The results of such a two-mode experiment[12] are shown in Fig. 2, in which the $l=2$ and $l=3$ modes, respectively, play the roles of mode 1 and mode 2 in Fig. 1. In the top picture neither of these modes is Landau damped, but the $l=1$ mode (driven by the nonlinear interaction of the $l=2$ and $l=3$ modes) is heavily Landau damped; energy flows from the two high-frequency modes into the $l=1$ mode so that the mode-coupling coefficients a_{ij} in (1) are negative, as noted above.[13] In the center picture the $l=2$ mode is Landau damped, but the amplitude of the $l=3$ mode has increased, as predicted by Fig. 1. In the bottom picture the $l=2$ mode is quenched and the $l=3$ mode is starting to show the effects of Landau damping (displacement of the line CC' toward the origin in Fig. 1).

It is of interest to examine these results in the light of an observation by Lamb[8] that the possibility of multimode operation in the helium-neon laser depends on the fact that the constituent elements (excited atoms) are distributed in velocity, so that each mode can be driven by a different velocity class, with only a small amount of interaction (weak coupling). This is in contrast with single-stream or fluid-like strong-coupling systems (such as the van der Pol vacuum-tube oscillator) in which multimode oscillation is not allowed under the conditions described here (incommensurate frequencies and "soft" self-excitation[6]). The present plasma "microinstability" (with the artificial damping used in these experiments) is evidently similar to the laser case, with the different unstable modes being driven by different resonant-electron velocity classes; this feature is a specific consequence of the fact that the plasma electrons are distributed in velocity. It also appears that the collective effect of nonlinear mode interactions can have an important influence on individual mode amplitudes. Finally, this work points up the desirability of having available a theoretical calculation of the self-damping coefficients (a_{ii}) and the mode-coupling coefficients (a_{ij}) for the universal instability in the weakly nonlinear case.

A more detailed report of this work is in preparation.

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[11]A standard result due to Lyapunov and Poincaré states that stability can be examined without considering these quadratic terms.[6]
[12]Experimental work performed at Plasma Physics Laboratory, Princeton University.
[13]Taking both a_{ij} to be negative is sufficient to recover the experimental results. In principle the a_{ij} can also be of opposite sign, but this case does not appear to apply here.

INFRARED ABSORPTION STRUCTURE IN RARE-EARTH METALS: RELATIONSHIP TO SPIN ARRANGEMENT AND BAND STRUCTURE

B. R. Cooper and R. W. Redington

General Electric Research Laboratory, Schenectady, New York

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Recently Schüler[1] reported the appearance of structure at about 0.35 eV in the infrared reflectivity for a holmium film in the temperature range where holmium has a spiral spin arrangement with periodicity along the c axis. Following the ideas of Miwa,[2] Schüler suggested that the structure observed is due to the optical absorption corresponding to energy gaps in the conduction bands at the magnetic Brillouin-zone boundaries associated with the spiral periodicity. It is the purpose of the present note to point out (1) that there is a definitive experiment for distinguishing whether this structure, or indeed any part of the

infrared absorption for the heavy rare-earth metals, is associated with the spiral periodicity; (2) that, as reported below, this experiment shows that for dysprosium, which also has spiral spin structure, the infrared absorption is completely independent of any periodicity associated with the spiral; and (3) that in the light of recent band calculations[3,4] there is an alternative explanation for the occurrence of such low-frequency structure for the heavy rare-earth metals.

The definitive experiment for seeing whether the infrared structure is associated with the spiral periodicity rests on the fact that the application of a magnetic field in the plane of the spiral can be used to change the spin arrangement to one differing only slightly from ferromagnetic alignment along the field. This change occurs quite sharply[5] at a critical field H_C. In general H_C is not very large. If the infrared structure is due to the extra spiral periodicity, this structure can be extinguished by applying a field greater than H_C in the plane of the spiral. This effect will occur very sharply as a function of field. Therefore one need only examine the change in reflection or absorption at a given temperature in the spiral regime on applying a field $H > H_C$. Dysprosium is particularly suitable for such a study since H_C is less than 10 kG throughout most of the spiral temperature region[6] in Dy. We have therefore examined transmission through Dy films for a range of frequency between 0.25 and 0.85 eV.

The measurements were made in a cryostat with calcium fluoride windows and special pole pieces which allowed a field of 22 kG to be applied perpendicular to the light path and in the plane of the film. The films studied had transmissions in the region of interest of a few percent, and were a few thousand angstroms thick. They were prepared by evaporation on sapphire substrates in a separate vacuum system. For protection during the transfer to the cryostat the films were given a protective evaporated layer of silicon monoxide 2000 to 3000 Å thick. Electrical measurements on auxiliary samples indicated that such a coat was effective in preserving the metallic nature of the film. No change was observed in the film properties in several days in the cryostat. However, after one to two weeks' exposure in air the transmission increased several times. As a final check on the experimental technique, the difference in transmission between a thick film and a thin film was studied, and this showed the same absorption as indicated in Fig. 1. A triangular five-point smoothing procedure was used on the raw data.

The results obtained on cooling below the Néel temperature, $T_N = 179°K$, are shown in Fig. 1. The figure shows the transmission at a given temperature normalized to that at 182°K (just above T_N). There is a distinct dip in the transmission for the 87°K curve at about 0.44 eV and at a somewhat lower frequency for 123°K. [The lowest temperature shown, 87°K, is quite close to that (≈85°K) at which bulk dysprosium has a thermal transition to a ferromagnetic phase.] Such structure is clearly absent for the 235°K curve where Dy has no magnetic ordering. (Structure is also absent in the corresponding room-temperature curve which lies between 1.3 and 1.5 on the scale of Fig. 1 for photon energies between 0.3 and 0.8 eV.) It is difficult to clearly distinguish whether there is structure in the frequency range observed at 138°K. For the spiral mechanism suggested by Schüler the absorption structure should go to zero frequency at T_N, while for the mechanism based on the band structure of the con-

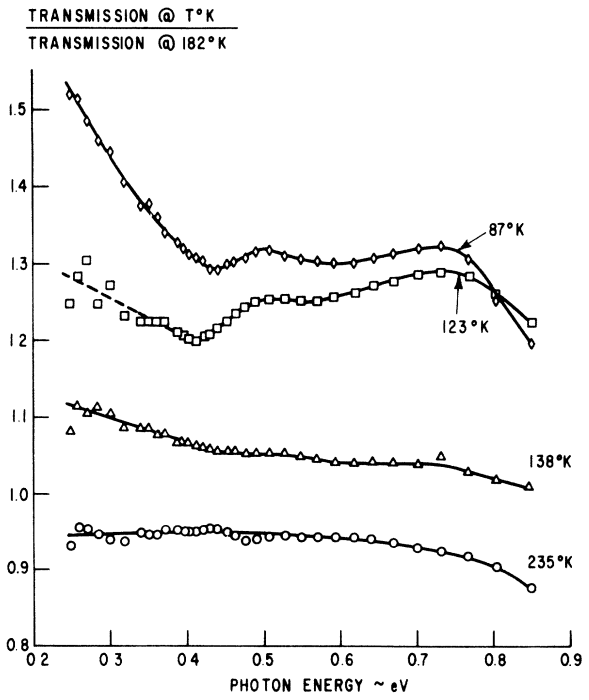

FIG. 1. Relative transmission for dysprosium films above and below the Néel temperature (179°K).

duction electrons discussed below it will probably shift to very low frequencies as temperature increases toward T_N. As the structure shifts to lower frequencies for temperatures increasing toward T_N, it is increasingly obscured by the free-carrier contribution to the absorption. This probably accounts for the difficulty in observing any well-defined structure at 138°K.

There is no measurable change in the curves shown in Fig. 1 when a field of 22 kG is applied in the plane of the film. Ideally, the experiment would be done with single crystals. However, the use of a polycrystalline film presents no great obstacle. For a polycrystalline sample, any given crystallite will have its c axis at some angle θ to H. The main effect of the applied field is to produce an effective field $H\sin\theta$ in the plane of the spiral which can distort the spiral.[7] Then crystallites with $\theta > \theta_c$ are essentially ferromagnetic, while those with $\theta < \theta_c$ are spirals. Here

$$\theta_c = \sin^{-1}(H_c/H). \tag{1}$$

Then the fraction of crystallites having a ferromagnetic arrangement for a given field is

$$f = \int_{\theta_c}^{\pi/2} \sin\theta d\theta \Big/ \int_0^{\pi/2} \sin\theta d\theta$$

$$= \left(\frac{H_c}{H}\right)\left[\left(\frac{H}{H_c}\right)^2 - 1\right]^{1/2}. \tag{2}$$

f increases rapidly for $H > H_c$. For example, $f = 0.87$ for $H = 2H_c$. For $T = 123$°K, for dysprosium[6] H_c is about 5 kG. Thus for an applied field of 22 kG, $H/H_c \gtrsim 4$, giving $f \gtrsim 0.97$. Thus in Dy, through most of the spiral region, one can obtain almost complete ferromagnetic alignment with a field such as the 22-kG one used in the present experiment. Then, just as for single crystals, one should be able to obtain almost complete extinction of any optical structure associated with the spiral periodocity by applying a field of this magnitude. The fact that there is no change in the infrared transmission structure upon application of the field clearly shows that the structure does not depend on the spiral periodicity and the resulting presence of magnetic Brillouin-zone boundaries.

There is an alternative explanation for the occurrence of such low-frequency structure for the heavy rare-earth metals. Recent work of Dimmock, Freeman, and Watson[3,4] indicates that the conduction-electron bands for the heavy rare-earth metals closely resemble those of transition metals rather than being free-electron-like. This leads to a large density of states at the Fermi level. The behavior for the rare-earth metals from Gd to Tm is expected to be similar in this regard. Since the situation for the heavy rare earths is closely analogous to that in nickel and iron with flat d-like bands lying close to the Fermi energy, it is quite reasonable to expect the same sort of low-frequency optical structure as occurs for nickel[8-10] and iron.[10] Indeed the frequency at which the structure occurs for Dy and Ho is quite similar to that for nickel and iron. The optical structure in nickel is attributed to transitions from a flat d band to the s band in the vicinity of the Fermi energy.[8,11-13] If a similar mechanism is the cause of the low-frequency optical structure in Dy and Ho, similar structure should occur for all the heavy rare earths including gadolinium, which is an ordinary ferromagnet. One would expect the structure to appear from very low frequencies on cooling below T_N as the d band for minority spin electrons in the ordered magnetic state is driven increasingly above the Fermi energy by the exchange with localized f electrons. The structure would shift to higher frequencies until the maximum d-band exchange splitting is obtained with thermal saturation of the magnetization. We also note that there is an indication of an additional dip in the relative transmission curves for 87 and 123°K at about 0.6 eV. As in the case of nickel,[12,13] additional structure at such frequencies would be quite reasonable for the model just discussed. To conclude we point out that the application of a magnetic field in the way done in the present experiment would be expected to have virtually no effect on the infrared structure for such a model. Thus our present observations on Dy are completely consistent with what one expects by considering the recent band calculations[3,4] for the heavy rare-earth metals and the known behavior[8-12] of Ni.

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HIGH-SENSITIVITY PIEZOREFLECTIVITY

W. E. Engeler

General Electric Research Laboratory, Schenectady, New York

and

H. Fritzsche

Institute for Study of Metals, University of Chicago, Chicago, Illinois

and

M. Garfinkel and J. J. Tiemann

General Electric Research Laboratory, Schenectady, New York

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This Letter describes an experimental technique that promises to be useful in the study of the energy band structure of solids. The technique is based on the fact that the optical reflectivity of a substance is in general changed by stress. The details of the changes depend upon the type of stress, the band structure of the material, and the wavelength of the light. These changes are most pronounced near the onset of direct interband transitions, and thus can yield information about these transitions. Since strain is a second-rank tensor, it is possible to obtain from single-crystal samples not only the position in energy of these transitions but also information about their location in the Brillouin zone. Further, the magnitude of the changes that are induced by a stress of known magnitude gives information about the deformation potentials of the bands involved. The potentialities of piezo-optics for yielding this type of detailed information have long been recognized.[1] In practice, however, the results have been severely limited by experimental difficulties. Previous static measurements of piezoreflectance[2] under uniaxial stress have required large ($\sim 10^{-2}$) sample strains. Since few materials may be strained this severely, this technique has been of limited usefulness. The experiment described below utilized oscillatory applied strain and synchronous detection at a frequency that is high in comparison with fluctuations in the photodetectors and other sources of instability. The resulting improvement in stability allows measurements to be made of relative reflectivity changes, $\Delta R/R$, as small as 5×10^{-6}. As a result, the method is useful for studying a wide range of solids. This sensitivity is similar to that reported by Seraphin and Hess[3] in measurements of electric-field-induced reflectivity changes in Ge. The ultimate sensitivity of this type of system is limited by photon shot noise. The sensitivity limit quoted above was in fact due to this source over much of the range covered. Even greater sensitivity could therefore be obtained if desired.

The experimental arrangement is shown schematically in Fig. 1. The sample to be studied is polished, and/or etched, and is mounted directly upon a piezoelectric transducer. An-