PION PRODUCTION BY POLARIZED VIRTUAL PHOTONS*

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In this paper we propose a new method for determining the spin and parity and the electromagnetic transformation form factors of nucleon isobars. We report on an experimental test of this technique using the well-known $N_{3/2}^*(1238)$ isobar.

Consider the pion electroproduction reaction $e+N\to e+N+\pi$, and assume that the exchange of a single photon dominates [see Fig. 1(a)]. To concentrate our attention on the more interesting strong-interaction physics, we express the electroproduction differential cross section in terms of known quantities derived from the electron-photon vertex and photon propagator, and the differential cross section for pion photoproduction by virtual photons[1]:

$$\frac{d^3\sigma}{dr_0'd\omega d\Omega}=\frac{\alpha}{2\pi^2}\frac{r_0'}{r_0}\frac{|\vec{k}|}{(-k^2)}\frac{d\sigma}{d\Omega}. \tag{1}$$

All quantities in (1) are defined in the laboratory frame except $d\Omega$, the differential pion solid angle in the pion-nucleon center-of-mass system; $d\omega$ is the laboratory electron solid-angle differential.[2] The exchanged photon is polarized transversely and longitudinally, the density matrix in the pion-nucleon center-of-mass frame[3] being given in terms of photon helicity states (+1, −1, and 0) by

$$\rho_{++}=\rho_{--}=\tfrac{1}{2}(1-\epsilon)^{-1},$$
$$\rho_{+-}=\rho_{-+}=-\tfrac{1}{2}\epsilon(1-\epsilon)^{-1},$$
$$\rho_{00}=(-k^2/K_0^2)\epsilon(1-\epsilon)^{-1},$$
$$\rho_{+0}=\rho_{0+}=-\rho_{-0}=-\rho_{0-}$$
$$=\tfrac{1}{2}[(-k^2)^{1/2}/K_0]\epsilon^{1/2}(1+\epsilon)^{1/2}(1-\epsilon)^{-1}, \tag{2}$$

where ϵ is the transverse linear polarization, given by

$$\epsilon=\frac{(-k^2/|\vec{k}|^2)\cot^2\frac{1}{2}\theta}{2+(-k^2/|\vec{k}|^2)\cot^2\frac{1}{2}\theta}. \tag{3}$$

The photoproduction cross section $d\sigma/d\Omega$, which is still a function of all five independent kinematic variables,[4] can be given more explicitly[5]:

$$d\sigma/d\Omega=(1-\epsilon)^{-1}[\sigma_U+\epsilon\sigma_P\sin^2\Theta\cos2\varphi$$
$$+(-k^2/K_0^2)\epsilon\sigma_L+[(-k^2)^{1/2}/K_0]\epsilon^{1/2}(1+\epsilon)^{1/2}$$
$$\times\sigma_I\sin\Theta\cos\varphi], \tag{4}$$

where σ_U, σ_P, σ_L, and σ_I are now functions only of k^2, W, and Θ. The first term σ_U is the differential photoproduction cross section for transverse unpolarized photons of four-momentum k^2, and is essentially the same as the ordinary photoproduction cross section except for form factor effects; the first two terms in the bracket correspond to virtual photoproduction by transversely polarized photons; σ_L is the cross section for longitudinal photons; and the final term corresponds to transverse-longitudinal interference.

At values of W close to the mass of a nucleon isobar, the pion photoproduction will be dominated by the three multipole amplitudes[6] with the J and parity of the isobar: magnetic

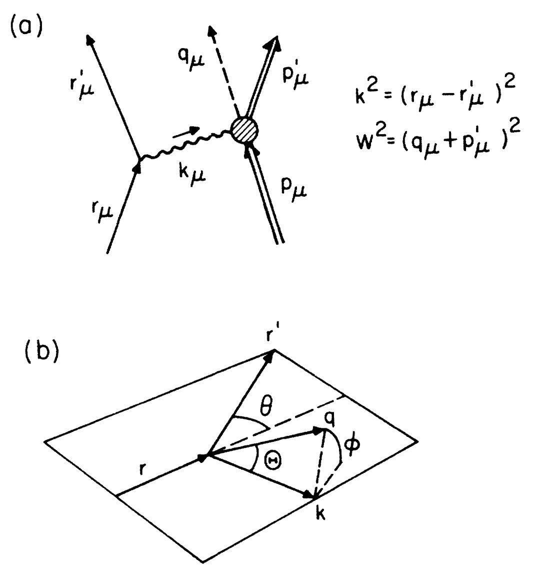

FIG. 1. (a) Diagram for the reaction $e+N\to e+N+\pi$ assuming single photon exchange. (b) A view of the momentum vectors in the laboratory frame illustrating the angle definitions used in this paper. Although the angle Θ between the pion momentum and the momentum transfer vector $\boldsymbol{k}$ is shown here in the lab frame, Θ is referred to the pion-nucleon center-of-mass frame throughout the text.

$M_{l,J}$, transverse electric $E_{l,J}$, and longitudinal electric $L_{l,J}$. The multipole order l is given by $l = J \pm \frac{1}{2}$ and parity $= (-1)^l$ for electric, $-(-1)^l$ for magnetic. For illustration we consider several examples.

1. $J = \frac{3}{2}+$isobar:

$$\sigma_U = (8|\vec{K}|^2)^{-1}[\tfrac{1}{2}(5-3\cos^2\Theta)|M_{1+}|^2 + \tfrac{3}{2}(1+\cos^2\Theta)|E_{2-}|^2 - \sqrt{3}(1-3\cos^2\Theta)\,\mathrm{Re}M_{1+}E_{2-}^*];$$

$$\sigma_P = (8|\vec{K}|^2)^{-1}[-\tfrac{3}{2}|M_{1+}|^2 + \tfrac{3}{2}|E_{2-}|^2 - \sqrt{3}\,\mathrm{Re}M_{1+}E_{2-}^*];$$

$$\sigma_L = (8|\vec{K}|^2)^{-1}(1+3\cos^2\Theta)|L_{2-}|^2;$$

$$\sigma_I = -(8|\vec{K}|^2)^{-1}\cos\Theta\,\mathrm{Re}(2M_{1+} + 6\sqrt{3}E_{2-})L_{2-}^*.$$

2. $J = \frac{3}{2}-$isobar:

$$\sigma_U = (8|\vec{K}|^2)^{-1}[\tfrac{1}{2}(5-3\cos^2\Theta)|E_{1+}|^2 + \tfrac{3}{2}(1+\cos^2\Theta)|M_{2-}|^2 + \sqrt{3}(1-3\cos^2\Theta)\,\mathrm{Re}E_{1+}M_{2-}^*];$$

$$\sigma_P = (8|\vec{K}|^2)^{-1}[\tfrac{3}{2}|E_{1+}|^2 - \tfrac{3}{2}|M_{2-}|^2 - \sqrt{3}\,\mathrm{Re}E_{1+}M_{2-}^*];$$

$$\sigma_L = (8|\vec{K}|^2)^{-1}(1+3\cos^2\Theta)|L_{1+}|^2;$$

$$\sigma_I = (8|\vec{K}|^2)^{-1}\cos\Theta\,\mathrm{Re}(-6E_{1+} + 2\sqrt{3}M_{2-})L_{1+}^*.$$

3. $J = \frac{3}{2}+$isobar dominated by magnetic dipole transition, plus nonresonant s-wave electric dipole background:

$$\sigma_U = (8|\vec{K}|^2)^{-1}[|E_{1-}|^2 + \tfrac{1}{2}(5-3\cos^2\Theta)|M_{1+}|^2 + 2\cos\Theta\,\mathrm{Re}E_{1-}M_{1+}^*];$$

$$\sigma_P = -(8|\vec{K}|^2)^{-1}\tfrac{3}{2}|M_{1+}|^2;$$

$$\sigma_L = (8|\vec{K}|^2)^{-1}|L_{1-}|^2;$$

$$\sigma_I = (8|\vec{K}|^2)^{-1}2\,\mathrm{Re}M_{1+}L_{1-}^*.$$

It is clear from these examples that the dependence on the electron scattering angle (contained in ϵ) and on the pion production angles Θ and φ can be used to distinguish the contributions of various multipoles, and thus determine the spin and parity of the isobar. In ordinary unpolarized photoproduction only the σ_U dependence on Θ is available,[7] which in the absence of other information is insufficient to determine the parity. In complicated cases where the isobar interferes with a sizable background from other resonances or nonresonant states, the additional information available from transversely and longitudinally polarized virtual photons will help to clarify the problem.

Each isobar matrix element has a Breit-Wigner W dependence in which the input channel width has the form of an invariant function of k^2 (a nucleon-to-isobar transition form factor) multiplied by a power of $|\vec{K}|$ which depends on the angular momentum.[8] The $|\vec{K}|$ dependence can then provide information on isobar spins, even in experiments in which only the electron is detected.[9]

As a prototype of a virtual photoproduction experiment, we have measured the azimuthal asymmetry of positive pion electroproduction,[10] $e^- + p \to e^- + n + \pi^+$, at $\Theta = 90°$ for three values of W in the vicinity of the $N_{3/2}^*(1238)$ isobar, and with $k^2 = -1.0$ and -3.5 F^{-2}. At the peak of the acceleration cycle the internal circulating beam of the Cornell 2.2-GeV synchrotron struck a liquid-hydrogen target mounted in the synchrotron vacuum chamber. The flux was monitored by observing the forward bremsstrahlung yield with a Quantameter.[11] Scattered electrons were magnetically analyzed and counted in three adjacent 4% momentum channels using a quadrupole spectrometer and counter system similar to that used in previous Cornell experiments.[12] The laboratory electron scattering angle θ was fixed at 15°, resulting in photon polarizations $\epsilon = 0.89$ and 0.94 at $k^2 = -1.0$ and -3.5 F^{-2}. The incident and scattered energies γ_0 and γ_0' were varied to give the desired k^2 and W:

$$k^2 = -2\gamma_0\gamma_0'(1-\cos\theta),$$
$$W^2 = M^2 + k^2 + 2M(\gamma_0 - \gamma_0'). \qquad (5)$$

Positive pions were counted in coincidence with electrons by a three-counter range telescope subtending 45 msr, periodically moved in azimuth around the momentum-transfer axis. Measurements were taken at $\varphi = 135°$, 180°, 225°, and 270° [see Fig. 1(b)] for each W and k^2. The elastic ep scattering cross section was monitored to check the flux normalization and the electron solid angle. Data were corrected for pion absorption and decay (<5%), for events originating in the 0.0005-in. polyimide film target walls (<20%), and for radiative effects[13] (<26%). Errors were not entirely statistical, but also resulted from normalization uncertainties and such effects as pulse pile-up, photomultiplier fatigue, and accidental coincidences due to the very high single rates in the pion counters.

At such low-momentum transfers the longitudinal contributions in Eq. (4) are small (our φ distributions are consistent with zero $\cos\varphi$ term), so that the measured cross sections can be fitted to

$$d\sigma/d\Omega = (1-\epsilon)^{-1}(\sigma_U + \epsilon\sigma_P \cos 2\varphi).$$

Figure 2 shows our results for σ_U and σ_P. The φ-dependent term demonstrates that the virtual photon is indeed strongly polarized and gives the sign and magnitude of σ_P expected for magnetic dipole excitation of a $J = \frac{3}{2}+$ resonance in the presence of predominantly s-wave background (see example 3). Over our limited k^2 range the variation in the multipole matrix elements is expected to be rather slight; the angular-momentum factor tends to compensate for the form-factor decrease. We expect then that the σ_U measured in electroproduction should be fairly close to the ordinary 90° photoproduction cross section. The agreement is satisfactory.

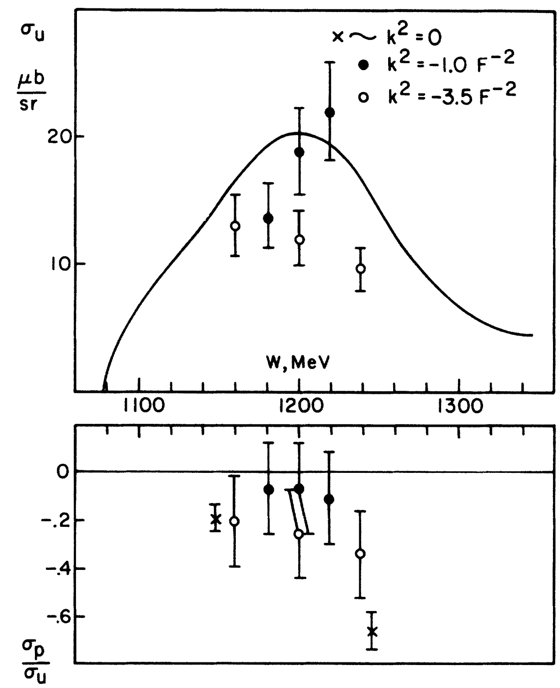

FIG. 2. Electroproduction results for σ_U and σ_P/σ_U [defined by (1) and (4)] at $\Theta = 90°$. For comparison we also show $d\sigma/d\Omega(90°)$ from unpolarized bremsstrahlung experiments (solid curve) and Smith and Mozley's (reference 7) results for σ_P/σ_U using transversely polarized bremsstrahlung (crosses).

In conclusion, besides demonstrating the feasibility of our proposal, the experiment confirms the following known properties of the $N_{3/2}^*(1238)$ isobar: (a) The parity is even; (b) it is excited primarily by magnetic dipole absorption, rather than electric quadrupole. A corresponding analysis in the region of the second or third nucleon isobar would certainly be interesting. Higher values of momentum transfer are also worth investigating to measure the nucleon-to-isobar transition form factors. However, such experiments require higher electron energies than were available at the Cornell synchrotron at the time of this experiment.

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[1]R. H. Dalitz and D. R. Yennie, Phys. Rev. 105, 1598 (1957); M. Gourdin, Nuovo Cimento 21, 1094 (1961); L. N. Hand, thesis, Stanford University, 1961 (unpub-

lished); S. M. Berman, Phys. Rev. 135, B1249 (1964).

[2]In general, lower case letters correspond to the laboratory system and capitals to the pion-nucleon center-of-mass system. For instance, γ_0 and γ_0' refer to the incident and scattered electron lab energies.

[3]In obtaining (2) and (4) we have followed Dalitz and Yennie's prescription (reference 1) for using charge-current conservation and gauge invariance to eliminate the scalar component of the photon. The extra factor k^2/K_0^2 required in the longitudinal interaction is absorbed into the density matrix. A discussion of the density matrix formalism is given by W. S. C. Williams, An Introduction to Elementary Particles (Academic Press, Inc., New York, 1961), p. 172.

[4]For instance, the square of the invariant four-momentum transfer k^2, the total energy in the pion-nucleon center-of-mass system W (also invariant), the polarization parameter ϵ, and the center-of-mass pion production angles Θ and φ relative to the momentum transfer direction [see Fig. 1(b)].

[5]Equation (4) is obtained by contracting the density matrix ρ_{jk} (2) with the nucleon transition current tensor $\frac{1}{2}\sum_{f,i}\langle f|J_j|i\rangle\langle f|J_k^*|i\rangle$, recognizing that the interference between states of opposite helicities necessarily produces a factor $\sin^2\Theta\cos 2\varphi$, and so on. A similar expression has been obtained by J. K. Randolph (unpublished).

[6]In our notation these are S-matrix elements; M_{1+} refers to $l=1$, $J=l+\frac{1}{2}$, and so on.

[7]Several pion photoproduction experiments using transversely polarized bremsstrahlung beams have been performed: R. C. Smith and R. F. Mozley, Phys. Rev. 130, 2429 (1963); G. Barbiellini, G. Bologna, J. DeWire, G. Diambrini, G. P. Murtas, and G. Sette, unpublished. Earlier experiments are listed in these papers.

[8]Arguing by analogy from the formalism developed for electron-nucleus inelastic scattering. For a recent review, see W. C. Barber, Ann. Rev. Nucl. Sci. 12, 1 (1962).

[9]A. A. Cone, K. W. Chen, J. R. Dunning, G. Hartwig, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters 14, 326 (1965). For earlier work see L. N. Hand, Phys. Rev. 129, 1834 (1963); G. G. Ohsen, Phys. Rev. 120, 584 (1960); W. K. H. Panofsky and E. A. Allton, Phys. Rev. 110, 1155 (1958).

[10]A single measurement of the reaction $e^-+p\to e^-+p+\pi^0$ has been reported by J. P. Perez y Jorba, P. Bounin, and J. Chollet, Phys. Letters 11, 350 (1964).

[11]R. R. Wilson, Nucl. Instr. 1, 101 (1957).

[12]K. Berkelman, J. M. Cassels, D. N. Olson, and R. R. Wilson, in Proceedings of the Tenth International Rochester Conference on High-Energy Physics, 1960, edited E. C. G. Sudarshan, J. H. Tinlot, and A. C. Melissinos (Interscience Publishers, Inc., New York, 1960), p. 757.

[13]We are indebted to D. R. Yennie for helpful suggestions on this point.

INTERFERENCE BETWEEN THE DECAYS $\rho^0\to\pi^++\pi^-$ AND $\omega\to\pi^++\pi^-$ IN THE REACTION $\pi+N\to\pi+\pi+N$†

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A number of authors have considered the possibility that the ω meson might decay with a significant rate into two pions, $\omega\to\pi^++\pi^-$.[1] Since this decay mode violates the isotopic-spin and G-parity selection rules, it presumably results from electromagnetic effects, and would probably be of negligible importance were it not for the suppression of the strong decay $\omega\to\pi^++\pi^-+\pi^0$ by angular-momentum barrier effects. The experimental situation regarding the $\pi^+\pi^-$ decay mode of the ω is unclear. Walker et al.[2] obtained a branching ratio $\Gamma_{\omega\to2\pi}/\Gamma_{\omega\to3\pi}$ of $1.8_{-0.6}^{+1.2}\%$ from a study of the di-pion mass spectrum in the reactions $\pi^-+p\to\pi^++\pi^-+n$ and $\pi^-+p\to\pi^++\pi^-+\pi^-+p$. This analysis assumed that the $\pi^+\pi^-$ states produced by the decays of intermediate ω and ρ^0 mesons interfered almost completely, and in addition, used data selected for moderate momentum transfers to the nucleon. The interference assumption was later criticized by Islam and Piñon,[3] who pointed out that the ω and ρ^0 states were completely incoherent if the ω was assumed to be produced by the exchange of a ρ meson between the incident pion and nucleon, and the ρ^0, by the exchange of a pion.[4] A second analysis by Lütjens and Steinberger[5] which included data from different types of experiments failed to show any effect, and suggests strongly that the branching ratio $\Gamma_{\omega\to2\pi}/\Gamma_{\omega\to3\pi}$ is small (<0.8%). There nevertheless appears to be a persistent anomaly in the $\pi^+\pi^-$ mass spectrum near the ω mass in the reactions $\pi^-+p\to\pi^++\pi^-+n$ and $\pi^++n\to\pi^++\pi^-+p$.[6]

In the present note, we wish to point out that the argument of Islam and Piñon concerning