ever, $v_\mu{}^i$ and $(1/\eta)a_\mu{}^i$ rather than $v_\mu{}^i$ and $a_\mu{}^i$ themselves would form a U(6)⊗U(6) algebra [R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964); K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 13, 698 (1964)], so that we chose $\eta = 1$.

It may also be interesting to note that in $M(12)$ the $\pi \to \mu + \bar{\nu}$ decay amplitude $f_\pi$ and the $\rho \to e^+ + e^-$ amplitude $\gamma_\rho$ are related by $\gamma_\rho = mf_\pi$, giving a reasonable $\rho = e^+ + e^-$ decay rate (which depends on $m^2$).

[7]Our extrapolation procedure $(1+t/2Mm \to 1+m/2M)$ corresponds to model (2) of reference 3, where $\mu_\rho = 2M_B/m$ rather than $\mu_\rho = 1 + 2M_B/m^1$. In the context of the less successful model (1) of reference 3 (pole dominance of Pauli and Dirac form factors), universality would take the form (5) and relation (1) would not obtain.

[8]P. G. O. Freund and Y. Nambu, Phys. Letters 12, 248 (1964).

[9]In view of the large width of the axial-vector mesons it may be more adequate to treat them as particles with a spectral mass distribution rather than having a given mass (δ function as spectral function). It is not difficult to see that our arguments can be reproduced also in such a treatment, whereas the condition (16) would be replaced by a condition on the spectral function describing the $1^+$ meson. It is also good to remember that the $B$ meson, if its $J^P = 1^+$, would be an abnormal axial-vector meson and therefore could not belong to the supermultiplet considered here. It probably belongs to 405 or 189, in which case its small width ($\Gamma_B \ll 1$ BeV) can be accounted for.

[10]L. M. Brown and P. Singer, Phys. Rev. 133, B812 (1964); F. Crawford, Jr., L. Lloyd, L. Price, and E. Fowler, Phys. Rev. Letters 11, 564 (1963); M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. Henri, B. Jongjeans, D. Leith, G. Lynch, F. Muller, and J. Perreau, Phys. Letters 12, 255 (1964); L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters 14, 329 (1965).

[11]T. K. Kuo and T. Yao, Phys. Rev. Letters 13, 415 (1964); M. Bég and V. Singh, Phys. Rev. Letters 13, 418 (1964).

[12]N. N. Bogoliubov, N. Van Hieu, D. Stoianov, B. Struminsky, A. Tarkhelidze, and V. Shelest, to be published.

[13]It should be pointed out that the treatment of quarks in the paper of Bogoliubov et al. and in the present paper differ. In their paper they consider quarks obeying a Klein-Gordon rather than a Dirac equation. Whereas such an approach preserves $M(12)$ invariance, it also runs into trouble with the requirement of definite metric in Hilbert space unless the quarks obey para-Fermi statistics [see O. W. Greenberg, Phys. Rev. Letters 13, 598 (1964)] rather than Fermi statistics. In the approach of Bogoliubov et al., universality does not extend to quarks. For triality-zero hadrons (baryons, mesons) our arguments are independent of whether universality does or does not extend to quarks so that a compatibility requirement for the two approaches is fair in this case. Our mass formula therefore applies only for triality-zero hadrons.

---

# BACKWARD PION-NUCLEON SCATTERING AND SPIN DETERMINATION OF PION-NUCLEON RESONANCES

Richard M. Heinz and Marc H. Ross*

Physics Department, University of Michigan, Ann Arbor, Michigan
(Received 6 May 1965)

Recent measurements[1,2] of the pion-proton total cross section between 2.1 and 5.5 BeV/$c$ have revealed four new resonances, two in each of the isospin states. In this paper we propose a method of determining the spins of the isospin-$\frac{3}{2}$ resonances.

The behavior of the differential $\pi N$ elastic-scattering cross section near a resonant energy can indicate the spin of the resonance if the background amplitude with which the resonance amplitude interferes is known. However, this method of finding the resonance spin is experimentally feasible only when the magnitude of the background amplitude is not much larger than the magnitude of the resonance amplitude. For high-energy $\pi N$ scattering the background amplitude is too large in the forward (c.m.) hemisphere, but the situation may be favorable in the backward hemisphere.

The existing data[3,4] for $\pi^+ p$ elastic scattering above 2 Bev/$c$ show that the differential cross section reaches a very sharp peak at $\cos\theta = -1$ ($\theta$ is the c.m. scattering angle). This suggests that the scattering amplitude for $\cos\theta < 0$ is dominated by a baryon exchange. The exchange of an isospin-$\frac{1}{2}$ baryon ($N$) contributes to $\pi^+ + p \to \pi^+ + p$, but not to $\pi^- + p \to \pi^- + p$, while the exchange of an isospin-$\frac{3}{2}$ baryon ($N^*$) contributes to both. Since the cross section at $\cos\theta = -1$ for $\pi^- + p \to \pi^- + p$ is approximately $\frac{1}{5}$ of that for $\pi^+ + p \to \pi^+ + p$,[3-6] and since the $N^*$ exchange amplitude in $\pi^- + p \to \pi^- + p$ is attenuated by a factor of $\frac{1}{3}$ for $\pi^+ + p \to \pi^+ + p$ because of the isospin coupling, it is plausible to assume that the backward $\pi^+p$ amplitude is primarily due to $N$ exchange. We further assume that the amplitude due to the exchange of higher mass isospin-$\frac{1}{2}$ baryon systems is negligible compared to the nucleon. We correct the nucleon-exchange amplitude for initial- and final-state absorption. With these assumptions, we will adjust the undetermined $\pi NN$ vertex form factor so that the model agrees with the existing data at 4 Bev/$c$.[3,4] Near a resonant energy, we will add the resonance amplitude to the $N$-exchange amplitude and exhibit the interference as a function of the spin of the resonance.

The one-nucleon-exchange $\pi^+p$ amplitude is

$$A_N = (M/4\pi E)\bar{u}(p_2, s_2)\sqrt{2}g_0 F(t^2)\gamma_5(\not{t} - M + i\epsilon)^{-1} \times \sqrt{2}g_0 F(t^2)\gamma_5 u(p_1, s_1), \quad (1)$$

where $p_1$ and $s_1$ ($p_2$ and $s_2$) are the momentum and spin-polarization four-vectors of the incoming (outgoing) proton, $t$ is the four-momentum of the exchanged nucleon, $M$ is the nucleon mass, $E$ is the total c.m. energy, $F(t^2)$ is the vertex form factor $[F(M^2)=1]$, and $g_0$ is the $\pi NN$ coupling constant ($g_0^2/4\pi \approx 14$). The spin-flip part of $A_N$ is negligible compared to the spin-nonflip part. In Eq. (1), therefore, we choose $s_1$ and $s_2$ to correspond to spin-nonflip scattering, and no nucleon spin sum is performed. (Both helicity states for the incoming proton yield the same spin-nonflip $A_N$.) We take into account absorption in the initial and final states[7] by suppressing each partial wave in $A_N$ by $\eta_l$, where $\eta_l = \exp(-2\delta_l)$ and $\delta_l$ is the imaginary part of the $l$th partial wave $\pi^+p$ phase shift. To determine the $\eta_l$'s we assumed the scattering amplitude

$$\sum_l \frac{2l+1}{2k}(1-\eta_l)P_l(\cos\theta) = \frac{k\sigma_{\text{tot}}}{4\pi}\exp[-Ak^2(1-\cos\theta)]. \quad (2)$$

In Eq. (2), $k$ is the magnitude of the c.m. three-momentum, $\sigma_{\text{tot}}$ is obtained from references 1 and 2, and $A$ has been determined by Damouth, Jones, and Pere.[8] After projecting out the partial wave amplitudes from Eq. (1),

$$A_N \sum_l \frac{2l+1}{2} A_{N,l} P_l(\cos\theta), \quad (3)$$

the final amplitude is

$$A_N{}' = \sum_l \frac{2l+1}{2} A_{N,l}\eta_l P_l(\cos\theta). \quad (4)$$

An important result of the calculation is that $A_N{}'$ is real and negative for all $\theta$.

For the function $F(t^2)$, we used the form suggested by Ferrari and Selleri,[9]

$$F(t^2) = \frac{1-D}{1-(t^2-M^2/\alpha^2)} + D. \quad (5)$$

The differential cross section at 4 Bev/$c$ corresponding to the amplitude $A_N{}'$, using $D = 0.14$ and $\alpha = 340$ MeV in Eq. (5), is shown in Fig. 1 along with the existing experimental data. The values of the parameters in the form factor are very reasonable. The experimental point at $\cos\theta = -0.89$ represents the average cross section from $\cos\theta = -0.79$ to $\cos\theta = -1.0$. The average of the theoretical curve over this interval is in agreement with the experimental result. No attempt was made to fit the data with precision, since the results we obtain

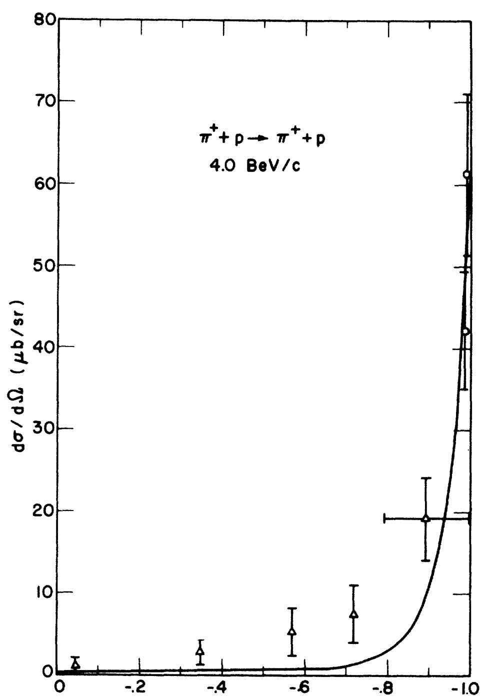


FIG. 1. One-nucleon-exchange model theoretical cross section for $\pi^+p$ backward elastic scattering at 4.0 BeV/$c$ pion laboratory momentum. The abscissa is the cosine of the c.m. scattering angle. The experimental data are taken from reference 3 (triangles) and reference 4 (circles).

are relatively insensitive to changes in $A_N'$.

For a resonance with a certain $j, l$, the non-spin-flip resonance scattering amplitude is

$$A_R^{NF} = \frac{1}{k} T_R \begin{bmatrix} l+1 \\ l \end{bmatrix} P_l(\cos\theta), \tag{6}$$

where the term in the brackets is $l+1$ if $j=l+\frac{1}{2}$, and $l$ if $j=l-\frac{1}{2}$; also,

$$T_R = \frac{\Gamma^\pi/2}{(E_0-E)-i\Gamma/2}. \tag{7}$$

In Eq. (7), $\Gamma$ and $E_0$ are the resonance width and energy and $\Gamma^\pi$ is the partial width for the resonance to decay into the $\pi^+p$ channel. The spin-flip resonance scattering amplitude is

$$A_R^F = \frac{1}{k} T_R \begin{bmatrix} +1 \\ -1 \end{bmatrix} \left[ l(l+1) \right]^{1/2} Y_l^1(\theta, \varphi). \tag{8}$$

We derive an expression for $\Delta\sigma_{\text{tot}}$, the total $\pi^+p$ cross section at $E=E_0$ minus the total cross section if there were no resonance, by adding the resonance amplitudes from Eqs. (6) and (8)-in which $T_R$ is pure imaginary, to the diffraction-scattering amplitude. One can show that a unitary amplitude is obtained by simply adding these contributions, in the limit of many channels.[10] We find

$$\Delta\sigma_{\text{tot}} = \frac{4\pi}{k^2} \begin{bmatrix} l+1 \\ l \end{bmatrix} (2-\eta_l) \frac{\Gamma^\pi}{\Gamma}. \tag{9}$$

This determines $\Gamma^\pi$, and the resonance amplitudes become

$$A_R^{NF} = \frac{k}{4\pi} \Delta\sigma_{\text{tot}} \frac{\Gamma/2}{(E_0-E)-i\Gamma/2} \frac{1}{2-\eta_l} P_l(\cos\theta), \tag{10}$$

and

$$A_R^F = \frac{k}{4\pi} \Delta\sigma_{\text{tot}} \frac{\Gamma/2}{(E_0-E)-i\Gamma/2} \frac{1}{2-\eta_l} \times \begin{bmatrix} (l+1)^{-1} \\ -l^{-1} \end{bmatrix} \left[ (l+1)l \right]^{1/2} Y_l^1(\theta, \varphi). \tag{11}$$

Using Eqs. (4), (10), and (11), $d\sigma/d\Omega$ for $\cos\theta < 0$ is

$$d\sigma/d\Omega = (A_N' + \text{Re}A_R^{NF})^2 + (\text{Im}A_R^{NF})^2 + |A_R^F|^2. \tag{12}$$

The effect of the resonance on $d\sigma/d\Omega$ will be the greatest at $E=E_0 \pm \Gamma/2$, where $|\text{Re}A_R^{NF}|$ is a maximum.

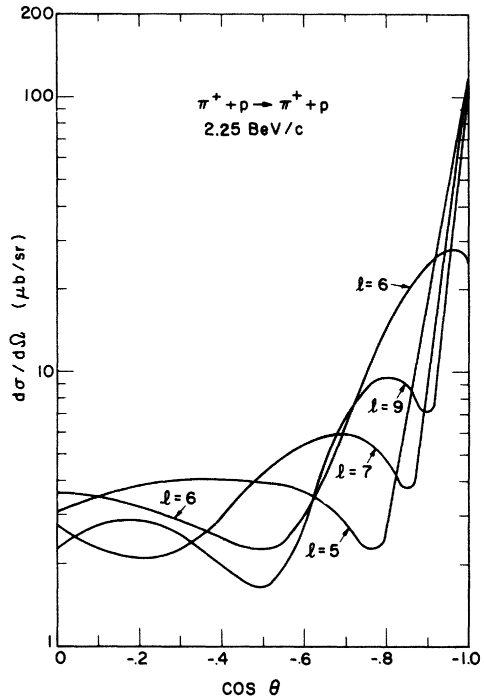


FIG. 2. Theoretical cross sections for $\pi^+p$ backward elastic scattering at 2.25 BeV/$c$ pion momentum. The curves are obtained from Eq. (12) for four different values of $l$ for the resonance at $E_0 = 2.36$ BeV.[1]

An important result of the calculation is a simple method of determining whether $l$ is odd or even. Since $P_l(-1) = (-1)^l$, if $l$ is even $\text{Re}A_R^{NF}$ is positive at $E=E_0-\Gamma/2$ ($\theta=180°$) and negative at $E=E_0+\Gamma/2$. Since $A_N'$ is negative, this results in constructive interference at $E=E_0+\Gamma/2$, and destructive interference at $E=E_0-\Gamma/2$. For $l$ odd, the interference is destructive at $E=E_0-\Gamma/2$. Thus, measuring $d\sigma/d\Omega$ ($\theta=180°$) near a resonant energy determines whether $l$ is even or odd. The value of $l$ is found from the angular distribution of $d\sigma/d\Omega$ from Eq. (12); however, the dependence of $d\sigma/d\Omega$ on whether $j=l+\frac{1}{2}$ or $j=l-\frac{1}{2}$, which is contained in $A_R^F$ [Eq. (11)], is very slight.

We will exemplify our procedure of finding $l$ by calculating $d\sigma/d\Omega$ near the $\pi^+p$ resonance at[1] $E_0 = 2.36$ BeV for several $l$'s. At $E=E_0-\Gamma/2$ (2.25 BeV/$c$ pion momentum), using $\Delta\sigma_{\text{tot}} = 2$ mb, $d\sigma/d\Omega$ is shown in Fig. 2 for $l=5$, 7, 9, and 6. The striking features of these angular distributions are the maxima and minima that occur for a specific $l$. After it has been determined whether $l$ is even or odd by measuring $d\sigma/d\Omega$ near 180°, the location of the maxima and minima in $d\sigma/d\Omega$ can be used to distinguish between $l$'s that differ by 2 or more. Figure 3 indicates how insensitive the positions of these extremes are to changes in $A_N'$; the $l=7$ curve at 2.25 BeV/$c$ is calculated for background amplitudes that are 50% smaller and larger than $A_N'$ from Eq. (4). The background amplitude could be consistent with experiment and be much less than 50% $A_N'$ from Eq. (4) in the region $\cos\theta \approx 0$ (but not near $\cos\theta \approx -1$). The shape of $d\sigma/d\Omega$ is sensitive to this ambiguity near $\cos\theta \approx 0$. However, the shape of $d\sigma/d\Omega$ in the region $\cos\theta < -0.7$ is very stable. Thus our method of determining $l$ does not depend significantly on the absorption parameters or form factor used in $A_N'$. Additional information can be obtained from $d\sigma/d\Omega$ at $E = E_0 + \Gamma/2$, although the effects observed in Fig. 2 are much less pronounced when the interference is destructive at $\cos\theta = -1$. Thus $d\sigma/d\Omega$ at $E = E_0 + \Gamma/2$ is most sensitive to $l$ when $l$ is even.

The results we have obtained depend quite strongly on our assumption that the background amplitude is primarily due to the one-nucleon exchange. However, this assumption results in very definite predictions. The one-nucleon-exchange amplitude is real and negative and primarily spin-nonflip. The resulting sharp peak in $d\sigma/d\Omega$ at $\theta = 180°$ becomes sharper as $E$ is increased [the value of $E^2 d\sigma(t^2)/dt^2$ is approximately energy independent], but $d\sigma/d\Omega$ ($\theta = 180°$) does not change much with $E$. Within the uncertainties of the absorption correction, our prediction at 8.0 BeV/$c$ is not in serious disagreement with the experimental results.[4] Once the $l$ of a resonance is known, experiments off resonance, on resonance, and at the half maxima could check this background amplitude in detail. When the background amplitude is well established, $j$ might also be determined.

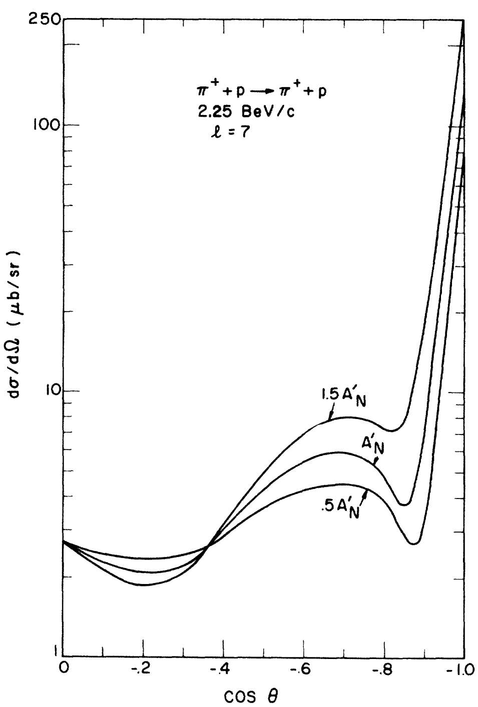


FIG. 3. Theoretical cross sections for $\pi^+ p$ backward elastic scattering at 2.25 BeV/$c$. $d\sigma/d\Omega$ is obtained from Eq. (12) for $l=7$ and three background amplitudes, $A_N'$ [Eq. (4)], 1.5 $A_N'$ and 0.5 $A_N'$.

The method we have developed for finding the $l$ of resonances may be in difficulty when applied to resonances at higher energies because the partial width $\Gamma^\pi$ is expected to decrease rapidly with increasing energy. However, the predicted resonance[2] at $E_0 = 2.825$ BeV (with $\Delta\sigma_{\rm tot} = 0.4$ mb) still shows the same effects seen in Fig. 2 in the region $\cos\theta > -0.8$ (i.e., away from 180° because the resonance amplitude is relatively small).

We wish to thank A. Krisch and D. Meyer for several discussions concerning the experimental data, and L. Jones[11] for his comments and support.

*Supported in part by the U. S. Office of Naval Research Contract No. Nonr 1224(23) and the U. S. Atomic Energy Commission.

[1]A. N. Diddens, E. W. Jenkins, T. F. Kycia, and K. F. Riley, Phys. Rev. Letters 10, 262 (1963).

[2]A. Citron, W. Galbraith, T. F. Kycia, B. A. Leontić, R. H. Phillips, and A. Rousset, Phys. Rev. Letters 13, 205 (1964).

[3]Aachen-Berlin-Birmingham-Bonn-Hamburg-London-Munich Collaboration, Phys. Letters 10, 248 (1964).

[4]R. Rubinstein, J. Orear, D. Scarl, D. White, A. Krisch, W. Frisken, A. Read, and H. Ruderman, Bull. Am. Phys. Soc. 10, 528 (1965); preliminary data.

[5]M. Perl, Y. Y. Lee, and E. Marquit, in Phys. Rev. 138, B707 (1965).

[6]C. Coffin, F. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams, Bull. Am. Phys. Soc. 10, 528 (1965).

[7]M. Ross and G. Shaw, Phys. Rev. Letters 12, 627 (1964); K. Gottfried and J. Jackson, Phys. Letters 8, 144 (1964); L. Durand, III, and Y. T. Chiu, Phys. Letters 12, 399 (1964).

[8]D. Damouth, L. Jones, and M. Perl, Phys. Rev. Letters 11, 287 (1963).

[9]E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450

(1963).
[10]M. Ross, to be published.
[11]M. Perl, L. Jones, and C. Ting, Phys. Rev. 132, 1252 (1963).

# SOME RARE DECAY MODES OF THE $\omega$ MESON AND A SEARCH FOR $C$-INVARIANCE VIOLATION*

Stanley M. Flatté,† Darrell O. Huwe, Joseph J. Murray, Janice Button-Shafer, Frank T. Solmitz, M. Lynn Stevenson, and Charles Wohl

Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 24 May 1965)

We present in this paper the following measurements of $\omega$ decay:

1. The widths of several decay modes have been determined:

(a) $\Gamma(\omega\to\text{neut.})/\Gamma(\omega\to\pi^++\pi^-+\pi^0)=(9.7\pm1.6)\times10^{-2}$;

(b) $\Gamma(\omega\to\eta(\text{all modes})+\text{neut.})/\Gamma(\omega\to\pi^++\pi^-+\pi^0)\leq1.7\times10^{-2}$;

(c) $\Gamma(\omega\to\pi^++\pi^-+\gamma)/\Gamma(\omega\to\pi^++\pi^-+\pi^0)<5\times10^{-2}$;

(d) $\Gamma(\omega\to\pi^++\pi^-)/\Gamma(\omega\to\pi^++\pi^-+\pi^0)=(0.17\pm0.03)^2=2.9\times10^{-2}$ ("complete coherence"), or $=(8.2\pm2.0)\times10^{-2}$ ("complete incoherence"). Glashow and Sommerfield[1] point out that $\omega\to\eta+\pi^0$ violates $C$ conservation. The branching ratio $b$ sets an upper limit of 0.14 MeV for $\Gamma(\omega\to\eta+\pi^0)$.

2. The $\omega\to\pi^++\pi^-+\pi^0$ decay has been tested for nonconservation of $C$ as suggested by Lee[2] and found to be consistent with $C$ conservation.

Following is a resume of experimental methods we used to determine these numbers. Details will be presented in a forthcoming article.[3]

Measurements have been completed on approximately 43 620 V + two-prong events in an exposure of the 72-inch hydrogen chamber to $K^-$ mesons with momenta between 1.2 and 1.8 BeV/$c$. Among these events we have identified 28 850 as $K^-+p\to\Lambda+2$ prongs, and of these, 10 242 are identified as

$$K^-+p\to\Lambda+\pi^++\pi^-+\pi^0. \quad (1)$$

By plotting the $M(\pi^+\pi^-\pi^0)$ histogram, we identify 4208 events as

$$K^-+p\to\Lambda+(\omega\to\pi^++\pi^-+\pi^0). \quad (2)$$

There are 1450 background events that lie in the $\omega$-peak region, 750 MeV $\leq M(\pi^+\pi^-\pi^0)\leq$ 815 MeV. In the control regions on either side of the peak region we find 1202 events in the interval 685 MeV $\leq M(\pi^+\pi^-\pi^0)\leq$ 750 MeV, and 1276 events in the interval 815 MeV $\leq M(\pi^+\pi^-\pi^0)\leq$ 880 MeV. We use these control-region events to estimate the behavior of the background events in the peak region.

Neutral decay mode.—All the measurements of 27 660 V + zero-prong events have been completed for the momentum settings 1.42, 1.51, 1.60, and 1.70 BeV/$c$. From these events, 12 351 have been identified as $K^-+p\to\Lambda$ + neutrals. Figure 1 is the histogram of the invariant mass squared of the neutral system that recoils against the $\Lambda$; we refer to this as the square of the missing mass. Peaks for the $\pi^0$, $\eta$, and $\omega$ mesons are clearly visible. Few of the $K^-$'s in this experiment had momenta

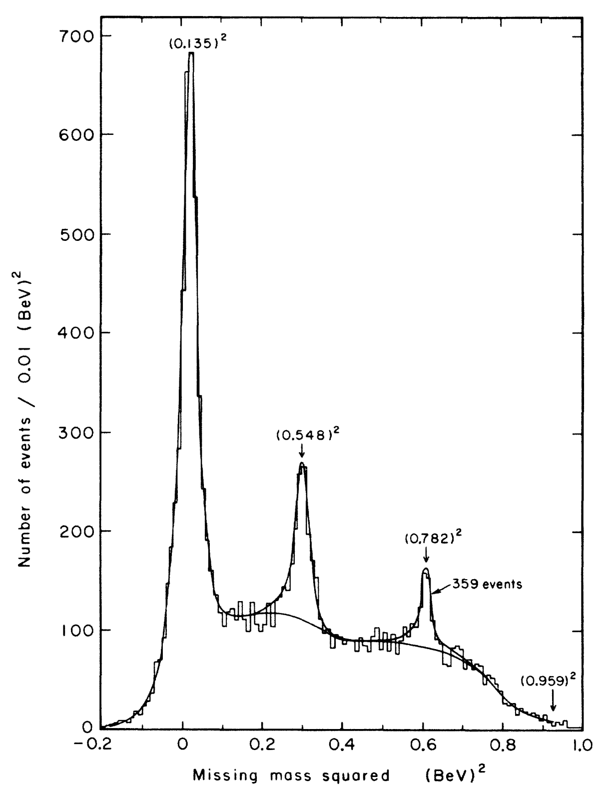


FIG. 1. Missing-mass squared distribution for 12 351 events of $K^-+p\to\Lambda$ + neutrals at 1.4, 1.5, 1.6 and 1.7 BeV/$c$.