and hence does not represent a real decrease in the nuclear susceptibility of the He^3. We therefore conclude that there is no transition to a superfluid state in He^3 above 3.5 mdeg. With a low-temperature limit of about 4 mdeg, this conclusion is supported by measurements of heat capacity and spin-lattice relaxation time which will be reported elsewhere.

The above results lead us to believe that the heat-capacity anomaly reported by Peshkov is not a property of bulk He^3. A further discussion will be given in a paper to be submitted shortly for publication.

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UNSTABLE ELECTROSTATIC PLASMA WAVES PROPAGATING PERPENDICULAR TO A MAGNETIC FIELD*

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In the past few years significant progress has been made toward the stabilization of plasmas against hydromagnetic disturbances by using suitable magnetic configurations.[1,2] This progress has made more urgent the conquest of a second broad class of instabilities which limit densities and containment times in fusion experiments; viz., microinstabilities.

It has been known for some time that in an infinite homogeneous plasma in a uniform magnetic field electrostatic waves may be unstable if the velocity distribution function is sufficiently anisotropic.[3] The criteria for this instability have been examined in considerable detail.[4-6] Recently, Rosenbluth and Post[7] have shown that distribution functions of the form $f(v_\perp, v_\parallel)$ which vanish for $v_\perp = 0$ can be unstable ($v_\perp$ and $v_\parallel$ are the components of velocity perpendicular and parallel to the magnetic field). Because distributions of this form are a natural consequence of mirror confinement and of loss mechanisms such as charge-exchange reactions, these instabilities may be quite serious in fusion experiments. However, in the approximate dispersion relation of Rosenbluth and Post, only those waves for which $k_\parallel \neq 0$ are unstable ($k_\parallel$ is the component of the wave-propagation vector parallel to the magnetic field). There is reason to believe that these waves will be strongly damped in the region of the mirrors where the plasma density falls to zero. Thus the pessimistic predictions made for an infinite homogeneous plasma may not prove correct for laboratory plasmas of finite size.

It is the purpose of this note to point out that if some of the approximations made by Rosenbluth and Post are not made, then unstable waves with $k_\parallel = 0$ can exist. Such waves do not propagate toward the mirrors and therefore are not damped either by the effect of the density gradient noted above or by Landau damping from cold electrons moving along the magnetic field. The limit imposed by finite machine length, as described by Hall, Heckrotte, and Kammash,[6] may not be applicable in this case. We also show that if the distribution of perpendicular speeds is sufficiently broad, then the plasma is stable against these $k_{\parallel}=0$ waves. A noteworthy feature of this instability is that it may occur with zero frequency.

Our starting point is the dispersion relation of reference 3 with $k_{\parallel}=0$. We include only the contribution of a single species (ions or electrons), since here the interaction between species is not important:

$$\frac{k^2}{\omega_p^{\,2}}=\sum_{n=-\infty}^{\infty}\frac{n\omega_c}{\omega-n\omega_c}D_n(k)=Y(\omega,k), \tag{1}$$

where

$$D_n(k)=\int d^3v\,J_n^{\,2}\left(\frac{kv_\perp}{\omega_c}\right)\frac{1}{v_\perp}\frac{\partial f_0}{\partial v_\perp}. \tag{2}$$

ω_c is the gyrofrequency, ω_p is the plasma frequency, and J_n is the Bessel function of order n. This dispersion relation with $f_0(v_\perp, v_\parallel)$ given by

$$f_0(v_\perp, v_\parallel)=(1/2\pi v_\perp)\delta(v_\perp-\alpha_\perp)\delta(v_\parallel) \tag{3}$$

was first studied by Malmfors[8] who found an instability. Subsequently, Gross[9] found an error in Malmfors's work and conjectured that no instability existed. Sen[10] analyzed the dispersion relation numerically and found unstable growth. One of us (E.G.H)[3] claimed to have shown that instabilities existed if $b=k_\perp v_\perp/\omega_c>1.8$, but, in fact, this was incorrect; the number should have been 2.4.

It is fairly easy to see that an approximate condition for instability in the frequency range $n<\omega/\omega_c<n+1$ is that $D_n<0$ while $D_{n+1}>0$. The frequency range $-1<\omega/\omega_c<+1$ is somewhat special. For such frequencies the condition is $D_0<0$ while $D_1>0$. Moreover, the symmetry properties of the dispersion relation are such that the real part of ω is zero. We shall refer to this unstable wave as the zero-frequency mode. If f_0 is given by Eq. (3), then the conditions, $D_0<0$ while $D_1>0$, for instability of the zero frequency mode require that $b(=k_\perp v_\perp/\omega_c)$ lie in the following bands: $2.40\leq b\leq 3.83$, or $5.52\leq b\leq 7.02$, or $8.65\leq b\leq 10.17$, etc. If, in addition,

$$\frac{\omega_c^{\,2}}{\omega_p^{\,2}}<\frac{1}{b}\frac{d}{db}J_0^{\,2}(b),$$

then the plasma is unstable. This leads to a density threshold for instability given by $\omega_p>4.13\omega_c$. Additional unstable ranges of b exist which will sustain growing modes with higher frequencies. Thus, in order that a wave with frequency $n<\omega/\omega_c<n+1$ be unstable, it is necessary that $j_{n,m}<b<j_{n+1,m}$, where $j_{n,m}$ is the mth zero of J_n. The density threshold in such cases may be as low as $\omega_p\approx 2.7\omega_c$.

Now the distribution given by Eq. (3) is very special. Moreover, it is known that a plasma in thermal equilibrium is stable with respect to $k_{\parallel}=0$ waves; and in fact, it can be demonstrated that any two-temperature Maxwellian distribution is also stable against such disturbances. Thus, we should like to know how much spread in $v_\perp$ is required for stability. To that end we have investigated the class of distribution functions

$$F_0^{(j)}(v_\perp)=\int_{-\infty}^{\infty}dv_\parallel f_0(v_\perp,v_\parallel)=\frac{1}{\pi\alpha_\perp^{\,2}j!}\left(\frac{v_\perp}{\alpha_\perp}\right)^{2j}\exp(-v_\perp^{\,2}/\alpha_\perp^{\,2}), \tag{4}$$

with $j=1,2,3,\cdots$. These distributions are peaked at $\langle v_\perp\rangle=j^{1/2}\alpha_\perp$ and have half-widths which

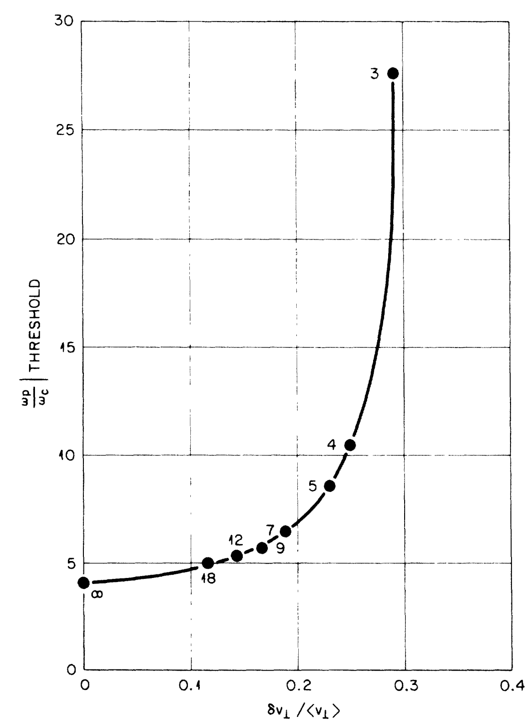

FIG. 1. Threshold value of ω_p/ω_c versus the relative half-width of the distribution function, $\delta v_\perp/\langle v_\perp\rangle$, for the zero-frequency mode. Point labels are j values for the distributions given by Eq. (4).

are approximated by $\delta v_\perp \approx \alpha_\perp/(4j)^{1/2}$. They are obtained by successive differentiations of $F_0^{(0)} = (1/\pi\alpha_\perp^2)\exp(-v_\perp^2/\alpha_\perp^2)$ with respect to $\alpha_\perp^2$. Thus, the integration of Eq. (2) is readily performed. We find the following: (1) For $j=0$, 1, 2, the distributions are stable with respect to $k_\| = 0$ waves. (2) For distributions with $j=3$, 4, 5, $\cdots$, the zero-frequency mode is unstable. The dependence of threshold density on relative half-width is shown in Fig. 1. Those waves are unstable whose propagation vectors fall in the band $2.5 \lesssim k\rho_g \lesssim 3.8$, where ρ_g is the gyroradius corresponding to the peak value of $v_\perp$. (3) The $j=6, 7, \cdots$ distributions sustain a growing wave whose real component of frequency is $\approx 1.2\omega_c$, with density threshold given by $\omega_p \approx 10\omega_c$ and $3.8 \lesssim k\rho_g \lesssim 5.0$. (4) As j increases, the results go over smoothly to those obtained with the distribution of Eq. (3). That is, higher frequency modes appear for threshold densities and bands of k which tend smoothly to those given by the distribution function of Eq. (3). We have observed unstable growth rates, dependent upon the density excess above the threshold value, which are typically some tenths of the gyrofrequency for excesses of the order of 10% of the threshold value.

The point to be emphasized here is that $k_\| = 0$ modes can be stabilized by a moderate amount of broadening and smoothing of an initially sharply peaked distribution. The absence of particles with small $v_\perp^2$ does not have to be eliminated completely.

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STUDY OF THE EXCHANGE INTEGRAL OF CRYSTALLINE ^{3}He AT 0°K

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The nuclear magnetic properties of crystalline ^{3}He have been studied extensively in recent years.[1] In particular, there have been many experimental[2-10] and theoretical[11-14] investigations aimed at obtaining an accurate value of the exchange integral J. All calculations of J have taken the effects of pair correlations into account. Bernardes and Primakoff[11] included them in a phenomenological way, whereas Saunders[12] derived an approximate differential equation for the correlation function. Recently Garwin and Landesman[14] have calculated J by means of an extension of Saunders's work.

The purpose of this note is to extend recent calculations of the ground-state energy E_0[15,16] to include the effects of exchange. The cluster expansion of E_0 used previously is generalized so that properly symmetrized wave functions may be treated. With an antisymmetrized version of the Jastrow-type wave function used previously, an expression is obtained for J which takes the effect of pair correlations into account in a systematic way. Calculations of J as a function of the nearest-neighbor distance are presented for both the bcc and hcp structures of crystalline ^{3}He. The effects of the pair correlations on J are analyzed.

The cluster development of the energy can