Λ_3-Λ_1 are expected.[11] The transition $\Gamma_{25'}$-Γ_{15} is expected near 3.0 eV. For both silicon and germanium additional structure is seen in the vicinity of 4.3 eV corresponding to the transitions X_4-X_1 and Σ_4-Σ_1.[10,11]

In copper, there is strong structure observed in the regions 1.9-2.3 V and 3.8-5.2 V, including abrupt changes in slope near 1.9, 3.8, and 4.9 eV. In the vicinity of 4.9 eV, transitions X_5-X_4, Σ_3-Σ_1, and between the two bands belonging to Σ_1 are expected. In the vicinity of 3.8 eV, the transition Σ_4-Σ_1 is expected, and the structure near 2 V is almost certainly related to transitions, from $Q+$, $Q-$, and Δ_5 to the Fermi surface.[12] Structure over this energy range has also been observed by reflectivity.[13]

This Letter describes piezoreflectance measurements 100 times more sensitive than previous results. This increase makes piezoreflectance measurements applicable to a much larger number of materials. The experimental data can be used to determine interband transition energies and deformation potentials, and can provide information about the location of the transitions in the Brillouin zone. The result of this experiment indicates that a similar peizo-optical transmission experiment is possible.

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RELATIVISTIC ENERGY BANDS FOR LEAD BY THE RELATIVISTIC AUGMENTED PLANE-WAVE METHOD*

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A method for calculating relativistic energy bands was recently developed by the author.[1] This method, which can be thought of as a generalization of the augmented plane-wave method,[2] was shown to give accurate results for a transition element in its first application to tungsten.[3] For that metal it was found that the spin-orbit splitting of degenerate levels as predicted by those calculations was in quantitative agreement with experimental results.

In this Letter we present the results of an ab initio energy-band calculation for lead using the relativistic augmented plane-wave (RAPW) method. These results are shown as solid curves in Fig. 1 and will be compared with the Fermi surface and band structure proposed by Anderson and Gold.[4-6] On the basis of de Haas–van Alphen measurements, these authors have given a very complete description of the Fermi surface using a pseudopotential interpolation scheme in which the spin-orbit interaction was taken into account. This parametrized Fermi surface was also found to be in very good agreement with experimental results obtained by other workers. The energy bands predicted by Anderson and Gold[6] are shown as dashed curves in Fig. 1 for comparison with the present results.

The theoretical Fermi energy was chosen such that intersections with the bands would give the best possible agreement with the parametrized Fermi surface; this surface was shown to very closely satisfy the requirement of equal hole and electron volumes. To facilitate comparison, the zero of energy for the RAPW bands has been shifted to bring the Fer-

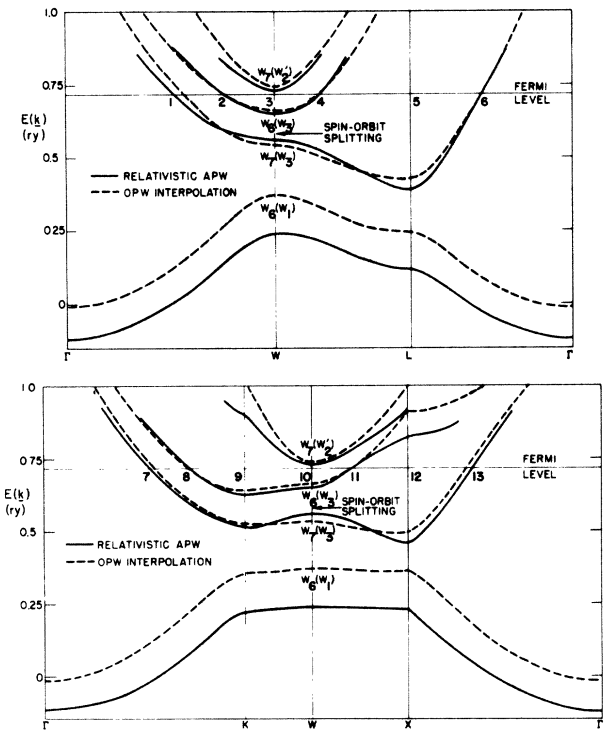

FIG. 1. Relativistic energy bands for lead using the RAPW method (solid curves) and energy bands from the four-parameter model used by Anderson and Gold[6] to fit experimental de Haas–van Alphen measurements (dashed curves).

mi levels for the two sets of bands into coincidence. The various dimensions of the Fermi surface as indicated on Fig. 1 are compared with those of the parametrized model in Table I. The agreement between these results can be seen to be very close. In addition there are two general features of the parametrized model which are retained in the theoretical bands: There is an energy gap between the first and second bands, and the fourth zone is empty. There are, however, noticeable differences in the two sets of energy bands. These are to be expected for several reasons. Firstly, only four orthogonalized plane waves were used in the parametrized fit of the Fermi surface to the experimental data, as compared to a basis set of 39 RAPW's in the theoretical calculation. Secondly, the fitting parameters in the pseudopotential scheme were determined from experimental information which reflects the electronic structure only in the immediate vicinity of the Fermi energy. For these reasons one would not necessarily expect the parametrized model to represent accurately the states away from the Fermi energy. In this respect it is perhaps unfortunate that the largest spin-orbit splitting occurs beneath the Fermi energy. The only degenerate level (nonaccidental) which is split by this interaction is the level W_3. Since this level is below the Fermi energy, the splitting is not directly reflected in the Fermi surface. There are, however, several bands around the symmetry points W, K, and X which (although nondegenerate) are spread further apart by the spin-orbit interaction, and this spreading certainly has an over-all effect on the Fermi surface. Finally, since no attempt was made toward self-consistency in the theoretical calculation, there is sufficient uncertainty in the potential to account for the differences between the two sets of results.

Table I. Dimensions of lead Fermi surface (a.u.$^{-1}$).

Dimension	Relativistic APW	Anderson and Gold
1-3	0.356	0.318
2-3	0.178	0.181
3-4	0.158	0.157
5-6	0.259	0.250
7-9	0.338	0.318
8-9	0.184	0.199
10-11	0.146	0.141
12-13	0.239	0.206

This is the first application of the RAPW method to a nearly free-electron metal. The agreement with experiment is very good, as it was in the first application of the method to the transition element tungsten. The method can therefore be recommended for theoretical studies of the electronic structures of any of the heavier elements.

It has been my pleasure to have many interesting discussions on this subject with Dr. J. R. Anderson and Dr. A. V. Gold. Also the cooperation of Dr. J. T. Waber at Los Alamos Scientific Laboratory in providing the results of relativistic atomic calculations for constructing the crystal potential is greatly appreciated.

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$K\bar{K}\pi$ RESONANCE AT 1280 MeV*

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The $K\bar{K}(n\pi)$ final states produced by annihilation of stopped antiprotons have been analyzed systematically by Armenteros et al.[1] They observed a strong enhancement at $M \simeq 1410$ MeV with $\Gamma \simeq 60$ MeV in the effective-mass distribution for the neutral $K\bar{K}\pi$ combinations from the reactions

$$\bar{p} + p \rightarrow K_1^0 + K^{\pm} + \pi^{\mp} + \pi^{+,0} + \pi^{-,0}$$

and

$$\rightarrow K_1 + K_{1,2} + \pi^0 + \pi^+ + \pi^-.$$

Since no analogous effect was apparent in either the singly or doubly charged combinations also accessible, they concluded that the enhancement at 1410 MeV most likely resulted from the production and subsequent decay of an unstable state (E meson) with isotopic spin $I = 0$; no determination of J^P was possible.[1] In this Letter we report the observation of a similar enhancement in the $K\bar{K}\pi$ systems produced in $\pi^- p$ interactions. The same final states show an additional peak in the neutral $K\bar{K}\pi$ combinations at $M = 1280 \pm 10$ MeV. We interpret this peak as evidence for a new $I = 0$ state (D meson) and discuss possible $I^G J^P$ assignments.

In a continuing study of resonant states produced in $\pi^- p$ interactions over the momentum interval 1.7 to 4.2 BeV/c, we have obtained 1062 events whose best fits are to the hypotheses

$$\pi^- + p \rightarrow K^+ + \bar{K}^0 + \pi^- + n, \quad (1a)$$

$$\rightarrow K^0 + K^- + \pi^+ + n, \quad (1b)$$

$$\rightarrow K^0 + \bar{K}^0 + \pi^- + p, \quad (1c)$$

$$\rightarrow K^0 + K^- + \pi^0 + p, \quad (1d)$$

$$\rightarrow K^+ + K^- + \pi^- + p. \quad (1e)$$

These final states are of particular interest since they may represent important decay modes for unstable mesons whose decay into two or three pions is forbidden. The data are summarized in Table I. In all cases track ionization on the film was checked for consistency with the calculated fits.

Table I. Momentum distribution for final states used in this analysis.

Final state	Momentum interval (BeV/c) 1.8 to 2.7	2.7 to 3.3	3.8 to 4.3	1.8 to 4.3
$K^+\bar{K}^0\pi^- n$	31	137	77	245
$K^0 K^- \pi^+ n$	8	116	92	216
$K^0\bar{K}^0\pi^- p$	20	159	103	282
$K^0 K^- \pi^0 p$	15	129	60	204
$K^+ K^- \pi^0 p$	9	76	30	115

The $M(K\bar{K}\pi)$ distribution for the charged com-

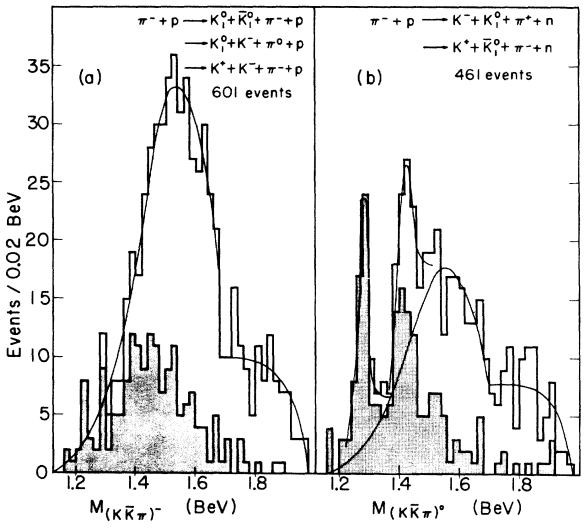

FIG. 1. Effective mass distributions for the charged and neutral $K\bar{K}$ combinations. To illustrate the difference in structure, events with $M(K\bar{K}) \leq 1.1$ BeV are shown separately in the shaded areas.