r}(p)\\ b_{\mu r}(p)\end{pmatrix}$$

$(\mu=1,2,3;\ r=1,2)$. Then H_0 can be rewritten as

$$H_0=\sum_{\alpha=1}^{12}\int d^3p\,E_pA_\alpha^*(p)A_\alpha(p),$$

which is manifestly invariant under the 12-dimensional unitary transformation: $A(p)\to UA(p)$, $U^\dagger U=1$. There is also the analog to Eq. (14) with U depending on the momentum p and the corresponding infinitesimal generators given by $X_\beta{}^\alpha=\int d^3p\,A_\alpha^*(p)f_{\alpha\beta}(p)\times A_\beta(p)$ $(\alpha,\beta=1,\cdots,12)$ for an arbitrary function $f_{\alpha\beta}(p)$ of p. These $X_\beta{}^\alpha$ commute with H_0 and the momentum operator $\vec{P}$, but will only commute with the rotation operator $\vec{L}$ if the $f_{\alpha\beta}(p)$ are functions of p^2 alone.

[10]The relative parity within this reduction depends on whether the quark and antiquark fields are bound in s or p states.

[11]Cf. S. Okubo, C. Ryan, and R. Marshak, Nuovo Cimento 34, 759 (1964).

[12]There will be terms in the total Hamiltonian which will break the charge-conjugation degeneracy.

[13]Cf. B. Sakita, Phys. Rev. Letters 13, 643 (1964); F. Gürsey, A. Pais, and L. A. Radicati, ibid. 13, 299 (1964), and subsequent papers.

ELECTROMAGNETIC MASS DIFFERENCES*

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In the past few years a number of authors have proposed sum rules for the medium-strong and electromagnetic mass differences of hadrons within the framework of the eightfold way. The purpose of this note is twofold. We first show how the various sum rules that have appeared in the literature can be discussed in a unified and coherent fashion. Second, we propose a few additional sum rules which are in agreement with observation.

The most general mass operator for the baryon octet can be written as[1,2]

$$m=m_0+s_1Y+s_2(\vec{T}^2-\tfrac{1}{4}Y^2)+s_3Y^2-e_1Q+e_2(\vec{U}^2-\tfrac{1}{4}Q^2)+e_3Q^2+aYQ+bYQ(Y+Q). \qquad (1)$$

The nine coefficients that appear in Eq. (1) can be expressed in terms of the eight baryon masses and the $\Lambda\Sigma^0$ transition mass denoted by m_T. Table I supplies the desired connection.

The s_1, s_2, and s_3 terms are isospin scalars

Table I. Coefficients of Eq. (1). The numerical values (taken from Rosenfeld et al.[a]) are in MeV.

m_0	$\Lambda+3^{1/2}m_T$	$1115.4\pm0.1+3^{1/2}m_T$
s_1	$\frac{1}{2}(n-\Xi^0)$	-187.5 ± 0.5
s_2	$\frac{1}{2}(\Sigma^0-\Lambda)-3^{-1/2}m_T$	$38.5\pm0.2-3^{-1/2}m_T$
s_3	$\frac{1}{2}[n+\Xi^0-\frac{1}{2}(\Sigma^0+3\Lambda)+3^{1/2}m_T]$	$-8.2\pm0.6+(3^{1/2}/2)m_T$
e_1	$\frac{1}{2}(\Sigma^--\Sigma^+)$[b]	3.8 ± 0.2
e_2	$-(2/\sqrt{3})m_T$	$-(2/\sqrt{3})m_T$
e_3	$\frac{1}{2}(\Sigma^++\Sigma^--2\Sigma^0)$	1.0 ± 0.2
a	$\frac{1}{2}(p-n+\Xi^--\Xi^0-\Sigma^+-\Sigma^-)+\Sigma^0-3^{1/2}m_T$	$1.5\pm0.8-3^{1/2}m_T$
b	$\frac{1}{4}(p-n+\Xi^0-\Xi^-+\Sigma^--\Sigma^+)$	0.1 ± 0.4
$a+2b$	$p-n-\Sigma^++\Sigma^0-3^{1/2}m_T$	$1.6\pm0.3-3^{1/2}m_T$

[a]A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz, and M. Roos, Rev. Mod. Phys. 36, 997 (1964).

[b]This is accurate to terms of order m_T^2/s_2.

whereas the e_1, e_2, and e_3 terms are U-spin scalars.[3] The a and b terms are due to interference between the electromagnetic interaction and the medium-strong interaction. They are expected to be small if the medium-strong interaction is relatively weak. The celebrated 1961 mass formula of Coleman and Glashow,[4]

$$\Xi^--\Xi^0=p-n+\Sigma^--\Sigma^+, \qquad (2)$$

which correctly predicted the $\Xi^--\Xi^0$ mass difference, is obtained by setting $b=0$. The sum rules for the $\Lambda\Sigma^0$ transition mass derived by Macfarlane and Sudarshan[2] and by Dalitz and von Hippel,[5,6]

$$m_T=-3^{-1/2}(\Sigma^+-\Sigma^0-p+n), \qquad (3)$$

is equivalent to $a+2b=0$. Complete octet dominance for the electromagnetic mass differences[7-9] leads to $e_3=0$ (as well as $a=b=0$), which gives an equal-spacing rule for the three Σ masses. If we set $a=s_3=0$, we reproduce the Gell-Mann–Okubo formula[2,10] in the form given by Nauenberg,[11]

$$n+p+\Xi^0+\Xi^-=3\Lambda+\Sigma^++\Sigma^--\Sigma^0. \qquad (4)$$

The art of setting some of the coefficients equal to zero does not end here. The mass relations derived by Sakita[12] and Chan and Sarker,[13] using an electromagnetic interaction with definite transformation properties under SU(6),[14-16]

$$p-n=\Sigma^+-\Sigma^0; \quad \Xi^--\Xi^0=\Sigma^--\Sigma^0, \qquad (5)$$

follow if we set $a=b=e_2=0$. In this way one gets

$$m_T=0, \qquad (6)$$

which means that there is no electromagnetic $\Lambda\Sigma^0$ mixing in this case.

Experimentally, the coefficient e_3 is definitely different from zero, since the equal-spacing rule for the Σ masses is now violated by about six standard deviations.[17] If we assume $a+2b=0$, then e_2 must be different from zero; this is because the Σ^+-Σ^0 mass difference is (-2.9 ± 0.3) MeV, while the p-n mass difference is -1.3 MeV.

We now take the point of view that the e_3 term can be well accounted for by the usual mechanism in which the electromagnetic self-energies are due to the emission and reabsorption of a single virtual photon by a baryon with elastic form factors at the relevant vertices. This view is supported by calculations of Coleman and Schnitzer,[18] who have shown that the conventional photon mechanism with the observed nucleon form factors and their unitary transforms gives the Q^2 rules to an accuracy of about 30%. Moreover, the magnitude of the calculated e_3 term also appears to be correct ($e_3\approx1$ MeV).[19]

To obtain further relations we now postulate the following: Apart from the e_3 term (well accounted for by the conventional photon mechanism), the electromagnetic mass shift is proportional to the corresponding medium-strong mass shift with a proportionality constant λ (expected to be of the order of 1/137), and, if it were not for the scale factor λ, the mass operator would be invariant under

$$Y\rightleftarrows-Q, \qquad \vec{T}\rightleftarrows\vec{U}. \qquad (7)$$

This postulate was first made plausible by Coleman and Glashow[8] and by Suzuki[9] on the basis of the tadpole model in which η' tadpoles are responsible for the medium-strong mass difference (a possibility pointed out earlier by one of us[20]) while $\pi^{0\prime}$ tadpoles give rise to the electromagnetic mass differences,[8,9] where η' and π' are the $T=0$ and 1 members of a hypothetical 0^+ octet for which there is no experimental evidence. Subsequently, we have shown[1] that everything the tadpole model can do can be done equally well by a model in which the medium-strong and electromagnetic mass differences are respectively due to ω_8-ω_1 mixing[20] and ρ^0-ω_1 mixing. In any case, the above postulate gives

$$e_1/s_1 = e_2/s_2 = \lambda, \tag{8}$$

which, together with $a+2b=0$, means[21]

$$\frac{\Sigma^- - \Sigma^+}{\Xi^0 - n} = \frac{4}{3}\frac{p-n+\Sigma^0-\Sigma^+}{\Sigma^0-\Lambda}, \tag{9}$$

$$\frac{\Sigma^- - \Sigma^+}{\Xi^0 - n} = (2.0\pm 0.1)\times 10^{-2},$$

$$\frac{4}{3}\frac{p-n+\Sigma^0-\Sigma^+}{\Sigma^0-\Lambda} = (2.7\pm 0.5)\times 10^{-2}, \tag{10}$$

in good agreement with observation.[22]

According to our postulate the parameter λ that characterizes the ratio of the electromagnetic mass shift to the medium-strong mass shift must be a universal constant common to all the SU(3) multiplets. By setting

$$\epsilon/\sigma = e_1/s_1 = \lambda, \quad \alpha = 0, \tag{11}$$

where ϵ, σ, and α are, respectively, the coefficients in the pseudoscalar mass formula analogous to e_2, s_2, and a, we obtain[23]

$$\frac{4}{3}\frac{K^+-K^0+\pi^0-\pi^+}{\pi^0-\eta} = \frac{\Sigma^- - \Sigma^+}{\Xi^0 - n}, \tag{12}$$

whose left-hand side is equal to $(2.4\pm 0.3)\times 10^{-2}$ when squared masses are used. So our formula is again in good agreement with observation despite the fact that here octet dominance (which demands a vanishing $\pi^\pm$-π^0 mass difference) is in poor agreement with observation.[22,24]

For the $J=\frac{3}{2}^+$ decuplet we obtain[23]

$$\frac{Y_1^{*-} - Y_1^{*+}}{\Xi_{1/2}^{*0} - N_{3/2}^{*0}} = \frac{\Sigma^- - \Sigma^+}{\Xi^0 - n}, \tag{13}$$

in addition to relations such as

$$Y_1^{*-} - Y_1^{*+} = N_{3/2}^{*+} - N_{3/2}^{*-}, \tag{14}$$

which follow from the $e_1'Q + e_2'Q^2$ rule of Rosen, Macfarlane, and Sudarshan.[2]

Equation (13) can be compared with other relations obtained by assuming covariance of the mass operator under SU(6). Sakita[12] and Chan and Sarker[13] obtain

$$Y_1^{*-} - Y_1^{*+} = \Sigma^- - \Sigma^+, \tag{15}$$

whereas Kuo and Yao,[25] using a different mass operator (corresponding to $a=b=0$), obtain

$$Y_1^{*-} - Y_1^{*+} = 2(n-p) + (\Sigma^- + \Sigma^+ - 2\Sigma^0). \tag{16}$$

Numerically, Eqs. (13), (15), and (16) predict, respectively,

$$Y_1^{*-} - Y_1^{*+} = (5.8\pm 0.3)\ \text{MeV},$$
$$= (7.4\pm 0.2)\ \text{MeV, or}$$
$$= (4.5\pm 0.4)\ \text{MeV}. \tag{17}$$

Experimentally, this mass difference is known to be (17 ± 7) MeV, according to Cooper et al.,[26] and (4.4 ± 2.2) MeV according to Huwe.[27]

We may remark that the octet enhancement theory of Dashen and Frautschi[28] also gives relation (9), and, if their enhanced eigenvector can be shown to be universal, (12) and (13) as well. Note, however, that the validity of our relations (9), (12), and (13) does not rest on octet dominance (which is known to fail experimentally for the pseudoscalar octet, and also, to a less serious extent, for the baryon octet).

In conclusion we wish to emphasize that the success of our mass relations does not necessarily favor one particular dynamical model over another (e.g., the tadpole model or the vector-meson mixing model); any theory of mass differences which satisfies invariance under $Y \rightleftarrows -Q$, $\vec{T} \rightleftarrows \vec{U}$ apart from a universal scale factor is equally satisfactory.

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SU(6)⊗O(3) STRUCTURE OF STRONGLY INTERACTING PARTICLES

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Recently there has been considerable interest in the possibility of an SU(6) structure of the strongly interacting particles. Significant success has been achieved in the application of the SU(6) structure to multiplet assignments,[1] mass spectra,[2] meson-baryon coupling structure,[1] and electromagnetic[3,4] and weak interaction[5] properties of baryons. The group SU(6), as defined in the present context, contains the intrinsic spin group $SU(2)_S$ and the internal symmetry group SU(3). The theory is thus naturally a nonrelativistic one. The basic group of a nonrelativistic theory (as contrasted with a Galilean or a Lorentz group) is the Newtonian group of space and time translations and rotations in the three-dimensional space. It is thus natural to combine the SU(6) structure with this Newtonian group to construct the relevant combined space-time and internal symmetry group. As the first step in this direction, we postulate the invariance of the strong interactions under SU(6)⊗O(3), where O(3) is a group of rotations in the three-dimensional space, independent of the spin group $SU(2)_S$ contained in SU(6). We then examine the various consequences of this postulate. We find the following results:

(i) A unique SU(6)⊗O(3)-invariant (parity-conserving) Yukawa coupling, bilinear in the baryon supermultiplet components, can be con-