INGOING-WAVE BOUNDARY-CONDITION ANALYSIS OF α-Ni^{62} ELASTIC-SCATTERING CROSS SECTIONS

G. Rawitscher*
Lawrence Radiation Laboratory, University of California, Berkeley, California,
and Max-Planck-Institut für Kernphysik, Heidelberg, Germany†
(Received 18 December 1964)

An investigation of the elastic-scattering cross section of α particles[1] on Ni^{62} at laboratory energies of 25, 33, 50, and 85 MeV is presented utilizing an ingoing-wave boundary condition (IWB).

The IWB method[2,3] is an elaboration of the old scheme of Feshbach and Weisskopf,[4] and, compared to optical-model descriptions, has several simplifying features. It is a surface description which represents diffraction from a potential surface surrounding a black[5] nucleus, while in the optical-model description additional diffraction may come from the waves which re-emerge from the interior of the nucleus. In the present IWB model a nuclear potential is added to the Coulomb potential. The resulting wave functions in the region "outside" of the potential well are very insensitive to the assumed value of the nuclear potential in the "inside" region because of the absence of interference between the outgoing and ingoing branches of the waves in the interior of the well.[6,3] The position of the boundary-condition radius also does not affect the wave functions as long as it lies in the region of validity of the JWKB approximation inside the well.[3]

Figure 1 shows IWB results for α-Ni^{62} scattering cross sections at four energies.[1] The nuclear potential adopted is of the form

$$V = V_0(1 + i\xi)\exp(-r/a), \qquad (1)$$

and the values obtained for the three parameters are listed in Table I. The goodness of the fits was judged "by eye," and the values of the parameters should be taken only as indications of the general trend. However, the nonapplicability of the 85-MeV parameters to the 33-MeV case, and vice versa, has been ascertained. The four potentials listed in Table I have the property that at about 8 fermis their values are all equal to −2.2 MeV $[8 = 1.7\times(62)^{1/3} + 1.3]$. The position of the Coulomb barrier, where the sum of nuclear and Coulomb potentials goes through a maximum, lies at approximately 8 fermis in the α-Ni case. Furthermore, any one of the potentials in Table I gives rise to interference maxima and minima in the cross section which lie at the appropriate experimental positions at all four energies. In other words, it is the position of the barrier rather than the slope of the potential or the value of ξ which determines the position of the diffraction minima. Since these diffraction minima change with energy in a nontrivial manner, the IWB procedure appears to provide a useful description. When plotted as a function of momentum transfer q, the positions of the maxima and minima in the cross section shift to smaller values of q for increasing val-

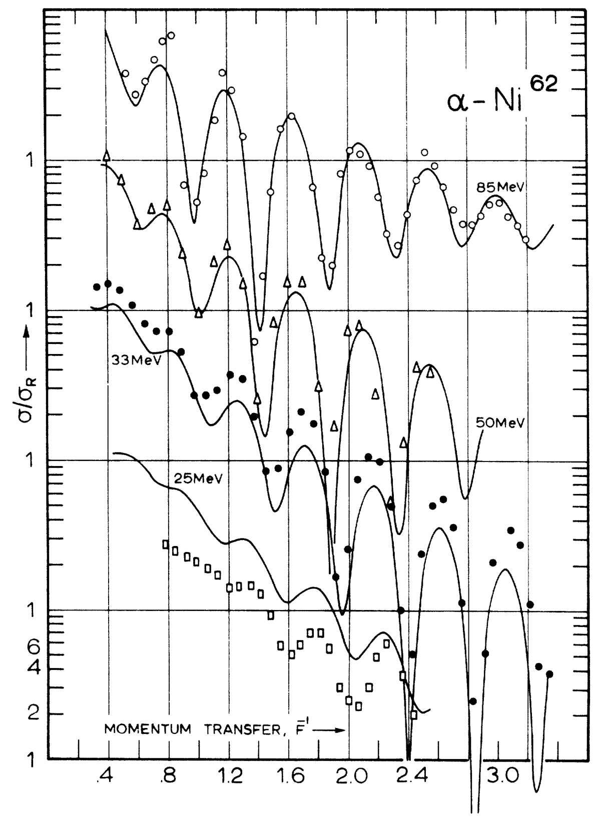

FIG. 1. Scattering of α particles by Ni^{62}. The ordinate represents ratio of cross section to Rutherford cross section. The abscissa represents momentum transfer $Q = 2(p/\hbar)\sin(\theta/2)$ in units of F^{-1}. The c.m. scattering angle is θ, the momentum is p. The experimental points are those of reference 1, and the solid curves are the theoretical results corresponding to the parameters listed in Table I.

Table I. Parameters used in Eq. (1) to obtain the fits shown in Fig. 1.

Energy (MeV)	V_0 (MeV)	a (F)	ξ (dimensionless)
85	-0.51×10^6	0.650	0.50
50	-0.51×10^6	0.650	0.37
33	-0.25×10^7	0.574	0.10
25	-0.195×10^8	0.500	0.00

ues of the energy. The shift is presumably due to the Coulomb repulsion which is relatively more important at the lower energies, and it is for this reason that the shift was called "nontrivial" in the sentence above.

Once the positions of the minima are fitted, the remaining two parameters a and ξ are determined from the overall slope of the cross section with angle and the peak-to-valley ratio. The visible discrepancy of the positions of the 50-MeV experimental and theoretical minima can be remedied by a 20% increase in the value of V_0. Since after this change the agreement with theory at all energies is better than the agreement shown at 50 MeV, it is felt that the accuracy in the determination of the real part of V at ~8 fermis is better than 20%.[7] The increase of a with energy is curious. The increase of ξ with energy indicates larger participation of the surface in the excitation of inelastic processes,[8] and may be quite reasonable.

In view of the interest in the determination of deuteron-nucleus potentials and distorted waves, an attempt was made to examine the 11.8-MeV deuteron copper elastic-scattering cross section[9] by means of the IWB procedure. It was not possible to find an acceptable fit. Not only was it impossible to reproduce the position of the minima and maxima, but also the experimental overall slope of the cross section with angle and the peak-to-valley ratios were mutually incompatible in the IWB description. Of course the model has very few parameters, spin zero is assumed for the deuteron, and additional effects such as the possible presence of Coulomb distortion and excitation at large distances are not allowed for. If inclusion of such effects still would lead to no agreement with experiment, then the discrepancy would be very interesting. It might then indicate the need of deuterons "returning" from the interior of the nucleus, much in agreement with present optical-model descriptions.[10,11] There the low-angular-momentum optical-model partial waves have the character of standing waves inside the potential well,[12] and these appear to be required to fit the data.[11]

The author wishes to thank Dr. John Meriwether for discussions which stimulated the α-Ni analysis. It is a pleasure to thank Professor John O. Rasmussen and Professor I. Perlman for their kind hospitality at the Lawrence Radiation Laboratory during the summer months, as well as the hospitality of Professor W. Gentner at the Max-Planck-Institut für Kernphysik.

---

*On leave of absence from the University of Connecticut, Storrs, Connecticut.

†The author's present stay at Heidelberg is supported by a grant from the Alexander v. Humboldt-Stiftung.

[1]J. R. Meriwether et al., University of California Radiation Laboratory Report No. UCRL-11484, 1964 (unpublished).

[2]N. Oda and K. Harada [Progr. Theoret. Phys. (Kyoto) 15, 545 (1956)] appear to be the first to use an IWB method to calculate α-particle scattering.

[3]G. Rawitscher, Phys. Rev. 135, B605 (1964). Other than imposing a boundary condition on the wave function at a radius away from the origin, the IWB procedure is numerically similar to optical-model calculations.

[4]H. Feshbach and V. F. Weisskopf, Phys. Rev. 71, 145 (1947); 76, 1550 (1949).

[5]When the wave number is complex the meaning of an ingoing current becomes difficult to define, even if the JWKB approximation is applicable. Therefore the word "black" should strictly be applied only to those cases for which the boundary-condition radius lies in a region where the potential is real. Helpful conversations with Professor Jensen concerning this point are gratefully acknowledged.

[6]By "inside" is meant that portion of the potential well where for each significant partial wave the JWKB approximation is valid. It can be shown by means of the JWKB approximation that changes in the inside potential do not affect the outside IWB wave function.

[7]In the optical-model fits of G. Igo [Phys. Rev. 115, 1665 (1959)] to the 40-MeV α-copper elastic-scattering cross sections, the tail of the real potential appears to be determined to an accuracy of about 20%. Igo's exponential potential is approximately 1.7 times larger than the 33-MeV potential listed in Table I. The difference may be due to the presence of an imaginary potential in Igo's calculation, but it may also be that for projectiles as light as the α particle, optical-model and IWB potentials are different.

[8]For $\xi = 0$ all absorption occurs in the "inside"

region.
[9]G. Igo, W. Lorenz, and U. Schmidt-Rohr, Phys. Rev. 124, 832 (1961).
[10]C. Perey and F. Perey, Phys. Rev. 132, 755 (1963).
[11]R. H. Bassel et al., to be published.
[12]As can be seen from the presence of zeros in the plots of the absolute value of the wave function versus distance, as shown for instance in Fig. 4 of reference 10.

$M2$ LIFETIMES AND CORE EXCITATION IN THE SCANDIUM ISOTOPES*

R. D. Lawson and M. H. Macfarlane
Argonne National Laboratory, Argonne, Illinois
(Received 30 December 1964)

A recent Letter[1] reports measurements of the $M2$ lifetimes of the positive-parity first excited states of Sc^{43}, Sc^{45}, and Sc^{47}. The binding energies of these levels[2] and their strong formation in (d, He^3) reactions[3] are consistent with their interpretation as $1d_{3/2}$ single-hole excitations of the neighboring even-mass Ti isotopes. Such an interpretation, however, implies $M2$ lifetimes shorter by factors of about 200 than the value measured by Holland, Lynch, and Nystén.[1] In this paper we show that the pertinent $M2$ matrix elements are acutely sensitive to admixtures of core-excited states in the single-hole wave functions and that Elliott's generating procedure[4] yields estimates of the amount of core excitation of the correct order of magnitude.

The transitions under consideration and the configurations involved are

$$(d_{3/2}^{\;7} f_{7/2}^{\;n+1})_{3/2^+} \xrightarrow{M2} (d_{3/2}^{\;8})_0 (f_{7/2}^{\;n})_{7/2^-}, \tag{1}$$

where $n = 3$, 5, and 7 refer to Sc^{43}, Sc^{45}, and Sc^{47}, respectively, and the isobaric spin of initial and final states is $\frac{1}{2}(n-2)$. Now core-excited states of isobaric spin $\frac{1}{2}(n-1)$ or $\frac{1}{2}(n-3)$ can contribute to the wave function of the $\frac{3}{2}^+$ state. Since the $T = \frac{1}{2}(n-1)$ states lie several MeV above the $T = \frac{1}{2}(n-3)$ states in the even-mass Ti core nuclei, we shall neglect higher T admixtures. The wave function of the $\frac{3}{2}^+$ state can therefore be written

$$\Psi(\tfrac{3}{2}^+) = a_0 (d_{3/2}^{\;7})_{3/2} (f_{7/2}^{\;n+1})_0 + a_2 [(d_{3/2}^{\;7})_{3/2} \times (f_{7/2}^{\;n+1})_2]_{3/2}, \tag{2}$$

where $[\cdots\times\cdots]_{3/2}$ indicates vector coupling to total spin $\frac{3}{2}$ and $(f_{7/2}^{\;n+1})_J$ symbolizes a core state of angular momentum J and isobaric spin $\frac{1}{2}(n-3)$.

To exhibit the dependence of the $M2$ lifetimes on the core-excitation probabilities $a_2^{\,2}$, we introduce the "hindrance factor" $\mathfrak{h}$ by which the $M2$ matrix elements are inhibited relative to the Moszkowski single-particle estimates.[5] In terms of reduced transition rates[5] we have

$$B(M2) = \frac{1}{\mathfrak{h}^2} B_{\mathrm{sp}}(M2). \tag{3}$$

The experimental hindrance factors (Table I) lie between 10 and 20. The wave functions given in Eqs. (1) and (2) lead to an expression

$$\mathfrak{h}^{-1} = \left[\frac{(n+1)(n-2)}{(n-1)}\right]^{1/2} \sum_{J=0,2} \alpha_J [4(2J+1)]^{1/2} W(\tfrac{3}{2}\tfrac{3}{2}\tfrac{7}{2}\tfrac{7}{2}; J2) \times [f_{7/2}^{\;n}, J_0 = \tfrac{7}{2}, T_0 = \tfrac{1}{2}(n-2) | \} f_{7/2}^{\;n+1}, J, T = \tfrac{1}{2}(n-3)] \tag{4}$$

for the hindrance factor, where W is a Racah coefficient and $[|\}]$ is a coefficient of fractional parentage[6] (cfp) connecting states of the $f_{7/2}$ shell. If we assume that there is no excitation of the core ($\alpha_2 = 0$) and that seniority[7] is a good quantum number within the configurations $f_{7/2}^{\;n}$, Eq. (4) yields $\mathfrak{h} = 2$, smaller than the experimental hindrance factors by an order of magnitude. Now it is clear[8] that the assumption of good seniority within $f_{7/2}^{\;n}$ is not valid and that seniority mixing can influence the $f_{7/2}$-shell cfp [and hence, by Eq. (4), the hindrance factors] by factors of the order of 2. It is quite clear, however, that it is impossible to understand the strong observed inhibition of the $M2$ decays without permitting core excitation.

To study the effects of core excitation, a shell-model calculation could be carried out and energy matrices diagonalized within the config-