density operators. The important point is the identification of the tensor character of the relevant dynamical variable, and the particle multiplet furnishing the representation. In such a case we have, in fact, the Hermitian representation of a (finite-dimensional) Lie algebra A. We could, therefore, equally well define the unitary representation of the Lie group G, by exponentiating the representation for A. And in the case of a compact Lie algebra the Hermitian representations are completely reducible into finite-dimensional Hermitian representations.

Given the current algebra and the multiplet we will, in general, still have ambiguities about the identification of their transformation properties. For example,[6] the SU(4) current algebra acting on a multiplet containing the nucleon can treat the nucleon as 4 or $\bar{4}$ or as part of the 20 or $\overline{20}$. We could eliminate the first two alternatives, if we so choose, by normalizing the isotopic spin current matrix elements of the nucleon resonances. Even then the last two alternatives are equally good and give an ambiguous prediction for the ratio of proton and neutron magnetic moments.[7] It is easy to show that this ambiguity is associated with the (outer) automorphism of the SU(4) current algebra generated by the extended charge-conjugation operator. The relevant observation is that this ambiguity is equally well present in the equivalent group-theoretic formulation.

We conclude that the formulation in terms of the algebra of currents is equivalent to the specific formulation of the group-theoretic scheme discussed above. In neither case is the invariance of the Hamiltonian relevant, and in either case the necessary physical assumptions are the same. It is perhaps appropriate to seek further elaborations of the group-theoretic formulation in the language of the algebra of the currents.

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[1]For a progress report, see Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, January 1965 (W. H. Freeman & Company, San Francisco, California, to be published).

[2]See, in particular, the discussion remark by J. R. Oppenheimer in reference 1.

[3]M. Gell-Mann, Physics 1, 63 (1964); R. P. Feynman, M. Gell-Mann, and G. Zweig, Phys. Rev. Letters 13, 678 (1964).

[4]B. W. Lee, Phys. Rev. Letters 14, 673, 850 (1965); C. H. Woo and A. J. Dragt, to be published; C. Ryan, to be published; S. Okubo, to be published.

[5]For some reason, this aspect of the SU(6) couplings is rarely mentioned!

[6]I thank Dr. C. H. Woo for a very helpful correspondence.

[7]C. Ryan, reference 4.

SUPERMULTIPLET SCHEMES AND MESON POLE MODELS FOR ELECTROMAGNETIC FORM FACTORS

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Supermultiplet schemes combining spin and internal symmetries generally lead to rather specific predictions for electromagnetic[1] and weak-interaction form factors. Some of these predictions depend only upon the assumed transformation properties of the electromagnetic or weak couplings. These are the predictions

$$G_M^{\,p}(q^2) = -\tfrac{3}{2}G_M^{\,n}(q^2), \quad G_E^{\,n}(q^2) \equiv 0, \tag{1}$$

where G_M and G_E are the Sachs form factors. Other predictions depend upon specific meson pole-dominance models:

$$\mu_p \approx \left(1 + \frac{2m}{\mu}\right), \quad \frac{G_M^{\,p}(q^2)}{G_E^{\,p}(q^2)} \approx \frac{1}{2m}\,\frac{1 + 2m/\mu}{1 - q^2/2m\mu}, \tag{2}$$

where m and μ are the central baryon and odd-parity meson masses, and μ_p is the total magnetic moment of the proton. We wish to point out a special feature of the pole-dominance model, which results in the possibility of one alternative set of predictions, namely

$$\mu_p \approx \frac{2m}{\mu}, \quad \frac{G_M^{\;p}(q^2)}{G_E^{\;p}(q^2)} \approx \frac{\mu_p}{2m}, \quad \left(\frac{\mu_p}{2m}\right)^2 \approx \frac{\langle r_p^{\;2}\rangle}{6}, \tag{3}$$

where r_p is the proton charge rms radius.

We are interested in the electromagnetic vertex for baryons and for SU(3) triplets (quarks). Since the calculations in both cases are quite analogous, we restrict our exposition to the baryon octet. As far as symmetries are concerned, we use the spurion scheme of broken $M(12)$ symmetry[2] $[M(12) \equiv \tilde{\mathrm{U}}(12) \equiv \mathrm{U}_{\mathfrak{L}}(12)]$. Since strict $M(12)$ symmetry[3] is a special case of this scheme, it also gives the final results obtained from the spurion scheme.

In general, the spurion scheme leads to the electromagnetic vertex

$$\bar{\Psi}_{ABC}(p')[\{i\gamma_\alpha F_1(q^2) + i\sigma_{\alpha\beta}q_\beta F_2(q^2)\}(1+q^2/4m^2)^{-1} \times \tfrac{1}{2}(\lambda_3 + \lambda_8/\sqrt{3})]_{A'}{}^{A}\Psi^{A'BC}(p), \tag{4}$$

where $q = p - p'$, and the factor $(1+q^2/4m^2)^{-1}$ has been introduced in order that upon the expansion of the expression, the functions F_1 and F_2 coincide with the Dirac and Pauli form-factors of the proton, respectively. The corresponding Sachs form factors are given by the formulas

$$G_E^{\;p}(q^2) = [F_1(q^2) - (q^2/2m)F_2(q^2)],$$

$$G_M^{\;p}(q^2) = (1/2m)[F_1(q^2) + 2mF_2(q^2)]. \tag{5}$$

In order to obtain a pole model for the electromagnetic vertex, we consider the meson-baryon coupling

$$i\bar{\Psi}_{ABC}(p')\Psi^{A'BC}(p)[g_0\bar{\Phi}_{A'}{}^{A}(q) + g_1(1 - i\gamma\cdot q/\mu)_D{}^{A}\bar{\Phi}_{A'}{}^{D}(q)], \tag{6}$$

where we have neglected a singlet term which is not relevant for our purpose, and insert

$$\bar{\Phi}(q) \to (1 + i\gamma\cdot q/\mu)(\gamma\cdot e)\tfrac{1}{2}(A_3 + \lambda_8/\sqrt{3})\frac{f\mu^2}{\mu^2+q^2}, \tag{7}$$

where f is the meson-photon transition amplitude.

A priori we do not require $q^2 = -\mu^2$ in the coefficients, but if we do so, the second term in Eq. (6) evidently vanishes.

We may now bring the resulting expression into the form (4) and obtain

$$F_1(q^2) \approx \mu F_2(q^2) \approx fg_0\frac{\mu^2}{\mu^2+q^2}\left(1 + \frac{q^2}{4m^2}\right). \tag{8}$$

On the other hand, we may bring Eqs. (6) and (7) into a form corresponding to the vertex (4), but with the bracket $\{\cdots\}$ replaced by

$$\frac{1}{2m}\left\{[G_E(q^2)P_\mu - G_M(q^2)ir_\mu]\left(1 + \frac{q^2}{4m^2}\right)^{-1}\right\}, \tag{9}$$

where $P = p + p'$ and $r_\alpha = \epsilon_{\alpha\beta\gamma\sigma}P_\beta q_\gamma \gamma_5\gamma_5$.

We then obtain, as is well known,

$$G_E(q^2) \approx fg_0\frac{\mu^2}{\mu^2+q^2}\left(1 - \frac{q^2}{2m\mu}\right)\left(1 + \frac{q^2}{4m^2}\right),$$

$$G_M(q^2) \approx fg_0\frac{\mu^2}{\mu^2+g^2}\;\frac{1}{2m}\left(1 + \frac{2m}{\mu}\right)\left(1 + \frac{q^2}{4m^2}\right). \tag{10}$$

Except for the factor $(1+q^2/4m^2)$, which will be discussed later, we now take the residue in Eq. (10) at $q^2 = -\mu^2$ and then extrapolate. Thus the factor $(1 - q^2/2m\mu)$ becomes $(1 + \mu/2m)$, and we find

$$G_E(q^2) \approx \mu G_M(q^2) \approx fg_0\frac{\mu^2}{\mu^2+q^2}\left(1 + \frac{\mu}{2m}\right)\left(1 + \frac{q^2}{4m^2}\right). \tag{11}$$

This possibility is the central point of our argument.

The two versions (8) and (11) of the pole model give quite different results for the form factors of baryons and quarks:

(1) If we make the pole approximation in the F functions, we can calculate the Sachs form factors from Eq. (8) and obtain[5] the predictions (2). With $fg_0 = 1$, the normalization $G_E^{\;p}(0) = F_1(0) = 1$ is in order.

(2) Requiring pole dominance for the G functions, we obtain directly from Eq. (11) the predictions (3), provided we choose $fg_0 = (1 + \mu/2m)^{-1}$ so that $G_E^{\;p}(0) = 1$.[6] In this model the Dirac-Pauli form factors must be computed from Eq. (5) which yields

$$F_1(q^2) \approx \left(1 + \frac{q^2}{2m\mu}\right)\frac{\mu^2}{\mu^2+q^2},$$

$$F_2(q^2) \approx \frac{1}{2m}\left(\frac{2m}{\mu} - 1\right)\frac{\mu^2}{\mu^2+q^2}; \tag{12}$$

we note that the condition $F_1(0)=1$ is satisfied.

The model (2) described above seems to be preferable from the phenomenological point of view because electron scattering experiments indicate that $G_M{}^p/G_E{}^p$ is essentially q^2-independent.[7] Also, the value $\mu_p = 2m/\mu$ for the total magnetic moment of the proton may be more favorable than $\mu_p = 2m/\mu$ in view of the experimental value $\mu_p = 2.79$ and the inequality $\mu < m$. Furthermore, the prediction[8] $(\mu_p/2m)^2 \approx \langle r_p{}^2\rangle/6$, which is obtained from Eq. (11) with the approximation $1/4m^2 \ll 1/\mu^2$, is in reasonable agreement with experiments.

We have not yet discussed the factor $(1+q^2/4m^2)$, which appears in the expressions for the form factors $G(q^2)$, etc. Although it may not be reasonable to extrapolate the pole-model expressions to $q^2=-4m^2$, if we nevertheless do so, we must consider the condition $G_E(-4m^2) = 2m\,G_M(-4m^2)=0$ for the baryon form factors. This condition follows from the requirement $G_E = 2m\,G_M$ at $q^2=-4m^2$, and the fact that G_E and G_M have different d/f ratios.[9] It may be reasonable, therefore, to retain the factor $(1+q^2/4m^2)$, even in case (2).

Considerations analogous to those given above for baryons can be made for quarks. A model of the type (1) gives $\mu_{Q\,\mathrm{anom}}/\mu_{p\,\mathrm{anom}} \approx e_Q/e$, whereas, with model (2), we obtain $\mu_Q/\mu_p \approx e_Q/e$. The factor $(1+q^2/4m^2)$ appearing in Eqs. (8) and (11) is not present in the case of quark form factors. For model (2) this would imply $G_E{}^Q/G_M{}^Q \approx \mu$ at $q^2=-4m^2Q$, and in order not to violate the requirement $G_E{}^Q - 2m_Q G_M{}^Q = 0$ at this point, we should introduce a q^2-dependent meson-quark form factor, which vanishes at $q^2 = -4m_Q{}^2$. Perhaps this is the reason for the suppression of the reaction $p+\bar{p} \to Q + \bar{Q}$.

Summing up, we see that the meson pole models of electromagnetic form factors in $M(12)$ theories give different results depending upon whether pole dominance is required for the Dirac and Pauli form factors (F), or for the Sachs (G) form factors. It appears that we obtain results which are empirically more favorable[10] if the pole dominance is assumed for the Sachs form factors.

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[1]See, for example, M. A. B. Bég, B. W. Lee, and A. Pais, Phys. Rev. Letters 13, 514 (1964); B. Sakita, Phys. Rev. Letters 13, 643 (1964); J. M. Charap and P. T. Matthews, Phys. Letters 13, 346 (1964); K. J. Barnes, P. Carruthers, and F. von Hippel, Phys. Rev. Letters 14, 82 (1965); M. Gell-Mann, Phys. Rev. Letters 14, 77 (1965); A. Salam, R. Delbourgo, and T. Strathdee, Proc. Roy. Soc. (London) A284, 146 (1965); B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 405 (1965); P. G. O. Freund, Phys. Letters 15, 352 (1965), and to be published; R. Oehme, Phys. Rev. Letters 14, 664 (1965), and to be published.

[2]Oehme, reference 1, and to be published; Freund, reference 1.

[3]K. Bardakci, J. M. Cornwall, P. G. O. Freund, and B. W. Lee, Phys. Rev. Letters 14, 48 (1965); Salam, Delbourgo, and Strathdee, reference 1; M. A. B. Bég and A. Pais, Phys. Rev. Letters 14, 264 (1965); Sakita and Wali, reference 1.

[4]F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. 119, 1105 (1960); R. G. Sachs, Phys. Rev. 126, 2256 (1962).

[5]Salam, Delbourgo, and Strathdee, reference 1; Sakita and Wali, reference 1, and Phys. Rev. (to be published).

[6]A recent paper by K. J. Barnes [Phys. Rev. Letters 14, 798 (1965)] describes a meson pole model of electromagnetic form factors which is based upon an algebraic structure called $P\tilde{U}(4)$. This scheme gives results corresponding to those of our case (2). The reason is the one we have described in the text; namely, the implicit requirement of pole dominance for the G form factors. The same scheme can also be adapted to our case (1), and it gives then the corresponding results.

[7]For a survey see, for example, Robert R. Wilson and J. S. Levinger, Ann. Rev. Nucl. Sci. 14, 135 (1964); L. N. Hand, D. C. Miller, and R. Wilson, Rev. Mod. Phys. 85, 335 (1963); also A. P. Balachandran, P. G. O. Freund, and C. R. Schumacher, Phys. Rev. Letters 12, 209 (1964). These papers contain further references.

[8]R. Dashen and M. Gell-Mann, private communication by R. Dashen; B. W. Lee, Phys. Rev. Letters 14, 676 (1965).

[9]Y. Hara, Phys. Rev. Letters 14, 603 (1965).

[10]In both models we may multiply all form factors by an arbitrary but common function of q^2 with the required analyticity properties. We could then consider these form factors for large values of q^2, where we find in model (1) $F_2(q^2)/F_1(q^2) \to \mu_{p\,\mathrm{anom}}$, $G_M(q^2)/G_E(q^2) \to 0'$; whereas, in model (2) we have $F_2(q^2)/F_1(q^2) \to 0$ and $G_M(q^2)/G_E(q^2) \to \mu_p$. Again the experimental evidence favors model (2).