ANTIPROTON-PROTON ANNIHILATION AT REST INTO TWO PSEUDOSCALAR MESONS AND SU(6) SYMMETRY

Michiji Konuma*

Scuola Normale Superiore, Pisa, Italy, and Centro Ricerche Fisica e Matematica, Pisa, Italy

and

Ettore Remiddi

Scuola Normale Superiore, Pisa, Italy

(Received 19 May 1965)

In this note we discuss, in the SU(6) symmetry model,[1-3] the proton-antiproton annihilation at rest into two pseudoscalar mesons:

$$\bar{p}+p\to M_1+M_2. \qquad (1)$$

This problem has been recently considered by Dyson and Xuong,[4] including in the final state all the mesons of the 35-plet. As our treatment and results are different from theirs, however, we think it would be worthwhile to report them.

For annihilation at rest, we take the initial and final states in Reaction (1) as ${}^3S_1$ and P wave, respectively.[5] Considering that the final two-meson state must be symmetrical under the exchange of the two particles, we see that in the odd angular momentum case it must be antisymmetrical in the SU(6) part, which therefore must, in general, be of the form

$$M_\alpha{}^\beta(1)M_\gamma{}^\delta(2)-M_\gamma{}^\delta(1)M_\alpha{}^\beta(2). \qquad (2)$$

The matrix element A for (1) can be written as

$$A=\chi_i{}^\dagger(\vec{\sigma}\vec{q})_j{}^i\chi^j\omega, \qquad (3)$$

where $\chi_i{}^\dagger, \chi^j$ are the antiproton and proton two-component spinors, $\vec{q}$ is the relative momentum of the mesons, and ω is a suitable expression involving internal SU(3) quantities. We introduce the hypothesis of minimal SU(6) violation by requiring that $\chi_i{}^\dagger\chi^j\omega$ behaves like the $(\underline{1}, \underline{3})$ component of the 35-plet.

As in reference 4, we complete $(\vec{\sigma}\vec{q})$ to a spurion $Q_\nu{}^\mu$ of the structure of the 35-plet of SU(6) by defining it as

$$Q_\nu{}^\mu=(\vec{\sigma}\vec{q})_n{}^m\delta_N{}^M$$

[with the usual correspondence $\mu=(m,M)$ between SU(6) and SU(2), SU(3) indices], so that A becomes formally SU(6) invariant. C invariance can be taken into account for the initial $S_3=0$ state by imposing

$$A=-A^T, \qquad (4)$$

where A^T means transpose of A in SU(6) indices.

There are eighteen 35-plets which can be constructed out of $\underline{56}\otimes\underline{\overline{56}}\otimes\underline{35}\otimes\underline{35}$, corresponding to the product of $\bar{B}_{\alpha\beta\gamma}$, $B^{\alpha\beta\gamma}$, $M_\alpha{}^\beta(1)$, and $M_\alpha{}^\beta(2)$ multiplets. Requiring the conditions (2) and (4) to hold, however, we are left with just four terms[6]:

$$I_{1\mu}{}^\nu=\bar{B}_{\alpha\beta\mu}B^{\alpha\gamma\nu}[M_\delta{}^\beta(1)M_\gamma{}^\delta(2)-M_\gamma{}^\delta(1)M_\delta{}^\beta(2)],$$

$$I_{2\mu}{}^\nu=\tfrac{1}{2}\{\bar{B}_{\alpha\beta\mu}B^{\alpha\beta\gamma}[M_\gamma{}^\delta(1)M_\delta{}^\nu(2)-M_\delta{}^\nu(1)M_\gamma{}^\delta(2)]-\bar{B}_{\alpha\beta\gamma}B^{\alpha\beta\nu}[M_\delta{}^\gamma(1)M_\mu{}^\delta(2)-M_\mu{}^\delta(1)M_\delta{}^\gamma(1)]\},$$

$$I_{3\mu}{}^\nu=\bar{B}_{\alpha\beta\gamma}B^{\alpha\beta\delta}[M_\mu{}^\gamma(1)M_\delta{}^\nu(2)-M_\delta{}^\nu(1)M_\mu{}^\gamma(2)],$$

and

$$I_{4\mu}{}^\nu=\tfrac{1}{2}\{\bar{B}_{\alpha\beta\mu}B^{\alpha\gamma\delta}[M_\gamma{}^\beta(1)M_\delta{}^\nu(2)-M_\delta{}^\nu(1)M_\gamma{}^\beta(2)]-\bar{B}_{\alpha\beta\gamma}B^{\alpha\delta\nu}[M_\delta{}^\beta(1)M_\mu{}^\gamma(2)-M_\mu{}^\gamma(1)M_\delta{}^\beta(2)]\},$$

so that, in general,

$$A=\sum_{i=1}^{4}g_iI_{i\mu}{}^\nu Q_\nu{}^\mu. \qquad (5)$$

Explicit calculation shows that I_4 does not contribute to final two-pseudoscalar-meson state, while I_2 and I_3 contributions are proportional. The final expression for the matrix elements is

$$A(\pi^+\pi^-) = g_1 + 5(g_2 + g_3),$$
$$A(K^+K^-) = -g_1 + 4(g_2 + g_3),$$
$$A(K^0\bar{K}^0) = -2g_1 - (g_2 + g_3) \qquad (6)$$

(apart from an irrelevant factor), where $A(\pi^+\pi^-)$ stands for the matrix element between the initial $\bar{p}p$ and the final $\pi^+(1)\pi^-(2) - \pi^-(1)\pi^+(2)$ state, etc. [see Eq. (2)]. Of course, $A(\pi^0\pi^0) = A(\eta\eta) = A(\pi^0\eta) = 0$, as it should be.

From (6) we obtain the triangular relation[7]

$$A(\pi^+\pi^-) - A(K^+K^-) + A(K^0\bar{K}^0) = 0. \qquad (7)$$

From the experimental data of reference 5, if we take the rate for Reaction (1) to be proportional to the meson momentum times the squared absolute value of the amplitude, we get, in arbitrary units, the following values:

$$|A(\pi^+\pi^-)| = 1.99 \pm 0.10,$$
$$|A(K^+K^-)| = 1.32 \pm 0.19,$$
$$|A(K^0\bar{K}^0)| = 0.87 \pm 0.06.$$

Equation (7) seems very well satisfied by these quantities.

We conclude that, at least for annihilation at rest into two pseudoscalar mesons, the SU(6)-symmetry model seems to give a rather satisfactory agreement with experiments.

The authors wish to thank Professor L. A. Radicati for constant encouragement, and Dr. R. Ferrari for useful comments and clarifying discussions. One of the authors (M.K.) wishes also to express his gratitude to Professor G. Bernardini and Professor L. A. Radicati for their hospitality at the Scuola Normale Superiore di Pisa.

---

*On leave of absence from the Department of Physics, University of Tokyo, Tokyo, Japan.

[1]F. Gürsey and L. A. Radicati, Phys. Rev. Letters 13, 173 (1964).

[2]A. Pais, Phys. Rev. Letters 13, 175 (1964).

[3]B. Sakita, Phys. Rev. 136, B1756 (1964).

[4]F. J. Dyson and N. Xuong, Phys. Rev. Letters 14, 654 (1965).

[5]R. Armenteros, L. Montanet, D. R. O. Morrison, S. Nilsson, A. Shapira, J. Vandermeulen, Ch. d'Andlau, A. Astier, J. Ballam, C. Ghesquière, B. P. Gregory, D. Rahm, P. Rivet, and F. Solmitz, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 351.

[6]The terms I_2 and I_4, apart from exchange antisymmetrization, correspond to the two independent terms of Eq. (4) in reference 4.

[7]It should be noted that Eq. (5) is fully symmetrical as far as the SU(3) part is concerned. In SU(3) symmetry alone, however, there is no relation for charged meson production, so Eq. (7) is a pure SU(6) prediction. We recall that the only prediction in SU(3) involves apparently S-wave neutral mesons, and gives no contribution in the low-energy region. See M. Konuma and Y. Tomozawa, Phys. Rev. Letters 12, 425 (1964).

EQUIVALENCE OF THE CURRENT ALGEBRA AND GROUP-THEORETIC FORMULATIONS IN PARTICLE PHYSICS*

E. C. G. Sudarshan

Physics Department, Syracuse University, Syracuse, New York

(Received 1 June 1965)

Recent developments in particle physics have led to a phenomenological but quantitatively successful correlation of a variety of properties of strongly interacting particles (including the low-lying resonances). The success of this approach raises fresh problems,[1] since the symmetry groups underlying these systemizations were by no means symmetry groups of the Hamiltonian of the system[2]; the apparent violations of the symmetry were sufficiently large to invalidate any traditional method of computing the effects of symmetry violation. And yet the simpler predictions of these (unitary) symmetry groups have had significant success. With a view to avoiding this embarrassment, several people have attempted to depart from a conventional approach to symmetry and to try to base it on the algebra of certain ("current") operators.[3] The formulation involving the (space)-integrated currents has been used recently[4] to rederive several results of the SU(6) and SU(4) theories in a more direct fashion. It is the purpose of this note to show that insofar as one uses only the integrated currents, the formulation in terms of the algebra of currents is completely equi-