lished); S. M. Berman, Phys. Rev. 135, B1249 (1964).

[2]In general, lower case letters correspond to the laboratory system and capitals to the pion-nucleon center-of-mass system. For instance, γ_0 and γ_0' refer to the incident and scattered electron lab energies.

[3]In obtaining (2) and (4) we have followed Dalitz and Yennie's prescription (reference 1) for using charge-current conservation and gauge invariance to eliminate the scalar component of the photon. The extra factor k^2/K_0^2 required in the longitudinal interaction is absorbed into the density matrix. A discussion of the density matrix formalism is given by W. S. C. Williams, An Introduction to Elementary Particles (Academic Press, Inc., New York, 1961), p. 172.

[4]For instance, the square of the invariant four-momentum transfer k^2, the total energy in the pion-nucleon center-of-mass system W (also invariant), the polarization parameter ϵ, and the center-of-mass pion production angles Θ and φ relative to the momentum transfer direction [see Fig. 1(b)].

[5]Equation (4) is obtained by contracting the density matrix ρ_{jk} (2) with the nucleon transition current tensor $\frac{1}{2}\sum_{f,i}\langle f|J_j|i\rangle\langle f|J_k^*|i\rangle$, recognizing that the interference between states of opposite helicities necessarily produces a factor $\sin^2\Theta\cos 2\varphi$, and so on. A similar expression has been obtained by J. K. Randolph (unpublished).

[6]In our notation these are S-matrix elements; M_{1+} refers to $l=1$, $J=l+\frac{1}{2}$, and so on.

[7]Several pion photoproduction experiments using transversely polarized bremsstrahlung beams have been performed: R. C. Smith and R. F. Mozley, Phys. Rev. 130, 2429 (1963); G. Barbiellini, G. Bologna, J. DeWire, G. Diambrini, G. P. Murtas, and G. Sette, unpublished. Earlier experiments are listed in these papers.

[8]Arguing by analogy from the formalism developed for electron-nucleus inelastic scattering. For a recent review, see W. C. Barber, Ann. Rev. Nucl. Sci. 12, 1 (1962).

[9]A. A. Cone, K. W. Chen, J. R. Dunning, G. Hartwig, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters 14, 326 (1965). For earlier work see L. N. Hand, Phys. Rev. 129, 1834 (1963); G. G. Ohsen, Phys. Rev. 120, 584 (1960); W. K. H. Panofsky and E. A. Allton, Phys. Rev. 110, 1155 (1958).

[10]A single measurement of the reaction $e^- + p \to e^- + p + \pi^0$ has been reported by J. P. Perez y Jorba, P. Bounin, and J. Chollet, Phys. Letters 11, 350 (1964).

[11]R. R. Wilson, Nucl. Instr. 1, 101 (1957).

[12]K. Berkelman, J. M. Cassels, D. N. Olson, and R. R. Wilson, in Proceedings of the Tenth International Rochester Conference on High-Energy Physics, 1960, edited E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 757.

[13]We are indebted to D. R. Yennie for helpful suggestions on this point.

INTERFERENCE BETWEEN THE DECAYS $\rho^0 \to \pi^+ + \pi^-$ AND $\omega \to \pi^+ + \pi^-$ IN THE REACTION $\pi + N \to \pi + \pi + N$†

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A number of authors have considered the possibility that the ω meson might decay with a significant rate into two pions, $\omega \to \pi^+ + \pi^-$.[1] Since this decay mode violates the isotopic-spin and G-parity selection rules, it presumably results from electromagnetic effects, and would probably be of negligible importance were it not for the suppression of the strong decay $\omega \to \pi^+ + \pi^- + \pi^0$ by angular-momentum barrier effects. The experimental situation regarding the $\pi^+\pi^-$ decay mode of the ω is unclear. Walker et al.[2] obtained a branching ratio $\Gamma_{\omega \to 2\pi}/\Gamma_{\omega \to 3\pi}$ of $1.8_{-0.6}^{+1.2}\%$ from a study of the di-pion mass spectrum in the reactions $\pi^- + p \to \pi^+ + \pi^- + n$ and $\pi^- + p \to \pi^+ + \pi^- + \pi^- + p$. This analysis assumed that the $\pi^+\pi^-$ states produced by the decays of intermediate ω and ρ^0 mesons interfered almost completely, and in addition, used data selected for moderate momentum transfers to the nucleon. The interference assumption was later criticized by Islam and Piñon,[3] who pointed out that the ω and ρ^0 states were completely incoherent if the ω was assumed to be produced by the exchange of a ρ meson between the incident pion and nucleon, and the ρ^0, by the exchange of a pion.[4] A second analysis by Lütjens and Steinberger[5] which included data from different types of experiments failed to show any effect, and suggests strongly that the branching ratio $\Gamma_{\omega \to 2\pi}/\Gamma_{\omega \to 3\pi}$ is small (<0.8%). There nevertheless appears to be a persistent anomaly in the $\pi^+\pi^-$ mass spectrum near the ω mass in the reactions $\pi^- + p \to \pi^+ + \pi^- + n$ and $\pi^+ + n \to \pi^+ + \pi^- + p$.[6]

In the present note, we wish to point out that the argument of Islam and Piñon concerning the lack of coherence of $\pi^+\pi^-$ states produced through the ρ^0 and ω fails when account is taken of initial- and final-state interactions in the production process. The interference term obtained using the modified single-particle exchange model[7] for ρ^0 and ω production is in fact quite large. As a result, rather small branching ratios for the decay $\omega \to \pi^+ + \pi^-$ lead to surprisingly large changes in the di-pion mass spectrum in the ω region. These effects vary with the di-pion production angle in the manner noted empirically by Fickinger, Robinson, and Salant[6] and Walker et al.,[2] that is, are small for small production angles, but quite large for angles at which the ρ^0 and ω production cross sections are comparable. We would like, therefore, to suggest (i) that the apparent anomalies in the di-pion mass spectra in the reaction $\pi + N \to \pi^+ + \pi^- + N$ may be real,[8] and (ii) that if real, they provide a sensitive method for detecting a small $\omega \to \pi^+ + \pi^-$ decay mode.

The calculations which we shall report are based on the single-pion-exchange model for ρ production, and the single-ρ-meson-exchange model for ω production, with the effects of competition from other inelastic channels included in both cases. The model for ρ production has been discussed in detail elsewhere.[9-11] The predicted production and decay angular distributions for the ρ are remarkably accurate, even for rather large production angles, over the entire range of incident momenta studied, 2-8 BeV/c, and the model seems correspondingly reliable. The modified ρ-exchange model for ω production has been studied by Jackson et al.,[11] and independently by the present authors,[12] at 1.7 and 3.3 BeV/c. Although the predictions for the ω-production angular distribution are only moderately successful, the ω decay distribution is fitted quite well. In particular, the distribution of the normal to the ω decay plane with respect to the direction of the incoming beam has the form $a + b\cos^2\theta$, $b > 0$, observed experimentally, and is strikingly different from the $\sin^2\theta$ distribution expected for the unmodified ρ-exchange model. The success of the modified ρ-exchange model is nevertheless surprising. We note first that the model retains the characteristic energy dependence of vector exchange models. The ω-production cross section therefore increases with increasing energy in contradiction to the experimental results, and different effective $\pi\rho\omega$ coupling constants are therefore needed to fit data at different energies. (This difficulty may be eliminated by incorporating the expected Regge behavior of the exchanged ρ meson into the model,[13] but this is still uncertain.) A second difficulty arises from the nucleon-exchange contributions, the associated forces being too strong to treat in the distorted-wave Born approximation.[14] Despite these theoretical problems, the modified ρ-exchange model seems to be sufficiently successful as a phenomenological model to be used in the following calculations. In particular, the magnitude of the ρ^0-ω interference term in the cross section is not changed drastically by reasonable changes in the model. The absolute predictions of our calculations should nevertheless be regarded with some suspicion, and taken only as indicative of the magnitude of the effects to be expected in a more refined theory. A final point of uncertainty concerns the neglect in our calculations of the small real part known to be present in the πN scattering amplitude, and which may be presumed to be present in the ρN and ωN amplitudes as well. If included, this would lead to additional contributions to the results to be presented, but it appears unlikely that the conclusions would be affected in a major way.

The cross section for the reactions $\pi^- + p \to \pi^+ + \pi^- + n$ or $\pi^+ + n \to \pi^+ + \pi^- + p$ calculated on the foregoing basis is given by[15]

$$(d^2\sigma/d\Omega ds') = \pi^{-1} s'^{1/2}[(s'-m_\rho^2)^2 + m_\rho^2\Gamma_\rho^2]^{-1}\{\Gamma_\rho(d\sigma_\rho/d\Omega)$$

$$+\Gamma_{\omega\to 2\pi}(d\sigma_\omega/d\Omega)[(m_\omega^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2][(s'-m_\omega^2)^2+m_\omega^2\Gamma_\omega^2]^{-1}$$

$$+(\Gamma_\rho\Gamma_{\omega\to 2\pi})^{1/2}(d\sigma_{\rho\omega}/d\Omega)(s'-m_\omega^2)[(m_\omega^2-m_\rho^2)^2+m_\rho^2\Gamma_\rho^2]^{1/2}[(s'-m_\omega^2)^2+m_\omega^2\Gamma_\omega^2]^{-1}\},$$

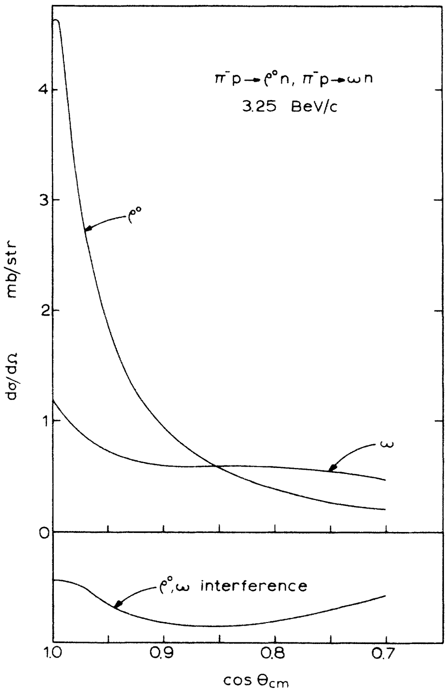

FIG. 1. Production angular distributions for the reactions $\pi^- + p \to \rho^0 + n$ and $\pi^- + p \to \omega + n$ at 3.25 BeV/c. The ρ^0-ω interference curve corresponds to 100% decay of the ω into the $\pi^+\pi^-$ state. The sign of the interference term depends on the unknown relative sign of the coupling constants $g_{\omega\rho\pi} g_{\rho NN}$ and $g_{\rho\pi\pi} \times g_{\pi NN}$.

where s' is the square of the di-pion mass. The differential ρ- and ω-production cross sections $d\sigma_\rho/d\Omega$ and $d\sigma_\omega/d\Omega$, and the interference term $d\sigma_{\rho\omega}/d\Omega$, are plotted in Fig. 1 for an incident pion momentum of 3.3 BeV/c.[16] The ρ and ω cross sections fit the experimental data at this momentum. The production and interference cross sections vary rather slowly with s', and may be treated as constants in the computation of the di-pion mass spectrum. It is evident from Fig. 1 that both the ratio of ω's to ρ's in production, and the ρ-ω interference term, are largest for $0.95 \gtrsim \cos\theta_{c.m.} \gtrsim 0.7$, and that the effects of the decay $\omega \to \pi^+ + \pi^-$ on the di-pion mass spectrum should consequently be largest in this region. (The models for ρ and ω production are not reliable, and background effects may be relatively large for larger angles.) The mass spectrum which results from a branching ratio $\Gamma_{\omega\to 2\pi}/\Gamma_\omega = 0.01$ is shown in Fig. 2(a) for the interval $0.95 \geq \cos\theta_{c.m.} \geq 0.75$. The sign of the interference term is not known, but positive interference $[d\sigma_{\rho\omega}/d\Omega > 0]$ is probably favored by the results of reference 6. The effect of the small admixture of $\omega \to \pi^+ + \pi^-$ decays is particularly prominent in this case, with deviations from the spectrum for a pure ρ ranging from -18 to $+30\%$ distributed over a fairly broad

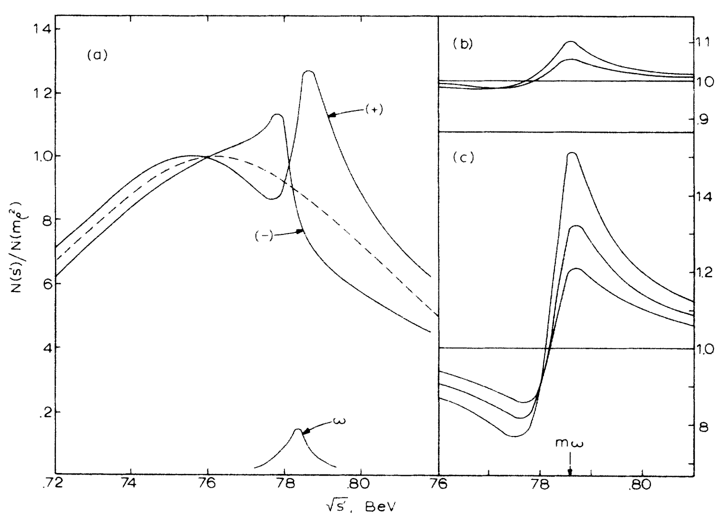

FIG. 2. Effects of a $\pi^+\pi^-$ decay mode of the ω on the $\pi^+\pi^-$ mass spectrum in the reaction $\pi^- + p \to \pi^+ + \pi^- + n$ at 3.25 BeV/c. (a) $\pi^+\pi^-$ mass spectra for a pure decay $\rho^0 \to \pi^+ + \pi^-$ (dashed curve) and for the predicted ρ-ω admixture with an ω branching ratio $\Gamma_{\omega\to 2\pi}/\Gamma_\omega = 0.01$ (solid curves). The results are given for both positive and negative ρ-ω interference, and have been averaged over the range of production angles $0.95 \geq \cos\theta_{c.m.} \geq 0.75$. All curves are normalized to unity at the mass of the ρ. The parameters used were $m_\rho = 760$ MeV, $\Gamma_\rho = 120$ MeV, $m_\omega = 783$ MeV, $\Gamma_\omega = 10$ MeV. (b) Ratio of $\pi^+\pi^-$ mass spectrum to that for pure ρ^0 decay for $\Gamma_{\omega\to 2\pi}/\Gamma_\omega = 0.02$ and 0.01, $1 \geq \cos\theta \geq 0.95$, positive interference. (c) Same as (b), for $\Gamma_{\omega\to 2\pi}/\Gamma_\omega = 0.02$, 0.01, and 0.005 and $0.95 \geq \cos\theta_{c.m.} \geq 0.75$.

mass range. The relatively narrow peak which would result from pure ω decays is shown for comparison.

It is evident from the curves in Fig. 1 that the relative contribution of the decay $\omega \rightarrow \pi^+ + \pi^-$ to the di-pion mass spectrum will be smaller for small production angles. In fact, the maximum deviation of the di-pion mass spectrum from the ρ spectrum is only 5.4% for di-pions produced in the angular interval with $1 \geq \cos\theta_{c.m.} \geq 0.95$. This diminution of the effects of the ω at small di-pion production angles is in accord with the behavior of the mass anomaly in the reactions $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ and $\pi^+ + n \rightarrow \pi^+ + \pi^- + p$ at 1.7 BeV/c noted by Fickinger, Robinson, and Salant.[6] The predicted deviations of the di-pion mass spectrum from a pure ρ spectrum are shown in Fig. 2(b) for the angular interval $1 \geq \cos\theta_{c.m.} \geq 0.95$ and branching ratios $\Gamma_{\omega \rightarrow 2\pi}/\Gamma_\omega = 0.02$ and 0.01, and in Fig. 2(c), for $0.95 \geq \cos\theta_{c.m.} \geq 0.75$ and $\Gamma_{\omega \rightarrow 2\pi}/\Gamma_\omega = 0.02$, 0.01, and 0.005. The average effects for the complete range of production angles are close to those of Fig. 2(b). For branching ratios smaller than 0.005, the pure ω contribution to the mass spectrum is nearly negligible. The ρ-ω interference effects, on the other hand, decrease only as $\Gamma_{\omega \rightarrow 2\pi}^{1/2}$, and thus remain significant for quite small branching ratios.

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†Work supported in part by the National Science Foundation.

[1]S. L. Glashow, Phys. Rev. Letters 7, 469 (1961); G. Feinberg, Phys. Rev. Letters 8, 151 (1962); J. D. Taylor, Phys. Rev. Letters 8, 219 (1962); J. Bernstein and G. Feinberg, Nuovo Cimento 25, 1343 (1962). Estimates for the branching ratio $\Gamma_{\omega \rightarrow 2\pi}/\Gamma_\omega$ range from the value 10^{-3} quoted in (G. Lütjens and J. Steinberger, Phys. Rev. Letters 12, 517 (1964), on the basis of a calculation by J. J. Sakurai, to $(5\text{-}7) \times 10^{-2}$ based on a broken SU(3) symmetry model [A. H. Zimmerman, Riazuddin, and S. Okubo, Nuovo Cimento 34, 1587 (1964)]. Although the last estimate is not consistent with experiment, it suggests that a branching ratio as large as 1-2% would not be unexpected.

[2]W. D. Walker, J. Boyd, A. R. Erwin, P. H. Slatterblom, M. A. Thompson, and E. West, Phys. Letters 8, 208 (1964).

[3]M. M. Islam and R. Piñon, Phys. Rev. Letters 12, 310 (1964).

[4]The exchange of a ρ (π) meson gives the longest-range interaction connecting the initial πN state with the final ωN (ρN) state, and hence is expected on geometrical grounds to yield the dominant part of the transition amplitude. In the crossed channel, the nucleon-antinucleon pair couples to the pion in a $^1S_0$ configuration, and to the ρ in a $^3S_1 + ^3D_1$ configuration. The lack of interference between the singlet and triplet spin states in the cross section leads to the result quoted in reference 3.

[5]Lütjens and Steinberger, reference 2.

[6]See, for example, W. J. Fickinger, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters 10, 457 (1963) [$\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 1.7 BeV/c]. Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento 25, 365 (1962) [$\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 1.6 BeV/c]; 35, 713 (1965) [$\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 2.75 BeV/c]. Walker et al., reference 2. The effect is also seen in the reaction $\pi^+ + d \rightarrow p + p + \pi^+ + \pi^-$ at 1.75 BeV/c (private communication from W. J. Fickinger). If a statistical fluctuation, the effect is perhaps remarkable (i) for its persistence, (ii) for the fact that it seems always to appear near the ω mass, not, for example, on the low-mass side of the ρ^0 peak in some experiments, and (iii) in that comparable effects apparently do not appear with the same persistence in the $\rho^\pm$ mass spectra as would be expected for a purely statistical effect.

[7]L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters 12, 399 (1964); 13, 45(E) (1964). K. Gottfried and J. D. Jackson, Nuovo Cimento 34, 735 (1964). The assumptions and theoretical background of the modified single-particle exchange model are reviewed in papers by L. Durand, III, and Y. T. Chiu, Proceedings of the International Conference on Particles and High Energy Physics, Boulder, Colorado, 1964 (University of Colorado Press, Boulder, Colorado, to be published), and Phys. Rev. (to be published).

[8]It is also tempting to ascribe the "double peaking" of the $\pi^+\pi^-$ mass spectrum observed in other types of experiments to ρ^0-ω interference [see for example, J. Button, G. R. Kalbfleisch, G. R. Lynch, B. C. Maglić, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. 126, 1858 (1962)]. We have not attempted to analyze these more complicated situations.

[9]Gottfried and Jackson, reference 7.

[10]L. Durand, III, and Y. T. Chiu, Phys. Rev. 137, B1530 (1965); Phys. Rev. Letters 14, 329, 680(E) (1965).

[11]J. D. Jackson, J. T. Donohue, K. Gottfried, R. Keyser, and B. E. Y. Svensson, Phys. Rev. (to be published).

[12]Because of somewhat different assumptions and calculational techniques, our results differ slightly from those of Jackson et al., reference 11. We have used a value of the tensor-to-vector ρNN coupling constants in the range suggested by the ρ contribution to pole-model fits to the nucleon-isovector electromagnetic form factors, $g^T/g^V = 2.6$ [see, for example, L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963)]. However, the theoretical uncertainties in the model becloud the significance of this ratio. Our results for the ω decay angular distribution are slightly better than those of reference 11, but the production angular distribution is not as good.

[13]Y. T. Chiu, thesis, Yale University, 1965 (unpub-

lished). L. Durand, III, and Y. T. Chiu, to be published.

[14]If this problem is ignored, and the nucleon-exchange contributions are included, the ω decay distribution is improved, but the production angular distribution acquires a large peak at backward angles which is apparently not observed.

[15]The decay $\omega \rightarrow \pi^+ + \pi^-$ was assumed to proceed through the conversion of the ω into a ρ through an effective interaction $g_{\rho\omega} F_{\mu\nu}{}^{\rho} F_{\mu\nu}{}^{\omega}$, followed by the decay of the ρ. In a more sophisticated theory than that used here, $g_{\rho\omega}$ could be complex (see, for example, the ρ-ω mixing theory of Bernstein and Feinberg, reference 1). This would result in a shift in the mass at which the ρ-ω interference term in the mass spectrum passes through zero, and some change in the magnitude of the effects. These points can only be examined on the basis of a detailed theory of the decay.

[16]We have omitted the small contributions of the ϵ^0 meson [$T^G = 0^+$, $J^P = 0^+$ $\pi\pi$ resonance near 700 MeV; see L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters 14, 329, 680(E) (1965), for a discussion of the indirect evidence for this particle. Some direct evidence has accumulated recently (private communications from J. Halpern and W. Selove)]. Although the ϵ^0 affects the di-pion decay distribution markedly, it has a negligible effect on the di-pion mass spectrum in the ω mass region. The angular distributions of the $\pi^+\pi^-$ pairs from the decay of the ρ^0 and ω are similar, and the overall distribution is affected rather little by the ρ^0-ω interference.

QUANTUM NUMBER ASSIGNMENTS FOR THE $N^*(1425)$ RESONANCE

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In November 1964 we reported evidence for the existence of a pion-nucleon resonance at a mass of ~1425 MeV/c^2.[1] We have now completed a full analysis of the events in which this resonance seems to occur and have evidence for the assignment of some pertinent quantum numbers.

Cocconi et al.[2] at CERN were the first to report any indications of a πN resonance in this energy region. Their momentum spectra, done for the reaction

$$p + p \rightarrow N + N^*, \quad N^* \searrow N + n\pi \qquad (1)$$

at nine values of incident proton momentum, showed a peak whose position varied with the incident momentum from 1.4 GeV/c at $P_0 = 3.60$ GeV/c to 1.51 GeV for $P_0 \geq 6.23$ GeV/c, as well as two other peaks, one at 1.24 GeV/c and one at 1.69 GeV/c, which remained constant with varying incident energies. They interpreted the peak at 1.24 GeV/c as the $N_{33}{}^*$ isobar and the one at 1.69 GeV/c as the appearance of the $N_{15}{}^*$ resonance. But they interpreted the shifting peak as an indication of a possible nucleon isobar at 1.4 GeV/c which gradually became submerged by the $N_{13}{}^*(1512)$ with increased incident energy and momentum.

At the same time, Bareyre et al.[3] at Saclay studied π-N total cross sections at 300 MeV $\leq T_{\pi\text{lab}} \leq$ 700 MeV and found an indication of a πN resonance in the $T = \frac{1}{2}$ state in the total cross-section curves at ~1400 MeV and in the invariant-mass spectrum of the final state

$$\pi^- + p \rightarrow \pi^+ + \pi^- + p + \pi^-. \qquad (2)$$

The Saclay group found their bump in the elastic cross section at $T_\pi = 430$ MeV, i.e., $M_{\pi N}$ ~1400 MeV; then, making the assumption that the residual cross section in that region of kinetic energy was dominated by a single partial wave, observed that the $T = \frac{1}{2}$ inelastic cross section reached its maximum value in that region in an angular momentum state $J = \frac{1}{2}$. In addition, they noted a shoulder in the mass spectrum (2) distinct from the bump at $M_{\pi N} = 1512$ MeV but of bare statistical significance.

Up to this time this was the only direct experimental evidence for a resonance in the $N\pi$ system at a mass approximating 1.4 GeV. But by far the most convincing work was the exhaustive phase-shift analysis of the pion-nucleon system between $T_\pi = 0$ and 700 MeV by Roper[4] at the Livermore Radiation Laboratory of the University of California at Berkeley. Roper and Wright reported[5] that the P_{11} phase shift was found to pass through 90° at a value of $T_\pi = 556$ MeV and of $M_{\pi N} = 1485$ MeV. It should be noted that this is distinct from the D_{13} phase