mal compressibility of $(4.7 \pm 0.3) \times 10^{-3}$ atm^{-1} given by Kidder,[12] a value of 3.3×10^{-3} atm^{-1} given by Grilly and Mills,[10] and values of (1.8 and 3.1)$\times 10^{-3}$ atm^{-1} given by Ahlers.[4] Since the thermal expansion coefficient is on the order of $10^{-3}\,{}^\circ\mathrm{K}^{-1}$,[4] the isothermal and adiabatic compressibilities should differ only very slightly.

In this work we have directly observed shear waves in solid He^4 and have obtained preliminary velocity measurements. A detailed study of the propagation of sound in solid helium will probably require a simultaneous determination of crystal orientation, perhaps using x-ray techniques or multiple transducers.

We take this opportunity to thank Dr. J. H. Vignos for suggesting this measurement and for the use of his thesis. We should also like to thank Dr. R. C. Hanson, Dr. E. H. Bogardus, Professor G. V. Chester, Professor J. A. Krumhansl, and Professor H. S. Sack for interesting and informative discussions.

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†Research supported by the Advanced Research Projects Agency and the National Science Foundation.

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MEASUREMENT OF THRUST IN A LINEAR HALL ACCELERATOR

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(Received 29 April 1965)

This Letter describes the measurement of the axial mechanical force or thrust acting on the magnetic field coil in a linear plasma Hall accelerator. The magnetic field components around a closed path linking the field coil are found by magnetic probes, and the force on the coil is evaluated by integrating the associated magnetic stress tensor. In the pulsed high-current accelerator used in this experiment, a typical value of this force is 13.9 Kg-wt ±7%.

The principle of the linear Hall accelerator can be described as follows: We consider any cylindrical conductor which possesses a Hall coefficient, and apply an axial electric field to it so as to cause current to flow in this direction. Around the conductor and coaxial with it is placed an independent current-carrying coil which produces radial and axial magnetic field components in the cylindrical conductor. The radial magnetic field interacts with the axial current in the conductor to produce an azimuthal Hall current i_θ. In turn, this current interacts with the radial magnetic field B_r to produce a magnetomotive force, $i_\theta B_r$, in the axial direction. Since the radial magnetic field and, hence, the azimuthal Hall current change sign through the plane of the coil, their product does not. Thus, despite the apparent symmetry about the plane of the coil, there is a net accelerating force in the direction of the axial current.

So far this acceleration has been used only in connection with plasma because of its high Hall coefficient. The linear or pure mode of Hall acceleration has been studied at Imperial College since 1961 in connection with a modified linear pinch (axial acceleration)[1,2] and with theta pinch rotation (azimuthal acceleration).[3] At present a toroidal accelerator is being studied. A coaxial or mixed mode of Hall acceleration, in which the azimuthal Hall currents result from the interaction of a radial current and an axial magnetic field, was proposed by Hess[4] in 1960; later, Hess independently considered also the linear mode.[5,6] Further work has been carried out by Cann and Marlotte.[7]

Next we consider where and how the necessary equal and opposite mechanical reaction is transferred. The induced azimuthal Hall current in the plasma is parallel to the external coil current on the anode side of the coil and antiparallel on the cathode side. The resulting magnetic field is asymmetric about the plane of the coil. This distortion of the magnetic field can be envisaged as the stretching of the lines of force by the current-carrying electrons flowing axially from the cathode to the anode. It is through the magnetic field distortion that the mechanical reaction is transferred to the external coil. This force may be calculated in terms of the magnetic stress tensor as follows.

The full expression for the instantaneous electromagnetic force acting on material in a volume V bounded by a surface S is, in mks units,[8]

$$\vec{F} = -\frac{\partial}{\partial t}\int_V \vec{D}\times\vec{B}\,dV + \oint_S(\vec{E}\vec{D}+\vec{H}\vec{B}-u\,\mathsf{I})\cdot\vec{n}\,dS, \qquad (1)$$

where $u=\frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})$, and I is the unit dyadic. For the experiment under consideration, shown in Fig. 1, we construct a volume bounded by two concentric cylindrical surfaces of equal length and two plane rings. This volume encloses the field coil, but excludes the cylindrical plasma conductor and discharge vessel. Assuming azimuthal symmetry, and neglecting the small contribution to the force by the electric field, we can reduce Eq. (1) to a line integral *abcda* around a cross section of the volume shown in Fig. 1. Then Eq. (1) becomes

$$F_z = -2\pi r_1\int_a^b H_zB_r\,dz + \pi\int_b^c(H_zB_z-H_rB_r-H_\theta B_\theta)r\,dr$$
$$-2\pi r_2\int_c^d H_zB_r\,dz + \pi\int_d^a(H_zB_z-H_rB_r-H_\theta B_\theta)r\,dr,$$

where r_1 and r_2 are, respectively, the radii of the inner and outer cylindrical surfaces of the volume.

The experimental apparatus consists of a cylindrical Pyrex discharge vessel 100 cm long and 15 cm in diameter, containing argon at 350 mm Hg pressure. The axial discharge current reaches a peak of 70 kA in 7.5 μsec, and the magnetic field produced at the center of the coil is 1.8 kG in vacuo. A range of radial magnetic fields and axial currents has been used, and the stress tensor and thrust calculated as a function of time for each condition. In this short communication we confine ourselves to one condition, namely, at $t=10$

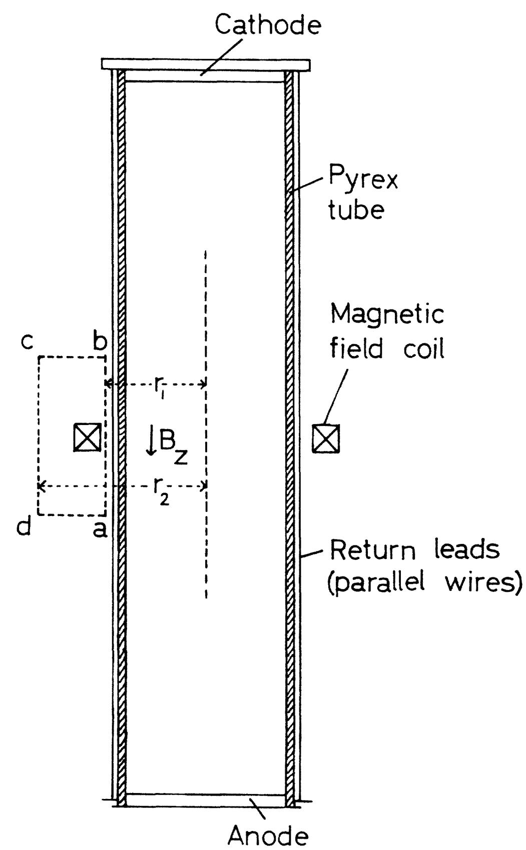

Fig. 1. Schematic diagram of apparatus indicating path of integration.

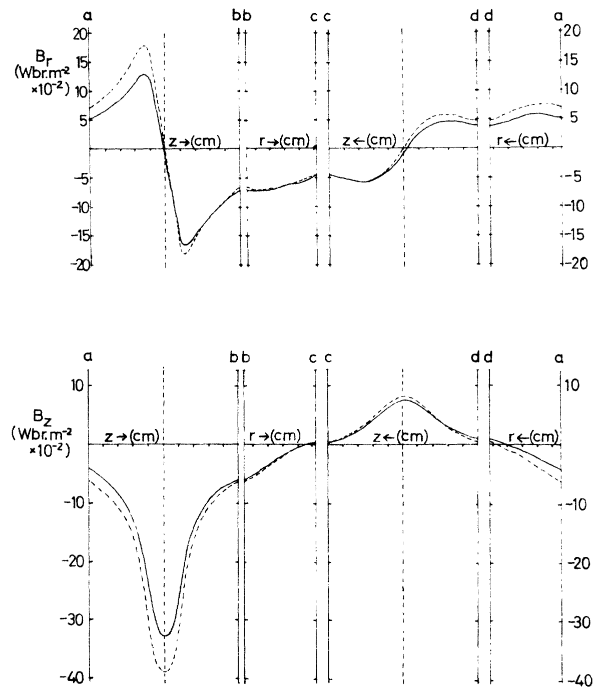

Fig. 2. (a) Radial induction along *abcda*. (b) Axial induction along *abcda*. Dashed line: Vacuum induction; solid line: induction at $t = 10$ μsec.

μsec and $I = 62$ kA, to demonstrate the method and the typical magnitude and direction of the axial force on the coil. The magnetic probes are small search coils 3 mm in diameter and 2.6 mm long. The length of the paths *ab* and *bc* in Fig. 1 are 10 cm and 4.7 cm, respectively. The radial, azimuthal, and axial components of the magnetic field are measured so as to yield a value of the thrust accurate to 7%.

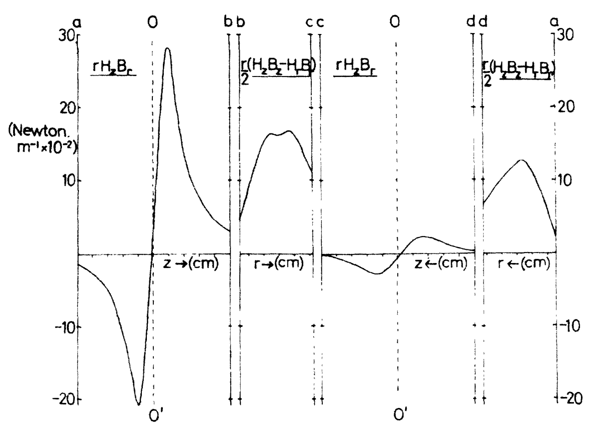

Fig. 3. Stress tensor components multiplied by r.

In Fig. 2 we plot the radial and axial magnetic field components along the closed path *abcda* at the time $t = 10$ μsec. For comparison, we include the vacuum magnetic field due to the coil alone. We note that there is a strong perturbation to the vacuum field and it is asymmetric about the plane of the coil OO'. This leads to a net axial force as can be seen on evaluation of the integrals in Eq. (2). It is found that the main contribution to the integral arises in the section *ab* along the inside of the magnetic field coil. The integrands rH_zB_r and $\frac{1}{2}(rH_zB_z - rH_rB_r)$ are plotted for the paths *ab*, *cd*, and *bc*, *da*, respectively, in Fig. 3. The term $rH_\theta B_\theta$ is, as expected, identical along *bc* and *da*. It therefore cancels out on integration and is neglected. For the particular experimental conditions the integrated axial force on the coil is 136 N ±7% (13.9 kg-wt). This mechanical reaction is correctly in the opposite direction to the discharge current and to the plasma acceleration detected by Measures,[2,9] using piezoelectric pressure probes.

We wish to acknowledge the Department of Scientific and Industrial Research for the provision of a maintenance grant to one of us (R.J.E.) and Dr. Latham, Dr. Folkierski, and Dr. Thonemann for useful help and discussions.

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