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DETERMINATION OF THE $\Delta_\delta^{++}-\Delta_\delta^0$ MASS DIFFERENCE

Martin G. Olsson*
Department of Physics, University of Wisconsin, Madison, Wisconsin
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The $\Delta_\delta^{++}-\Delta_\delta^0$ mass difference has been found to be -0.45 ± 0.85 MeV. This measurement is compared to an SU(6) prediction and to various dynamical theories of SU(3) splitting.

It is well known that the particle masses of an isotopic spin multiplet are split by electromagnetic effects. Within an SU(3) supermultiplet general relationships may be derived between these electromagnetic mass differences.[1] More specific predictions of these splittings may be made with dynamical models such as the "tadpole" model of Coleman and Glashow[2] or the calculation of Socolow[3] which uses only known particles. The agreement with experiment of these predicted mass differences has been quite satisfactory with the pseudoscalar and baryon octets[2]; thus it would be of interest also to test the decuplet mass-splitting predictions.

Accurate measurements of the decuplet masses are difficult for the following reasons:

(1) Rather large statistics are required since the Δ_δ, Σ_δ, and[4] Ξ_δ have level widths comparable to or larger than the electromagnetic mass differences.

(2) Σ_δ and Ξ_δ are not seen experimentally in two-body reactions of the form

$$m+B\rightarrow B^*\rightarrow m+B,$$

but must be studied in production reactions such as

$$m+B\rightarrow m+B^*\rightarrow m+m+B.$$

In production reactions the exact position of the B^* on the Dalitz plot has no simple relation to its mass, but depends on the largely unknown details of the production mechanism. However, if charge-symmetric reactions are studied, mass differences may, in principle, be measured.[5]

Although the width of the Δ_δ is about two orders of magnitude larger than the accuracy to which we wish to determine its mass, we have the advantage that there already exists a large body of experimental data on Δ_δ formation from low-energy pion-nucleon scattering experiments. The reactions which have been extensively studied are

$$\pi^+ + p \rightarrow \Delta_\delta^{++} \rightarrow \pi^+ + p;$$
$$\pi^- + p \rightarrow \Delta_\delta^{0} \rightarrow \pi^- + p$$
$$\rightarrow \pi^0 + n.$$

Thus by an analysis of these reactions we hope to determine, among other things, the $\Delta_\delta^{++}-\Delta_\delta^0$ mass difference.

For two reasons we have used only total cross-section measurements in our analysis. First, statistical accuracy is assured since the beam-attenuation measurement is the simplest and most often performed scattering experiment.[6] Second, the nonresonant partial waves contribute incoherently to the total cross section, thus minimizing their effect upon the dominant resonant state.

The experimental data are taken from the compilation of Klepikov, Meshcheryakov, and Sokolov,[7] with the addition of several later experiments.[8] These data have been fitted to an expression of the following form:

$$\sigma_{\text{tot}}(\pi^+p)=\sigma_r+c_{3/2}\sigma_b(\tfrac{3}{2});$$

$$\sigma_{\text{tot}}(\pi^-p)=\tfrac{1}{3}\sigma_r+\tfrac{1}{3}c_{3/2}\sigma_b(\tfrac{3}{2})+\tfrac{2}{3}c_{1/2}\sigma_b(\tfrac{1}{2}).$$

The meanings of the individual terms are as follows[9]:

$$\sigma_r=8\pi\lambda^2\frac{(\Gamma/2)^2}{(m-m_0)^2+(\Gamma/2)^2},$$

where

$$\Gamma/2=\gamma\eta^3\left(\frac{1+ac^2}{1+a\eta^2}\right),$$

$\eta = P$ (center-of-mass momentum)$/m_\pi$, m = invariant pion-nucleon mass, m_0 = resonant mass, γ is a reduced half-width, a is an asymmetry parameter related to the interaction distance, and $c = 1.25$[10]; $\sigma_b(\frac{3}{2})$ is the nonresonant background term in the isotopic spin-$\frac{3}{2}$ channel, derived from phase-shift analyses; and $c_{3/2}$ is a normalization parameter.

The parameters m_0, γ, a, and $c_{3/2}$ (e.g., in the π^+p case) are now varied until a chi-square minimum is achieved.

The nonresonant background could be unambiguously calculated from available phase-shift analyses if these were sufficiently accurate and consistent. The backgrounds calculated from the analyses of McKinley[11] and Roper[11] are qualitatively similar but differ by about a factor of two. Thus, we must limit ourselves to energy regions in which the background is relatively constant and then vary $c_{3/2}$ and $c_{1/2}$ until the best fit is achieved.

As can be seen from Fig. 1, the background in the π^+p case is only about 1%, and consequently has little effect upon the resonance parameters. The results of the π^+p fit are summarized in Table I.

In analyzing the π^-p data we note from Fig. 1 that the background is 5% at resonance and rises rapidly above 250 MeV; therefore, we shall use π^-p data only in the 100- to 225-MeV energy range. The results of this fit are given in Table I. The final results (including an estimate of systematic errors) are

$$m(\Delta_\delta^{++}) = 1236.0 \pm 0.55 \text{ MeV},$$

$$m(\Delta_\delta^{0}) = 1236.45 \pm 0.65 \text{ MeV},$$

$$m(\Delta_\delta^{++}) - m(\Delta_\delta^{0}) = -0.45 \pm 0.85 \text{ MeV};$$

$$\Gamma(\Delta_\delta^{++}) = 120.0 \pm 2.0 \text{ MeV},\quad \Gamma(\Delta_\delta^{0}) = 119.6 \pm 2.4 \text{ MeV},$$

$$\Gamma(\Delta_\delta^{++}) - \Gamma(\Delta_\delta^{0}) = 0.4 \pm 3.1 \text{ MeV};$$

where Γ is the full width evaluated at the resonance energy.

The status of decuplet mass-splitting measurements is still in a primitive state. The mass difference $\Sigma_\delta^+ - \Sigma_\delta^-$ has been measured

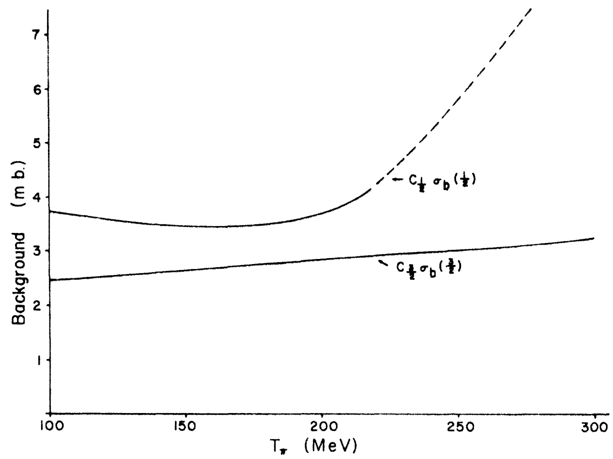

FIG. 1. Nonresonant background. The total cross section at resonance is about 200 mb in the π^+p case and about one-third of this in the π^-p case.

by Cooper et al.[12] to be -17 ± 7 MeV, and by Huwe[13] to be -4.4 ± 2.2 MeV. The Σ_δ^0 mass is also currently being investigated.[14] Gidal, Kernan, and Kim,[5] by use of charge-symmetric reactions, have found that $\Delta_\delta^{++} - \Delta_\delta^- = 0.6 \pm 5.0$ MeV.

In SU(3) there are only two independent decuplet electromagnetic mass differences, and there is one relation among the Δ_δ differences.[1] Therefore, by using the weighted mean of the $\Sigma_\delta^+ - \Sigma_\delta^-$ mass-difference measurements ($\Sigma_\delta^+ - \Sigma_\delta^- = -5.5 \pm 2.1$ MeV), and the SU(3) relation, $\Delta_\delta^+ - \Delta_\delta^- = \Sigma_\delta^+ - \Sigma_\delta^-$, together with our results, we can specify the level structure of the Δ_δ multiplet. This is shown in Fig. 2.

Coleman and Glashow[2] are able to predict this level structure under the assumption that tadpole diagrams are dominant. The tadpole splitting is shown in Fig. 3(a). Other types of mass-splitting diagrams have been calculated by Socolow.[3] In Fig. 3(b) the tadpole splitting plus the effect of self-energy diagrams with a baryon-octet member and a photon in the intermediate state are shown. The effect of a decuplet-member-plus-photon intermediate state in addition to the previous two splittings is shown in Fig. 3(c). The "decuplet" correction is difficult to calculate and may be only qualitatively correct.[3]

Recently Sakita[15] has considered electromagnetic mass splitting in the 56 representation of SU(6) which contains the baryon octet and the decuplet. The following relevant relations

Table I. Resonance parameters from fit to π^+p and π^-p data.

	Δ_δ^{++}	Δ_δ^{0}
Mass	1236.0 ± 0.5 MeV	1236.45 ± 0.55 MeV
Width at resonance	120.0 ± 1.6 MeV	119.6 ± 1.7 MeV
Asymmetry parameter	0.91	0.99
Number of "measurements"[a]	53	43
Chi square of fit[b]	93.2	67.2

[a]Where more than one measurement has been made at a given energy, a weighted average is used, as in the compilation of Klepikov, Meshcheryakov, and Sokolov (see reference 7).

[b]The relatively large chi square is the result of systematic errors between different experiments, but because of the large number of different experiments, the effect on the resonance parameters is small. The errors given in Table I, however, reflect increased effective measurement errors.

have resulted:

$$\Delta_\delta^{+}-\Delta_\delta^{0}=p-n=\Sigma^+-\Sigma^0,$$

$$\Delta_\delta^{-}-\Delta_\delta^{0}=\Xi^--\Xi^0=\Sigma^--\Sigma^0,$$

$$\Delta_\delta^{++}-\Delta_\delta^{0}=3(\Delta_\delta^{+}-\Delta_\delta^{0})+(\Delta_\delta^{-}-\Delta_\delta^{0}).$$

Using the average baryon mass differences,

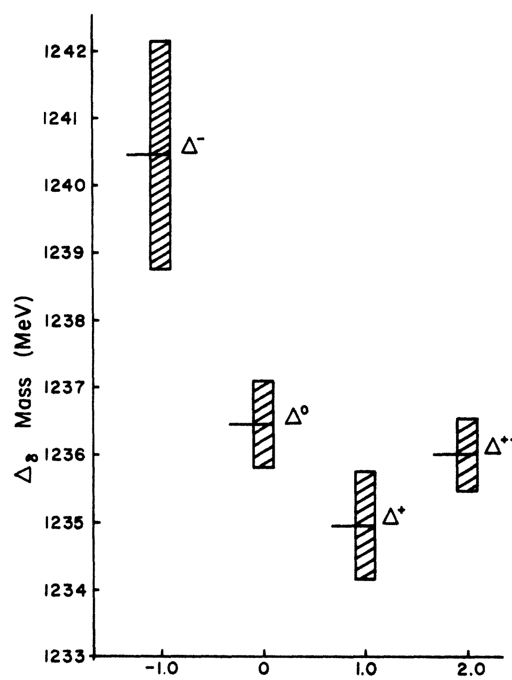

FIG. 2. Δ_δ mass structure. The Δ_δ^{++} and Δ_δ^0 masses are those given in this paper. The $\Delta_\delta^+-\Delta_\delta^-$ mass difference is given by SU(3) from $\Sigma_\delta^+-\Sigma_\delta^-$ mass-difference measurements. The absolute mass values of Δ_δ^+ and Δ_δ^- and their individual error assignments are given by the above data and the SU(3) relation[1] $m=m_0+aQ+bQ^2$. Note that the measurement of Gidal, Kernan, and Kim[5] is in agreement (within the error) with these mass values.

the first two Sakita relations would predict that

$$\Delta_\delta^{+}-\Delta_\delta^{0}=-2.1 \text{ MeV},$$

$$\Delta_\delta^{-}-\Delta_\delta^{0}=5.4 \text{ MeV},$$

from which we can derive (using the third relation) that

$$\Delta_\delta^{++}-\Delta_\delta^{0}=-0.9 \text{ MeV},$$

which is in good agreement with our result of -0.45 ± 0.85 MeV.

We might speculate whether the other Δ_δ masses (i.e., Δ_δ^- and Δ_δ^+) can be determined by two-body scattering reactions. It has been

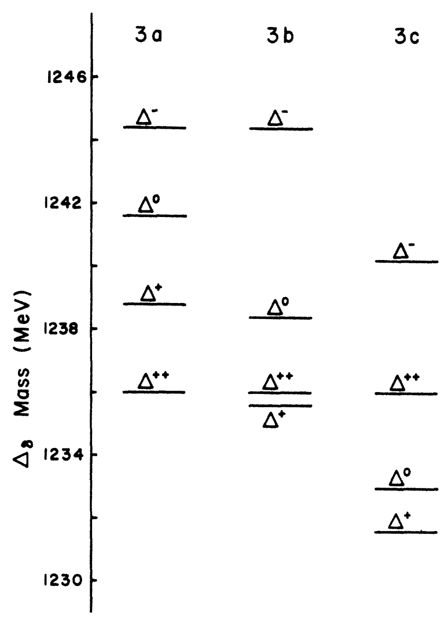

FIG. 3. Δ_δ mass structure predicted by various dynamical models. (a) Tadpole alone; (b) tadpole plus octet corrections; (c) tadpole plus octet plus decuplet corrections. The Δ_δ^{++} mass has been arbitrarily fixed at 1236 MeV.

shown by Derrick et al.[16] that it is feasible to measure the $\pi^- n$ cross section using a deuterium target. A series of $\pi^- n$ cross-section measurements could then be used to find the Δ_δ^- mass. In addition, an analysis of photoproduction experiments might determine the Δ_δ^+ mass with sufficient accuracy.

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POSSIBLE NEW BARYON STATES OF SU(6) SYMMETRY*

I. P. Gyuk and S. F. Tuan
Department of Physics, Purdue University, Lafayette, Indiana
(Received 26 October 1964)

Recent advances in SU(6) symmetry for strong and electromagnetic interactions[1-6] have yielded a substantial amount of predictions in agreement with known experimental data. In particular, in the area of mass splitting of spin–unitary-spin supermultiplets for baryon and meson states, there appears now a much deeper understanding of the role these states play in the framework of the 56- and 35-dimensional representations of SU(6). Pais[2] has emphasized the importance of the next other "small" representation $\underline{70}^-$ for consideration; indeed, it is felt that filling of the $\underline{70}$ baryon states should be as important for SU(6) symmetry as the existence of the decuplet $\underline{10}$ is for SU(3) symmetry. The experimental consequences here are rich because many of the states of the $\underline{70}$ are yet to be identified. In this note we propose a set of solutions to the mass formulas of Bég and Singh[4,6] for this representation, based in part upon the postulated existence of an η octet of baryon states (to be discussed below) as well as certain other input experimental information.

For convenience of reference we write down the seven mass relations obtained by Bég and Singh for the $\underline{70}^-$ representation $[\underline{70} = (\underline{1}, \underline{2}) \oplus (\underline{8}, \underline{4}) \oplus (\underline{10}, \underline{2}) \oplus (\underline{8}, \underline{2})]$ as follows:

$$3\Lambda_\gamma + \Sigma_\gamma = 2(N_\gamma + \Xi_\gamma); \tag{1}$$

$$4(\tilde{Y}_1^* + \tilde{\Sigma}) - 2(\tilde{N}^* + \tilde{N} + \tilde{\Xi}^* + \tilde{\Xi}) = 6(\tilde{N}^* - \tilde{N}) - 3(\tilde{Y}_1^* + \tilde{\Sigma} - \tilde{\Lambda} - \Lambda'); \tag{2}$$

$$2(\tilde{\Omega} - \tilde{N}^*) = 3(\tilde{\Xi}^* + \tilde{\Xi} - \tilde{Y}_1^* - \tilde{\Sigma}); \tag{3}$$

$$2(\tilde{\Omega} - \tilde{N}^*) = 3(\Sigma_\gamma + \Lambda_\gamma) - 6N_\gamma; \tag{4}$$