ties involved in determining the approximate shell-model wave functions. For example, in order to increase the calculated $\mathrm{Sc}^{43}$ hindrance factor from 8.7 to the experimental value of 13.8 , it is only necessary to increase the coreexcitation probability $\alpha_{2}{ }^{2}$ from 0.36 to 0.41 . This strikingly illustrates the sensitivity of the $M 2$ matrix elements to the probability of core excitation in the single-hole wave functions.

We conclude that the inhibited $M 2$ decays of the first excited states of $\mathrm{Sc}^{43}, \mathrm{Sc}^{45}$, and $\mathrm{Sc}^{47}$ imply core-excitation probabilities of 0.4 or greater in the positive-parity wave functions. By the same token, $40 \%$ or more of the $d_{3 / 2^{-}}$ hole strength in the ( $d, \mathrm{He}^{3}$ ) reaction on $\mathrm{Ti}^{44}$, $\mathrm{Ti}^{46}$, and $\mathrm{Ti}^{48}$ should proceed to higher excited $\frac{3}{2}^{+}$states in the Sc isotopes. Thus a direct experimental test of our interpretation of the $M 2$ lifetime is possible.

[^0]Phys. Rev. Letters 13, 241 (1964).
${ }^{2}$ R. K. Bansal and J. B. French, Phys. Letters 11, 145 (1964).
${ }^{3}$ J. L. Yntema and G. R. Satchler, Phys. Rev. 134, B976 (1964).
${ }^{4}$ J. P. Elliott, Proc. Roy. Soc. (London) A245, 128 (1958).
${ }^{5}$ S. A. Moszkowski, in Beta- and Gamma-Ray Spectroscopy, edited by Kai Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 373.
${ }^{6}$ A. deShalit and I. Talmi, Nuclear Shell Theory (Academic Press, Inc., New York, 1963), p. 268.
${ }^{7}$ Here we use "seniority" as a convenient abbreviation for the more cumbersome "symplectic symmetry," which is the generalization of seniority to mixed systems of neutrons and protons. See B. H. Flowers, Proc. Roy. Soc. (London) A212, 106 (1952).
${ }^{8}$ R. D. Lawson and B. Zeidman, Phys. Rev. 128, 821 (1962).
${ }^{9}$ R. D. Lawson, Phys. Rev. 124, 1500 (1961).
${ }^{10}$ J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964); J. N. Ginocchio, to be published.
${ }^{11}$ E. U. Baranger and C. W. Lee, Nucl. Phys. 22, 157 (1961).

## DETERMINATION OF THE $\Sigma^{+} \rightarrow p+\gamma$ DECAY RATE*

M. Bazin, H. Blumenfeld, U. Nauenberg, and L. Seidlitz<br>Princeton-Pennsylvania Accelerator, Princeton University, Princeton, New Jersey<br>and<br>C. Y. Chang<br>Columbia University, New York, New York<br>(Received 17 November 1964; revised manuscript received 11 January 1965)

In a continuation of a study of the rare decay modes of the $\Sigma$ hyperons, we have determined the rate of the decay mode $\Sigma^{+} \rightarrow p+\gamma$. Previous experiments based on very few events indicated that the rate could be as large as $1 \% .^{1}$

The sigmas were produced with a stopping $K^{-}$beam in the $30-\mathrm{in}$. Columbia-Brookhaven hydrogen bubble chamber via the reaction

$$
\begin{aligned}
& K^{-}+p \rightarrow \Sigma^{+}+\pi^{-} \\
& \measuredangle p+\text { uncharged particles. }
\end{aligned}
$$

The experimental technique was as follows: We considered only events where the $\Sigma^{+}$and $\pi^{-}$were colinear; that is, the reaction occurred when the $K^{-}$was captured at rest. We recorded during scanning all events which satisfied the following three criteria: (1) The $\Sigma^{+}$was clearly visible; (2) the decay track stopped in the chamber; (3) the decay track did not itself de-
cay. Using the fiducials both of the front glass and of the back mirror of the chamber, it was possible to tell whether a track went out of the chamber or stopped inside. All these events were then measured on standard digitizing machines. The range of the proton in the hydrogen, the length of the $\Sigma^{+}$, and the direction of the production pion were used to determine the momentum of the proton in the $\Sigma^{+}$center of mass $\left[P_{p}(\right.$ c.m. $\left.)\right]$. Since $P_{p}$ (c.m.) in the normal decay mode $\Sigma^{+} \rightarrow p+\pi^{0}$ and the rare decay mode $\Sigma^{+} \rightarrow p+\gamma$ differ by only $35.4 \mathrm{MeV} / c$, it is important to measure all the kinematical quantities accurately so that the peaks can be separated. Hence, after the spatial reconstruction of the event, we accepted only events which satisfied the following criteria: (a) Both the dips of the $\Sigma^{+}$and of the proton were required to be $\leqslant 60^{\circ}$. (b) The length of the $\Sigma^{+}$was required to be $\leqslant 1.10 \mathrm{~cm}$ (the maximum range
of such $\Sigma^{+\prime} s$ is 1.27 cm and the cutoff is necessary to be able to tell the momentum of the $\Sigma^{+}$accurately enough). (c) The momentum of the production pion was required to be $\geqslant 150$ $\mathrm{MeV} / c$ (this was done to avoid including the lambdas which decayed at the point of their production in $K^{-}-p$ reactions and might be confused as very short $\Sigma^{+}$decays). The momentum of the pion in Reaction (1) is $181.5 \mathrm{MeV} / c$. (d) The length of the proton was required to be $\geqslant 1.0 \mathrm{~cm}$ (this was done because the direction of the proton had to be known accurately). This last cutoff imposes corrections to the data which are different for the two decay modes. These and other corrections are discussed in detail in another report. ${ }^{2}$

The protons from the decay $\Sigma^{+} \rightarrow p+\gamma$ have different ranges than those from the decay $\Sigma^{+}$ $\rightarrow p+\pi^{0}$. Since only stopping protons are used in the analysis, the size of the bubble chamber imposes a bias on our data. We systematically avoided this bias by the following method applied to each event: The measured direction of the proton (but not its length) was used to make a zero-constraint class fit to the decay hypothesis $\Sigma^{+} \rightarrow p+\gamma$. The computed momentum was then used to determine what the range of such a proton would be. This was done by means of range-energy tables. Making use of the knowledge of the magnetic field, the end point of such a proton was then determined. If the end point of the proton was found to be either within 1.0 cm of the top or bottom of the chamber, or outside a cylinder of radius 25.0 cm with the $z$ axis of symmetry, the event was not included in the analysis. Subsequently, we will refer to this test as our fiducial cutoff. In this manner we systematically required that, for every proton from the decay $\Sigma^{+} \rightarrow p+\pi^{0}$ which stops in the hydrogen chamber, a proton from the decay $\Sigma^{+} \rightarrow p+\gamma$ moving in the same direction would also have stopped, and hence would have been recorded in scanning.

Due to the fact that we do not include in the analysis those protons whose lengths are $<1.0$ cm , and because of the fiducial cutoff, we have to apply a correction to the number of observed events. We find that the number of events of the type $\Sigma^{+} \rightarrow p+\pi^{0}$ rejected by the cutoffs is $78 \%$ of the number of events that satisfy all our cut-off criteria. The corresponding correction to the observed $\Sigma^{+} \rightarrow p+\gamma$ decay mode is $62 \%$.

All events with $P_{p}$ (c.m.) $<170 \mathrm{MeV} / c$ or
$P_{p}($ c.m. $)>200 \mathrm{MeV} / c$ were systematically recorded and remeasured. An event was not included in the final analysis if the $\Sigma$ was produced by a clear $K^{-}$interaction in flight or if the $\Sigma$ scattered. In Fig. 1, we show the events rejected by these criteria. We have $5830 \Sigma^{+} \rightarrow p$ $+\pi^{0}$ events that satisfy all our cut-off criteria. We have found 24 events of the type $\Sigma^{+} \rightarrow p+\gamma$. The distribution of events as a function of $P_{p}$ (c.m.) is shown in Fig. 1. One of the $\Sigma^{+} \rightarrow p$ $+\gamma$ events with $P_{p}(c . m)=.215 \mathrm{MeV} / c$ has a Dalitz pair and is most consistent with the hypothesis $\Sigma^{+} \rightarrow p+e^{+}+e^{-}$. The event with $P_{p}$ (c.m.) $=213 \mathrm{MeV} / c$ has $l_{\Sigma}+<0.1 \mathrm{~cm}$ and is considered a background event. The event with $P_{p}$ (c.m.) $=119 \mathrm{MeV} / c$ is consistent with the decay mode $\Sigma^{+} \rightarrow p+\pi^{0}+\gamma$. Of the 24 events $\Sigma^{+} \rightarrow p+\gamma$, we had only three cases which could be regarded (if we neglected any knowledge of the $\Sigma^{+}$direction) as $\Sigma^{+} \rightarrow p+\pi^{0}$ decays where the $\Sigma^{+}$was produced by a $K^{-}$with a residual momentum anywhere between 50 and $80 \mathrm{MeV} / c$. These are events where both the angle between the $\Sigma^{+}$


FIG. 1. Distribution of the proton momentum in the $\Sigma$ center of mass.
and $K^{-}$and the angle between the $\Sigma^{+}$and proton were $45^{\circ}$ or $135^{\circ}$. Nevertheless, the three events found are consistent with the number of events expected in these angular regions of production and decay, and we noted no deviation from colinearity between the $\Sigma^{+}$and $\pi^{-}$, where a deviation of up to $15^{\circ}$ was expected. We retained these three events as $\Sigma^{+} \rightarrow p+\gamma$ decay modes. We feel certain that these 24 events do not contain any background, and that the systematic errors do not contribute to the overall error. Hence we obtain for the branching ratio

$$
\frac{R\left(\Sigma^{+}-p+\gamma\right)}{R\left(\Sigma^{+} \rightarrow p+\pi^{0}\right)}=\frac{24 \times 1.62}{5830 \times 1.78}=(0.37 \pm 0.08) \times 10^{-2} .
$$

This rate is in reasonable agreement with the value determined theoretically with the use of simple dynamical models. ${ }^{3}$

The Princeton group would like to thank Pro-
fessor Jack Steinberger and Professor R. J. Plano for the use of the film to do this experiment. We would like to thank Professor R. Blankenbecler, Professor S. Treiman, and Professor M. Nauenberg for various comments and discussions.

[^1]
# MAXIMAL UNITARY GROUP FOR THE QUARK MODEL* 

S. Okubo and R. E. Marshak<br>Department of Physics and Astronomy, University of Rochester, Rochester, New York

(Received 14 December 1964)

Recently, the idea that the strong interactions among the hadrons might have an approximate symmetry group as high as $\mathrm{W}(6)=\mathrm{U}(6) \otimes \mathrm{U}(6)$ was proposed by the present authors ${ }^{1}$ and, independently, by Bardakci et al., ${ }^{2}$ and by Feynman, Gell-Mann, and Zweig. ${ }^{3}$ In our paper, we used the three-field model and made a "Casimir" decomposition ${ }^{4}$ of the four-component quark fields; we found that the scalar (S) and the pseudoscalar ( $P$ ) four-fermion interactions each led to invariance under a different $\mathrm{W}(6)$ group, and that taking $S$ and $P$ together, the underlying symmetry group is reduced to $\mathrm{U}(6)$. In reference 2 the three-field method is used, but a chiral decomposition ${ }^{5}$ is made, and this leads the authors into serious difficulty; Bardakci et al. state that certain linear combinations of the four-fermion interactions are invariant under the group GL(6) and that, by means of Weyl's "unitary trick," one may obtain the group $\mathrm{W}(6)$. Unfortunately, if one insists on maintaining the invariance of the equaltime commutation relations among the quark fields, the symmetry reduces to the $\mathrm{U}(6)$ [and not the $W(6)$ ] group, a result which is in agreement with our previous paper. ${ }^{6}$

Reference 3 does not use the three-field methof but arrives at $W(6)$ by considering the group generated by the space components of vector and axial-vector octets; chiral decomposition of the quark fields is implicit, but one can avoid the dilemma of reference 2 by arguing that the group properties of the current-octet approach bear no relationship to the three-field model. The trouble now is that one can receive no guidance from the three-field model regarding the tensorial behavior of the symmetry-breaking terms and must accept parity doublets. In contrast, the three-field model with "Casimir" decomposition not only maintains the commutation relations among the quark fields under $\mathrm{W}(6)$, but can predict the tensorial behavior of the symmetry-breaking terms and need not lead to parity doublets.

In this paper, we show that there is a much larger group than $\mathrm{W}(6)$ under which the $S$ or $P$ four-fermion interaction is invariant for a triplet of quark fields. This group is actually a unitary gauge group, which we call $\mathrm{V}(12)$ [since it contains $\mathrm{U}(12)$ as a subgroup], and can be obtained from an infinite-dimensional Lie algebra. Hereafter, we shall follow the


[^0]:    *Work performed under the auspices of the U.S. Atomic Energy Commission.
    ${ }^{1}$ R. E. Holland, F. J. Lynch, and K. E. Nystén,

[^1]:    *Work supported by the U. S. Atomic Energy Commission.
    ${ }^{1}$ G. Quarini et al., Nuovo Cimento 14, 1179 (1959); J. Schneps and Y. W. Kang, Nuovo Cimento 19, 1218 (1961); R. A. Burnstein, T. B. Day, F. Martin, M. Sakitt, R. G. Glasser, N. Seeman, and A. J. Herz, Phys. Rev. Letters 10, 307 (1963); R. Carrara, M. Cresti, A. Grigoletto, S. Limentani, L. Perruzzo, R. Santangelo, and R. B. Willmann, to be published.
    ${ }^{2}$ Princeton-Pennsylvania Accelerator Report No. PPAD-527E, 1964 (unpublished).
    ${ }^{3}$ R. E. Behrends, Phys. Rev. 111, 1691 (1958).

