shown by Derrick et al.<sup>16</sup> that it is feasible to measure the  $\pi^-n$  cross section using a deuterium target. A series of  $\pi^-n$  cross-section measurements could then be used to find the  $\Delta_{\delta}^-$  mass. In addition, an analysis of photoproduction experiments might determine the  $\Delta_{\delta}^+$  mass with sufficient accuracy.

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the  $\pi^+ p$  case and 10<sup>5</sup> events in the  $\pi^- p$  case.

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## POSSIBLE NEW BARYON STATES OF SU(6) SYMMETRY\*

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Recent advances in SU(6) symmetry for strong and electromagnetic interactions<sup>1-6</sup> have yielded a substantial amount of predictions in agreement with known experimental data. In particular, in the area of mass splitting of spin-unitary-spin supermultiplets for baryon and meson states, there appears now a much deeper understanding of the role these states play in the framework of the 56- and 35-dimensional representations of SU(6). Pais<sup>2</sup> has emphasized the importance of the next other "small" representation 70<sup>-</sup> for consideration; indeed, it is felt that filling of the 70 baryon states should be as important for SU(6) symmetry as the existence of the decuplet 10 is for SU(3) symmetry. The experimental consequences here are rich because many of the states of the 70 are yet to be identified. In this note we propose

a set of solutions to the mass formulas of Bég and Singh<sup>4,6</sup> for this representation, based in part upon the postulated existence of an  $\eta$  octet of baryon states (to be discussed below) as well as certain other input experimental information.

For convenience of reference we write down the seven mass relations obtained by Bég and Singh for the 70<sup>-</sup> representation [70 = (1, 2) $\oplus (\underline{8}, \underline{4}) \oplus (\underline{10}, \underline{2}) \oplus (\underline{8}, 2)]$  as follows:

$$3\Lambda_{\gamma} + \Sigma_{\gamma} = 2(N_{\gamma} + \Xi_{\gamma}); \qquad (1)$$

$$4(\tilde{Y}_1^* + \tilde{\Sigma}) - 2(\tilde{N}^* + \tilde{N} + \tilde{\Xi}^* + \tilde{\Xi})$$
  
= 6( $\tilde{N}^* - \tilde{N}$ ) - 3( $\tilde{Y}_1^* + \tilde{\Sigma} - \tilde{\Lambda} - \Lambda'$ ); (2)

$$2(\tilde{\Omega}-\tilde{N}^*)=3(\tilde{\Xi}^*+\tilde{\Xi}-\tilde{Y}_1^*-\tilde{\Sigma}); \qquad (3)$$

$$2(\tilde{\Omega} - \tilde{N}^*) = 3(\Sigma_{\gamma} + \Lambda_{\gamma}) - 6N_{\gamma}; \qquad (4)$$

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$$16\bar{\Lambda}\Lambda' = \{2(N_{\gamma} + \Xi_{\gamma}) - 4(N_{\gamma} - \tilde{N}) - [(\tilde{Y}_{1}^{*} + \tilde{\Sigma}) - (\tilde{\Lambda} + \Lambda')] + 3(\Sigma_{\gamma} - \Lambda_{\gamma})\}$$

$$\times \{2(N_{\gamma} + \Xi_{\gamma}) - 4(N_{\gamma} - \tilde{N}) + 16(\tilde{N}^{*} - \tilde{N}) - (\Sigma_{\gamma} - \Lambda_{\gamma}) - 9[(\tilde{Y}_{1}^{*} + \tilde{\Sigma}) - (\tilde{\Lambda} + \Lambda')]\}$$

$$- \{3[(\tilde{Y}_{1}^{*} + \tilde{\Sigma}) - (\tilde{\Lambda} + \Lambda')] - 4(\tilde{N}^{*} - \tilde{N})\}^{2};$$

$$(5)$$

$$16\tilde{Y}_{*} * \tilde{\Sigma} = \{2(N_{*} + \Xi_{*}) - 4(N_{*} - \tilde{N}) + 8(\tilde{N}^{*} - \tilde{N}) - 3[(\tilde{Y}_{*}^{*} + \tilde{\Sigma}) - (\Lambda' + \tilde{\Lambda})] + 3(\Sigma_{*} - \Lambda_{*})\}$$

$$SY_{1} = \{2(N_{\gamma} + \Xi_{\gamma}) - 4(N_{\gamma} - N) + 8(N^{2} - N) - 3[(Y_{1}^{2} + \Sigma) - (\Lambda^{2} + \Lambda)] + 8(\tilde{N}^{2} - \tilde{N}) - (\Sigma_{\gamma} - \Lambda_{\gamma})\}$$

$$\times \{2(N_{\gamma} + \Xi_{\gamma}) - 4(N_{\gamma} - \tilde{N}) - 3[(\tilde{Y}_{1}^{*} + \tilde{\Sigma}) - (\Lambda^{2} + \tilde{\Lambda})] + 8(\tilde{N}^{*} - \tilde{N}) - (\Sigma_{\gamma} - \Lambda_{\gamma})\}$$

$$- \{3[(\tilde{Y}_{1}^{*} + \tilde{\Sigma}) - (\Lambda^{2} + \tilde{\Lambda})] - 4(\tilde{N}^{*} - \tilde{N})\}^{2}; \qquad (6)$$

$$16\tilde{\Xi}^*\tilde{\Xi} = \left\{ 2(N_{\gamma} + \Xi_{\gamma}) - 4(2N_{\gamma} - \tilde{N}) + 8(\tilde{N}^* - \tilde{N}) + (5\Sigma_{\gamma} - \Lambda_{\gamma}) - 3[(\tilde{Y}_1^* + \tilde{\Sigma}) - (\Lambda' + \tilde{\Lambda})] \right\} \\ \times \left\{ 2(N_{\gamma} + \Xi_{\gamma}) - 4(2N_{\gamma} - \tilde{N}) + 8(\tilde{N}^* - \tilde{N}) + (\Sigma_{\gamma} + 3\Lambda_{\gamma}) - 3[(\tilde{Y}_1^* + \tilde{\Sigma}) - (\Lambda' + \tilde{\Lambda})] \right\} \\ - \left\{ 3[(\tilde{Y}_1^* + \tilde{\Sigma}) - (\Lambda' + \tilde{\Lambda})] - 4(\tilde{N}^* - \tilde{N}) \right\}^2.$$

$$(7)$$

The notation here is that particle label = particle mass;  $\Lambda'$ ,  $(\tilde{N}, \tilde{\Lambda}, \tilde{\Sigma}, \tilde{\Xi})$ , and  $(\tilde{N}^*, \tilde{Y}_1^*, \tilde{\Xi}^*, \tilde{\Omega})$ are, respectively, the (1, 2), (8, 2), and (10, 2)baryon states of spin and parity  $\frac{1}{2}^-$ . The " $\gamma$ " octet of  $\frac{3}{2}^-$  resonant states is denoted by  $(N_{\gamma}, \Lambda_{\gamma}, \Sigma_{\gamma}, \Xi_{\gamma})$ . The above seven equations involve 13 masses;

hence six input masses have to be inserted to obtain the complete mass spectrum for the representation. In the light of current experimental data, it is reasonable to assume that the  $Y_0^*(1405)$  can be a candidate for the  $\Lambda'$  member as the  $\frac{1}{2}$  resonant state sought for in the interpretation of  $(K^-, p)$  data.<sup>7</sup> Concerning the status of the  $\gamma$  octet there is some room for maneuver. The evidence that  $\Lambda_{\gamma}(1520)$  and  $N_{\gamma}(1512)$  are  $\frac{3}{2}$  resonances is good,<sup>8</sup> while the evidence that  $\Sigma_{\gamma}(1660)$  and  $\Xi_{\gamma}(1810)$  are  $\frac{3}{2}$  states is at least possible.<sup>9,10</sup> The latter two assignments become more compelling if we take into consideration that dynamical theories<sup>11,12</sup> have tended to converge in their prediction that  $\Sigma_{\gamma}(1660)$  and  $\Xi_{\gamma}(1810)$  as well as  $N_{\gamma}(1512)$  are  $\frac{3}{2}$  resonant states. These same theories (see especially reference 11) say, however, that  $\Lambda_{\gamma}(1520)$  is dynamically distinct from the above-mentioned three states. In the language of SU(3),  $\Lambda_{\gamma}(1520)$  is a unitary singlet rather than a member of the octet; in the language of SU(6) it must be assigned to a representation other than the 70. We shall come back to this point at a later stage. Thus we have two alternatives here, a new  $\gamma$  octet made up of  $N_{\gamma}(1512)$ ,  $\Lambda_{\gamma}(1661)$ ,  $\Sigma_{\gamma}(1660)$ , and  $\Xi_{\gamma}(1810)$ , where  $\Lambda_{\gamma}(1661)$  follows from Eq. (1);

or the standard Glashow-Rosenfeld<sup>13</sup> assignment of  $(N_{\gamma}(1512), \Lambda_{\gamma}(1520), \Sigma_{\gamma}(1660), \Xi_{\gamma}(1598))$ in which  $\Lambda_{\gamma}(1520)$  is not regarded as an SU(3) singlet. We are aware that the  $\Xi_{\gamma}(1598)$ , if it exists at all,<sup>14</sup> has a production at most ~1-2% of  $\Xi^*(1530)$  from current data On the other hand, the existence of a  $\Lambda_{\gamma}(1661)$ , being masked by the 1660-MeV  $\Sigma_{\gamma}$ , cannot perhaps be ruled out since the width of  $\Sigma_{\gamma}(1660)$  is not presently known to a great accuracy. Both cases will be considered in the following analysis.

It has been known for some time that the  $\eta$ production<sup>15</sup> from the reaction  $K^- + p \rightarrow \eta + \Lambda$ showed a sharp peak in the neighborhood of its threshold: more recent data<sup>16</sup> indicate that this threshold phenomenon is consistent with a  $(T=0, J=\frac{1}{2})$  state due to an S-wave interaction of  $\eta + \Lambda$ . Again, Hand and Schaerf<sup>17</sup> found some years back an anomalously sharp peak in  $\gamma + p \rightarrow \pi^+ + n$  at  $E_{\gamma} = 700$  MeV; this has been interpreted by Sakurai<sup>18</sup> as an  $\eta + N$  cusp effect due to the Ball-Frazer mechanism,<sup>19</sup> since the position of the sharp peak coincided with the  $\eta + N$  threshold. Indeed recent experiments on<sup>20</sup>  $\pi^+ + n \rightarrow \eta + p$  and on<sup>21</sup>  $\pi^- + p \rightarrow \eta + n$  do show a sharp rise in  $\eta$  production above threshold, while the pion-nucleon phase-shift analysis of Auvil et al.<sup>22</sup> lends support to a possible baryon "state"  $(\overline{T} = \frac{1}{2}, J = \frac{1}{2})$  in a neighborhood of the  $\eta + N$  threshold. In view of the fact that  $\eta$  production does not choose apparently to distinguish dynamically between  $\Lambda$  and N, it seems physically plausible to postulate the existence

of an  $\eta$ -baryon octet of  $\frac{1}{2}$  states with the remaining members associated with the  $\eta + \Sigma$  and  $\eta + \Xi$  thresholds. Such an " $\eta$  octet" will satisfy the octet-type mass formula

$$\frac{1}{2}(\tilde{N}+\tilde{\Xi}) = \frac{1}{4}(3\tilde{\Lambda}+\tilde{\Sigma}), \qquad (8)$$

with  $\tilde{B} = B + \eta$ ,  $B = (N, \Lambda, \Sigma, \Xi)$ . In particular, with  $\eta = 548$  MeV, we have

$$\tilde{N} \sim 1488 \text{ MeV}, \quad \tilde{\Lambda} \sim 1663 \text{ MeV},$$
 (9a)

$$\tilde{\Sigma} \sim 1740 \text{ MeV}, \quad \tilde{\Xi} \sim 1866 \text{ MeV}.$$
 (9b)

Since there exists some experimental and theoretical support<sup>15-22</sup> for baryon states  $\tilde{N}$ and  $\tilde{\Lambda}$ , it is evidently sensible to use these states in conjunction with  $\Lambda'(1405)$  and three members of the  $\gamma$  octet [Eq. (1) then determines the fourth member] as the six input masses to determine the remaining occupants of 70 from Eqs. (1) to (7). Because of possible mixing between states of the same (T, Y, J), like those between  $(\bar{\Sigma}, \bar{\Xi})$  of (8, 2) and the corresponding members  $(\tilde{Y}_1^*, \tilde{\Xi}^*)$  of (10, 2) as well as between  $\overline{\Lambda}$  of (8, 2) and  $\Lambda'$  of  $\overline{(1, 2)}$ , it is at first sight at least not obvious that the 70 mass formulas will bear out the conjectured  $\eta$  octet [Eqs. (8) and (9b)] even approximately. We thus regard the numerical results presented below as highly encouraging.

In Table I we have listed a few of the possible solutions of the 70<sup>-</sup> mass formulas for the remaining states  $(\tilde{\Sigma}, \tilde{\Xi})$  of (8, 2) and the complete set  $(\tilde{N}_{3/2}^*, \tilde{Y}_1^*, \tilde{\Xi}_{1/2}^*, \tilde{\Omega}^-)$  of the  $(\underline{10}, \underline{2})$ .

Since the exact mass values for  $\tilde{N}$  and  $\tilde{\Lambda}$  are not known to great accuracy, we have allowed for some typical variations of their values away from the threshold determination (9a). Solutions  $(a_{\pm})$ ,  $(b_{\pm})$ , and  $(c_{\pm})$  are derived with input information involving  $\Lambda'(1405)$ ,  $\tilde{N}$ ,  $\tilde{\Lambda}$ , and the new  $\gamma$  octet, while solution (d±) is obtained using the Glashow-Rosenfeld  $\gamma$  octet.<sup>13</sup> For each case two solutions  $(\pm)$  are possible because of the quadratic nature of Eqs. (5) to (7). In the framework of the new  $\gamma$  octet, complex solutions are realized if (i) we allow  $\tilde{N}$ to increase or  $\bar{\Lambda}$  to decrease by more than 50 MeV from their threshold values, and (ii) if  $\Lambda'$  is increased substantially ( $\gtrsim 250$  MeV) from its presently assigned (1405) value. Both considerations strengthen our confidence that we have correctly assigned  $\tilde{N}$ ,  $\tilde{\Lambda}$ , and  $\Lambda'$  to (8, 2) and (1, 2), respectively.<sup>23</sup>

It is evident from Table I that, for either choice of  $\gamma$ -octet assignment, the minus solutions in all four cases listed have no resemblance whatsoever to the conjectured  $\eta$  octet [Eqs. (8) and (9)]. Indeed, the  $(\tilde{N}, \tilde{\Lambda}, \tilde{\Sigma}, \tilde{\Xi})$  sets here do not satisfy the Gell-Mann-Okubo mass formula (8), nor is the situation improved if we make the substitution  $\tilde{\Sigma} \leftrightarrow \tilde{Y}_1^*$ ,  $\tilde{\Xi} \leftrightarrow \tilde{\Xi}^*$  in Table I [the multiplicative mass formulas (5) to (7) do not differentiate between the pairs  $(\tilde{\Sigma}, \tilde{Y}_1^*)$  and  $(\tilde{\Xi}, \tilde{\Xi}^*)$ ]. Some of the low-lying states predicted by the minus solutions are already in an energy region accessible to cur-

Table I. Possible solutions of the <u>70</u><sup>-</sup> mass formulas. Solutions (a±), (b±), and (c±) are derived with input information involving  $\Lambda'$ ,  $\tilde{N}$ ,  $\tilde{\Lambda}$ , and the <u>new</u>  $\gamma$  octet, while solution (d±) is obtained using the Glashow-Rosenfeld  $\gamma$  octet (cf. reference 13).

Input: $\Lambda' = 1405$ ; $(N_{\gamma}, \Sigma_{\gamma}, \Xi_{\gamma}) = (1512, 1660, 1810)$ ; $(\tilde{N}, \tilde{\Lambda}) = (1488, 1663)$	
$ \begin{array}{c} (a^+) \\ \Lambda \gamma = 1661, \ (\tilde{\Sigma}, \tilde{\Xi}) = (1696, 1845) \end{array} $	(2-) $\Lambda_{\gamma} = 1661, (\tilde{\Sigma}, \vec{\Xi}) = (1452, 1601)$
$(N^*, Y_1^*, \Xi^*, \Omega^-) = (1793, 1941, 2090, 2239)$	$(\bar{N}^*, \bar{Y}_1^*, \bar{\Xi}^*, \bar{\Omega}^-) = (1386, 1535, 1683, 1832)$
Input: $\Lambda' = 1405$ ; $(N_{\gamma}, \Sigma_{\gamma}, \Xi_{\gamma}) = (1512, 1660, 1810)$ ; $(\tilde{N}, \tilde{\Lambda}) = (1500, 1685)$	
(b+)	(b-)
$\Lambda \gamma = 1661, \ (\Sigma, \Xi) = (1727, 1875)$	$\Lambda_{\gamma} = 1661, \ (\Sigma, \Xi) = (1445, 1593)$
$(N^*, Y_1^*, \Xi^*, \Omega^-) = (1837, 1986, 2135, 2283)$	$(N^*, Y_1^*, \Xi^*, \Omega^-) = (1367, 1516, 1665, 1813)$
Input: $\Lambda' = 1405; (N_{\gamma}, \Sigma_{\gamma}, \Xi_{\gamma}) = (1512, 1660, 1810); (\tilde{N}, \tilde{\Lambda}) = (1488, 1688)$	
$\Lambda_{\gamma} = 1661, \ (\tilde{\Sigma}, \tilde{\Xi}) = (1738, 1887)$	$\Lambda_{cv} = 1661  (\tilde{\Sigma} = \tilde{\Xi}) = (1425 - 1574)$
$(\tilde{N}^*, \tilde{Y}_1^*, \tilde{\Xi}^*, \tilde{\Omega}^-) = (1837, 1986, 2135, 1183)$	$(\tilde{N}^*, \tilde{Y}_1^*, \tilde{\Xi}^*, \tilde{\Omega}^-) = (1316, 1465, 1614, 1762)$
Input: $\Lambda' = 1405$ ; $(N_{\gamma}, \Lambda_{\gamma}, \Sigma_{\gamma}) = (1512, 1520, 1660)$ ; $(\tilde{N}, \tilde{\Lambda}) = (1490, 1665)$	
	~ ( <u>d</u> -)
$\mu_{\gamma} = 1598, (\Sigma, \mu) = (1765, 1843)$	$\Xi_{\gamma} = 1598, \ (\Sigma, \Xi) = (1411, 1489)$
$(N^{+}, Y_{1}^{+}, \Xi^{+}, \Omega^{-}) = (1920, 2019, 2097, 2154)$	$(N^*, Y_1^*, \Xi^*, \Omega^-) = (1406, 1551, 1629, 1640)$

rent experiments,<sup>24</sup> and there does not appear to be strong support for these predictions. We shall not consider these solutions further in the present paper.

Solutions (a+), (b+), and (c+) for the new  $\gamma$ octet (with typical variations for the input  $\tilde{N}$ and  $\bar{\Lambda}$  members), on the other hand, exhibit highly attractive features. The  $(\bar{N}, \bar{\Lambda}, \bar{\Sigma}, \bar{\Xi})$ members of (8, 2) obey the Gell-Mann-Okubo equation (8) to  $\overline{4}$ , 8, and 13 MeV, respectively; the conjectured  $\eta$  octet is borne out to between 3% [solution (a+)] and 1% [solution (c+) accuracy; the (10, 2) members  $(\tilde{N}^*, \tilde{Y}_1^*, \tilde{\Xi}^*, \tilde{\Omega}^-)$ are equally spaced (to within 1 MeV) with mean spacing of 149 MeV. It is quite likely that even better agreement with the conjectured  $\eta$  octet can be obtained if we allow some variation of mass values for the  $\gamma$  octet away from the canonical values used here,<sup>23</sup> or perhaps included other symmetry-breaking terms (like  $M_{189}^{(8)}$ ) in the derivation of the mass formula. $^{4,6}$  It must be pointed out, however, that tolerable agreement with the  $\eta$  octet can also be obtained using the Glashow-Rosenfeld  $\gamma$  octet as input. Solution (d+) for this case satisfies Eq. (8) only moderately well (to about 23 MeV) though; in addition, predictions for the (10, 2) are not equally spaced and have a mean spacing of only 78 MeV.

The following remarks deserve emphasis:

(1) Mass regularities inherent in  $70^{-}$  formulas. -One can show that the set of equations (1) to (7) gives rise to two further linear relationships

$$\tilde{\Xi} - \tilde{\Sigma} = \alpha,$$
 (10a)

$$\tilde{\Xi}^* - \tilde{Y}_1^* = \alpha', \qquad (10b)$$

with  $\alpha = \alpha' = \frac{1}{2}(\Sigma_{\gamma} + \Lambda_{\gamma} - 2N_{\gamma}) = \Xi_{\gamma} - \Lambda_{\gamma}$ . They are independent of the set of linear mass formulas (1) to (4) and could therefore be used instead of one of the multiplicative equations such as (7). Two very instructive equations can be derived from (10) by adding (10a) and (10b) and using (3). We obtain

$$2(\tilde{\Omega} - \tilde{N}^*) = 3(\alpha + \alpha') \tag{3'}$$

and, immediately,

$$\frac{1}{2} [(\tilde{\Omega} - \tilde{\Xi}^*) + (\tilde{Y}_1^* - \tilde{N}^*)] = \frac{1}{4} [3(\tilde{\Xi}^* - \tilde{Y}_1^*) + (\tilde{\Xi} - \tilde{\Sigma})].$$
(11)

Equation (11) is a Gell-Mann-Okubo formula for mass differences, mixing the  $\eta$  octet and the decuplet. However, since  $\alpha = \alpha'$ , we also have the simple mass-spacing rule

$$(\tilde{\Omega} - \tilde{\Xi}^*) + (\tilde{Y}_1^* - \tilde{N}^*) = 2(\tilde{\Xi}^* - \tilde{Y}_1^*);$$
(12)

thus the modification of the usual equal-spacing law is that the mean of the  $(\tilde{\Omega}, \tilde{\Xi}^*)$  and  $(\tilde{Y}_1^*, \tilde{N}^*)$  mass differences is equal to the  $(\tilde{\Xi}^*, \tilde{Y}_1^*)$  mass difference. Indeed, detailed analysis shows that deviation from equal spacing is multiplicatively dependent on  $(\Lambda_{\gamma} - \Sigma_{\gamma})$ , which for the new  $\gamma$  octet is very small, because of the near mass degeneracy between  $\Lambda_{\gamma}$  and  $\Sigma_{\gamma}$ . Thus we observe almost precise equal spacing for solutions (a), (b), and (c). For the Glashow-Rosenfeld  $\gamma$  octet, on the other hand, this difference is large and hence equal spacing is badly broken.

Note that Eq. (3') in conjunction with Eq. (10) is formally similar to Eq. (21a) of Kuo and Yao<sup>25</sup>; however, solutions (a) to (d) presented here cannot then be made to satisfy their Eqs. (21b) and (21c) simultaneously as well. This is probably not surprising, since Kuo and Yao have emphasized that their basic mass formula is a special case of that of Bég and Singh.<sup>4</sup>

Pais<sup>2</sup> proposed recently a set of intuitive mass rules involving, among other things, octet-decuplet relations within the 70. The basic dynamical premise is that SU(6) - factorized  $[SU(3) \otimes SU(2)]$  (first stage) - broken SU(3) (second stage) is additive in the first- and second-stage breakdowns. Solutions (a+), (b+), and (c+) are remarkably consistent with Pais's proposal. To take an example, using  $\eta$ -octet mass values of (c+) as input to Pais's theory. we obtain a spacing for the (10, 2) of 149 MeV; this agrees embarrassingly well with the spacing among the (10, 2) members of solution (c+)! In addition, using the new  $\gamma$  octet as input to the Pais theory we obtain a decuplet spacing of 150 MeV, which in turn would imply a sum rule for  $(8, 2)^-$ ; to wit,  $\Xi \sim \Sigma + 150$  MeV. The latter sum rule has a ready parallel in Eq. (10) of the present discussion. We see, therefore, that the overall internal consistency of the Pais scheme, as well as its corroboration with predictions from the Bég-Singh mass formulas, is very much in evidence. This is all the more remarkable since mixing between states was not taken into account in the former work. It must also be emphasized at the same time that Pais's proposal cannot be made consistent with the existence of both the  $\eta$  octet and the Glashow-Rosenfeld (GR)  $\gamma$  octet within the 70,

in the following sense:

$$GR(\underline{8},\underline{4})^{-} \rightarrow (\underline{10},\underline{2})^{-}$$
 equal spacing -60 MeV  
 $\rightarrow (\underline{8},\underline{2})^{-}$  sum rule  $\tilde{\Xi} \sim \tilde{\Sigma} - 60$  MeV

whereas the  $\eta$  octet of say, for instance, solution (d+) gives

$$\eta(\underline{8},\underline{2})^- \rightarrow (\underline{10},\underline{2})^-$$
 equal spacing 78 MeV  
 $\rightarrow (\underline{8},\underline{2})^-$  sum rule  $\tilde{\Xi} \sim \tilde{\Sigma} + 78$  MeV.

Coupled with the lack of experimental evidence for  $\Xi_{\gamma}(1598)$ , the above consideration strengthens our belief that the new  $\gamma$  octet is probably the correct assignment for the 70.

(2) Possible assignment for  $\Lambda_{\gamma}(1520)$  in SU(6). -If we believe, as seems plausible, that  $\Lambda_{\gamma}(1520)$  is not a member of the  $\gamma$  octet, it must be assigned to a representation of SU(6)other than the 70. An elementary solution would be to group  $\Lambda_{\gamma}(1520)$  [or  $Y_0^*(1520)$ ] together with other baryon states<sup>26</sup> for which theoretical interpretations<sup>12,27</sup> can allow  $\frac{3}{2}^{-1}$ assignments into a higher representation like, say, 700. However, it is physically appealing if the low-lying state  $Y_0^*(1520)$  can be accommodated in the unfilled baryon representation 20 with content  $20^{-} \rightarrow (8, 2)^{-} \oplus (1, 4)^{-}$ . The question arises naturally at this stage whether the  $\eta$  octet may not belong to the 20 [where the  $(8, 2)^{-}$  members obey an unmixed Gell-Mann-Okubo octet mass formula] rather than the 70. Our expectation is that the  $\eta$  octet is dynamically a quasi or virtual bound state of the Swave  $\eta$ -baryon system,<sup>28</sup> with possible cusp manifestations in the meson-baryon systems coupled to it; decays into these two-body channels are expected to be important. On the other hand, 20 is not contained in  $\{35 \otimes 56\}_L$ . This is perhaps especially detrimental for the assignment of  $\eta$  octet to the 20, since for the Swave "resonances" here there are no spinorbit forces which might lead to some recoupling.<sup>29</sup> Pais<sup>2</sup> has emphasized that the 20 is a baryon-two-meson state; hence one should look at  $Y_0^*(1520)$  and the possible existence of another  $(8, 2)^-$  set in terms of production processes involving three-body final-state interaction.

There has been evidence for some time that the di-pion mass distribution in  $\pi^- + p \rightarrow \pi^+ + \pi^-$ +*n* is distorted over phase space in the vicinity of  $\pi + N_{33}^*(1238)$  threshold.<sup>30</sup> An earlier interpretation in terms of a triangle singularity effect is apparently discredited because of the large  $N^*$  width.<sup>31</sup> Indeed, the more recent evidence suggests a  $T = \frac{1}{2}$ ,  $J = \frac{1}{2}$  enhancement for  $\pi^{-}p$  at this rough energy. A di-pion system  $\sigma(\pi^+\pi^-)(T=0, J=0^+)$  of mass ~400 MeV in an L = 1 orbital with the nucleon can participate in a  $S_{11}$  enhancement. Such a model has proved useful in interpreting features of the final-state system from  $K^- + p \rightarrow \Lambda + \pi^+ + \pi^-$  at c.m. energy 1520 MeV<sup>33</sup> where again the anomaly associated with the di-pion mass distribution is noted.<sup>34</sup> To summarize, there is at least a hint of possible  $\frac{1}{2}$  "states" associated with  $n + \sigma(\pi\pi)$  and  $\Lambda + \sigma(\pi\pi)$  which can then be completed with  $\Sigma$  $+\sigma(\pi\pi)$  and a  $\Xi$  member (around 1720 MeV) to fill the remaining occupancy of the octet in 20. We have avoided here the deeper implications of possible supermixing in SU(6) between  $(8, 2)^{-1}$ members of 20 and 70, respectively.

In conclusion, we cannot emphasize too strongly the importance of obtaining experimental information on the remaining members of the  $\eta$  octet and the decuplet  $(\underline{10}, \underline{2})^-$ . Of particular interest would be the discovery of a new  $\overline{\Omega}^-$  ( $T = 0, Y = -2, J = \frac{1}{2}^-$ ), unstable against decay into, say,  $\Omega^-(\underline{1680}) + 2\pi$ , in the 2.2- to 2.3-BeV region.

One of us (S.F.T.) takes great pleasure in thanking Professor A. Pais and Professor M. A. B. Bég for their enthusiastic encouragement as well as invaluable suggestions on this work. He would like to thank Professor G. Goldhaber, Professor S. Goldhaber, Professor B. Moyer, Professor L. Rosenson, Professor S. P. Rosen, Professor N. Samios, and Professor W. J. Willis for helpful discussions and communications, and Professor R. M. Steffen and Professor T. T. Wu for their interest.

Note added in Proof. – It has been pointed out to us by Professor Freeman J. Dyson, Dr. N. Xuong, and Dr. S. Pakvasa that the  $\eta + N$  decay mode of  $\tilde{N}$  is forbidden in strict SU(6) symmetry; no such problem arises for the remaining members of the  $\eta$  octet. This raises the attractive possibility that the  $\tilde{N}$  member is below the  $\eta + N$  threshold and decays into the SU(6)-allowed mode  $\pi + N$ . Such an interpretation will favor solutions of type (c+) in Table I where  $\tilde{N}$  is assigned to a low mass value. Professor **M**. A. B. Bég has emphasized that coupling of  $\tilde{N}$  to  $\eta + N$  can, in general, occur via broken SU(3) in the chain SU(6) – SU(3) $\otimes$ SU(2) – SU(3).

<sup>\*</sup>Work supported in part by the U. S. Air Force Office of Scientific Research and the National Science

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## ERRATUM

MONTE CARLO INTRANUCLEAR CASCADE CALCULATIONS ON C12 WITH MEDIUM-EN-ERGY PROTONS. E. Gradsztajn [Phys. Rev. Letters 13, 240 (1964)].

"PROTONS"; thus the title as given above is correct. The correct version also was given in the Table of Contents and in the Author Index. The reference numbers for the experimental

points as shown in the figures should be advanced by one; thus, Fig. 1 should refer to reference 9 and Fig. 2 to reference 10.

The word "PHOTONS" in the title should read

Foundation.