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$M2$ LIFETIMES AND CORE EXCITATION IN THE SCANDIUM ISOTOPES*

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A recent Letter[1] reports measurements of the $M2$ lifetimes of the positive-parity first excited states of Sc^{43}, Sc^{45}, and Sc^{47}. The binding energies of these levels[2] and their strong formation in (d, He^3) reactions[3] are consistent with their interpretation as $1d_{3/2}$ single-hole excitations of the neighboring even-mass Ti isotopes. Such an interpretation, however, implies $M2$ lifetimes shorter by factors of about 200 than the value measured by Holland, Lynch, and Nystén.[1] In this paper we show that the pertinent $M2$ matrix elements are acutely sensitive to admixtures of core-excited states in the single-hole wave functions and that Elliott's generating procedure[4] yields estimates of the amount of core excitation of the correct order of magnitude.

The transitions under consideration and the configurations involved are

$$(d_{3/2}{}^{7} f_{7/2}{}^{n+1})_{3/2^+} \xrightarrow{M2} (d_{3/2}{}^{8})_0 (f_{7/2}{}^{n})_{7/2^-}, \qquad (1)$$

where $n=3$, 5, and 7 refer to Sc^{43}, Sc^{45}, and Sc^{47}, respectively, and the isobaric spin of initial and final states is $\frac{1}{2}(n-2)$. Now core-excited states of isobaric spin $\frac{1}{2}(n-1)$ or $\frac{1}{2}(n-3)$ can contribute to the wave function of the $\frac{3}{2}^+$ state. Since the $T=\frac{1}{2}(n-1)$ states lie several MeV above the $T=\frac{1}{2}(n-3)$ states in the even-mass Ti core nuclei, we shall neglect higher T admixtures. The wave function of the $\frac{3}{2}^+$ state can therefore be written

$$\Psi(\tfrac{3}{2}^+) = a_0 (d_{3/2}{}^{7})_{3/2} (f_{7/2}{}^{n+1})_0 + a_2 [(d_{3/2}{}^{7})_{3/2} \times (f_{7/2}{}^{n+1})_2]_{3/2}, \qquad (2)$$

where $[\cdots\times\cdots]_{3/2}$ indicates vector coupling to total spin $\frac{3}{2}$ and $(f_{7/2}{}^{n+1})_J$ symbolizes a core state of angular momentum J and isobaric spin $\frac{1}{2}(n-3)$.

To exhibit the dependence of the $M2$ lifetimes on the core-excitation probabilities $a_2{}^2$, we introduce the "hindrance factor" $\mathfrak{h}$ by which the $M2$ matrix elements are inhibited relative to the Moszkowski single-particle estimates.[5] In terms of reduced transition rates[5] we have

$$B(M2) = \frac{1}{\mathfrak{h}^2} B_{\mathrm{sp}}(M2). \qquad (3)$$

The experimental hindrance factors (Table I) lie between 10 and 20. The wave functions given in Eqs. (1) and (2) lead to an expression

$$\mathfrak{h}^{-1} = \left[\frac{(n+1)(n-2)}{(n-1)}\right]^{1/2} \sum_{J=0,2} \alpha_J [4(2J+1)]^{1/2} W(\tfrac{3}{2}\tfrac{3}{2}\tfrac{7}{2}\tfrac{7}{2}; J2) \times [f_{7/2}{}^{n}, J_0=\tfrac{7}{2}, T_0=\tfrac{1}{2}(n-2) | \} f_{7/2}{}^{n+1}, J, T=\tfrac{1}{2}(n-3)] \qquad (4)$$

for the hindrance factor, where W is a Racah coefficient and $[|\}]$ is a coefficient of fractional parentage[6] (cfp) connecting states of the $f_{7/2}$ shell. If we assume that there is no excitation of the core ($\alpha_2=0$) and that seniority[7] is a good quantum number within the configurations $f_{7/2}{}^{n}$, Eq. (4) yields $\mathfrak{h}=2$, smaller than the experimental hindrance factors by an order of magnitude. Now it is clear[8] that the assumption of good seniority within $f_{7/2}{}^{n}$ is not valid and that seniority mixing can influence the $f_{7/2}$-shell cfp [and hence, by Eq. (4), the hindrance factors] by factors of the order of 2. It is quite clear, however, that it is impossible to understand the strong observed inhibition of the $M2$ decays without permitting core excitation.

To study the effects of core excitation, a shell-model calculation could be carried out and energy matrices diagonalized within the config-

Table I. Hindrance factors $\mathfrak{h}$, defined by Eq. (1) in the text, for the $M2$ decays of the $\frac{3}{2}^+$ first excited states of various isotopes of Sc. The column headed $\mathfrak{h}$ (observed) gives the measured hindrance factors of reference 1; the column headed $\mathfrak{h}$ (generated) gives the hindrance factors calculated as described in the text by projection from intrinsic states with positive deformation. The last column gives the probability that, in the projected wave functions, the $d_{3/2}$ hole is coupled to the lowest excited 2^+ state of the core.

Nucleus	$\mathfrak{h}$ (observed)	$\mathfrak{h}$ (generated)	Core-excitation probability
Sc^{43}	13.8	8.7	0.36
Sc^{45}	14.5	9.2	0.42
Sc^{47}	20.2	9.0	0.40

urations $d_{3/2}{}^7 f_{7/2}{}^{n+1}$ and $d_{3/2}{}^8 f_{7/2}{}^n$. The dimensions of the matrices involved are quite large and, furthermore, the effective interaction in the $T=0$ states of $(d_{3/2}f_{7/2})$ is subject to large uncertainties. We therefore prefer to assume that most of the residual shell-model interaction simply serves to deform the effective single-particle potential in which the active nucleons move; Elliott's generating procedure[4] can then be used to obtain approximate shell-model wave functions. It has already been shown[9] that such wave functions give multipole moments and transition rates for normal-parity states of $f_{7/2}$-shell nuclei in good agreement with experiment. It has also been shown[10] that the generated normal-parity wave functions agree closely with those obtained from detailed shell-model calculations. Approximate wave functions for both $\frac{3}{2}^+$ and $\frac{7}{2}^-$ states will therefore be determined by projecting angular-momentum eigenstates from appropriate determinants of deformed-well eigenfunctions. Further, since the $\frac{3}{2}^+$ state is assumed to arise from the excitation of a $d_{3/2}$ particle to the $f_{7/2}$ level, there will be no spurious center-of-mass motion in the wave function describing this state.[11]

The relevant deformed single-particle orbits are shown in Fig. 1. For negative deformations the $M2$ decays in question are forbidden by a K selection rule; it is impossible by the action of the $M2$ operator to transform an $f_{7/2}$ proton with $K=\frac{7}{2}^-$ to a $d_{3/2}$ proton with $K=\frac{1}{2}^+$. However, it is probable that positive deformation is more appropriate[9] for the nuclei under consideration. In that case the intrinsic or generator wave function for the Sc^{43} ground state is

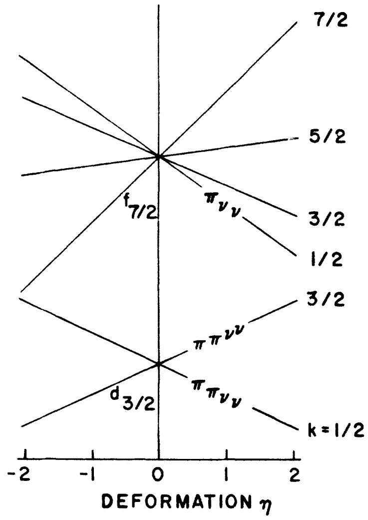

FIG. 1. Energy-level diagram for the $1d_{3/2}$ and $1f_{7/2}$ levels as a function of the deformation η. π denotes a proton and ν stands for a neutron. The ground state of Sc^{43} is generated from a determinant in which the levels indicated above are occupied.

a normalized 11-particle determinant $\chi(K=\frac{1}{2}^-)$, in which the orbits occupied by the five protons and six neutrons are indicated in Fig. 1. The intrinsic wave function $\chi(K=\frac{3}{2}^+)$ for the hole state is obtained by promoting a proton from the $K=\frac{3}{2}^+$ to the $K=\frac{1}{2}^-$ orbit. Similar considerations apply to Sc^{45} and Sc^{47}. The approximate shell-model wave functions are then obtained from the intrinsic states χ_K by the projection operation[4]

$$\Psi_M{}^I(x)=\left[\frac{2I+1}{8\pi^2}\right]N_1\int dR\, D_{MK}{}^I(R)\chi_K(Rx), \quad (5)$$

where N_1 is a constant that normalizes the projected angular-momentum eigenstate and $D_{MK}{}^I(R)$ is a rotation matrix. The methods of reference 9 can be used to evaluate the mixture coefficients α_J and the cfp for the generated wave functions. The hindrance factor is then obtained from Eq. (4); for positive deformations the result is

$$\mathfrak{h}=14(N_{7/2}/N_{3/2}), \quad (6)$$

independent of the normalization factors N_J for the projected core states.

The hindrance factors and core-excitation probabilities obtained with the generated wave functions are compared with experiment in Table I. Hindrance factors of the correct order of magnitude are obtained; indeed, with the possible exception of the Sc^{47} decay, agreement with experiment is within the uncertainties involved in determining the approximate shell-model wave functions. For example, in order to increase the calculated Sc^{43} hindrance factor from 8.7 to the experimental value of 13.8, it is only necessary to increase the core-excitation probability α_2^2 from 0.36 to 0.41. This strikingly illustrates the sensitivity of the $M2$ matrix elements to the probability of core excitation in the single-hole wave functions.

We conclude that the inhibited $M2$ decays of the first excited states of Sc^{43}, Sc^{45}, and Sc^{47} imply core-excitation probabilities of 0.4 or greater in the positive-parity wave functions. By the same token, 40% or more of the $d_{3/2}$-hole strength in the (d, He^3) reaction on Ti^{44}, Ti^{46}, and Ti^{48} should proceed to higher excited $\frac{3}{2}^+$ states in the Sc isotopes. Thus a direct experimental test of our interpretation of the $M2$ lifetime is possible.

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DETERMINATION OF THE $\Sigma^+ \to p + \gamma$ DECAY RATE*

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In a continuation of a study of the rare decay modes of the Σ hyperons, we have determined the rate of the decay mode $\Sigma^+ \to p + \gamma$. Previous experiments based on very few events indicated that the rate could be as large as 1%.[1]

The sigmas were produced with a stopping K^- beam in the 30-in. Columbia-Brookhaven hydrogen bubble chamber via the reaction

$$K^- + p \to \Sigma^+ + \pi^-$$
$$\Sigma^+ \to p + \text{uncharged particles.}$$

The experimental technique was as follows: We considered only events where the Σ^+ and π^- were colinear; that is, the reaction occurred when the K^- was captured at rest. We recorded during scanning all events which satisfied the following three criteria: (1) The Σ^+ was clearly visible; (2) the decay track stopped in the chamber; (3) the decay track did not itself decay. Using the fiducials both of the front glass and of the back mirror of the chamber, it was possible to tell whether a track went out of the chamber or stopped inside. All these events were then measured on standard digitizing machines. The range of the proton in the hydrogen, the length of the Σ^+, and the direction of the production pion were used to determine the momentum of the proton in the Σ^+ center of mass $[P_p(\text{c.m.})]$. Since $P_p(\text{c.m.})$ in the normal decay mode $\Sigma^+ \to p + \pi^0$ and the rare decay mode $\Sigma^+ \to p + \gamma$ differ by only 35.4 MeV/c, it is important to measure all the kinematical quantities accurately so that the peaks can be separated. Hence, after the spatial reconstruction of the event, we accepted only events which satisfied the following criteria: (a) Both the dips of the Σ^+ and of the proton were required to be $\leq 60°$. (b) The length of the Σ^+ was required to be ≤ 1.10 cm (the maximum range